

1. solve: $2 \sin^2(x) - \sin(x) - 1 = 0$ in $[0, 2\pi)$

2. rewrite: $2(\sin(x))^2 - \sin(x) - 1 = 0$

3. define: $u = \sin(x)$

4. replace $\sin(x)$ with u : $2u^2 - 1u - 1 = 0$

5. factor :

a. we need two numbers that multiply to -2 : $-2(1) = -2$

b. the same two numbers must add to -1 : $-2+1 = -1$

4. rewrite middle term: $2u^2 - 2u + 1u - 1 = 0$

5. Factor by grouping: $2u(u-1) + 1(u-1) = 0$

6. $(2u+1)(u-1) = 0$

7. Replace u back with $\sin(x)$: $(2\sin(x)+1)(\sin(x)-1) = 0$

8. Set each factor equal to 0: $2\sin(x)+1=0$ or $\sin(x)-1=0$

9. Solve each equation for $\sin(x)$: $\sin(x) = -\frac{1}{2}$ or $\sin(x) = 1$ using the unit circle

10. So the solutions are $x = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{\pi}{2}$

