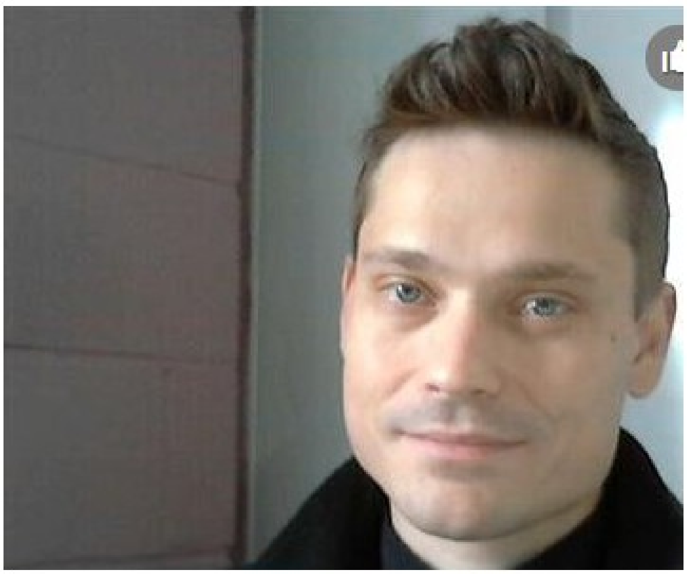
Dear Reader,
This booklet shows you how to perform each of the following:
1) How to construct a picture of the complex plane
2) How to draw a complex number
3) How to draw add complex numbers
4) How to show addition of complex numbers visually
5) How to stretch complex numbers
6) How to rotate complex numbers
7) Besides getting this booklet, you get a PDF version so you can download it and practice.
8) You also get access to a library of more than 420 HD math videos on a great variety of topics.

If you're somebody who likes to see what mathematics means in pictures, then this booklet is for you. In any case, my experience as a teacher suggests you should always draw pictures, for everything if you wish your comprehension to skyrocket. My experience is that virtually everything can be drawn, regardless of how abstract it might seem. If you wish to truly understand, it's best to slooooooooooow down, and pay extremtely close attention to each step. If you do this, you'll begin to see how remarkable, and yet how truly simple, mathematics is.
Your fellow explorer,
Tom

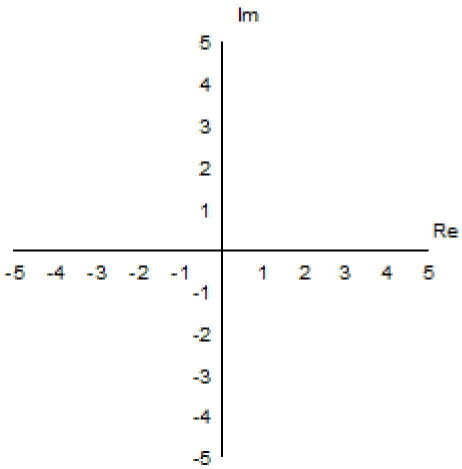


1) How to draw complex numbers

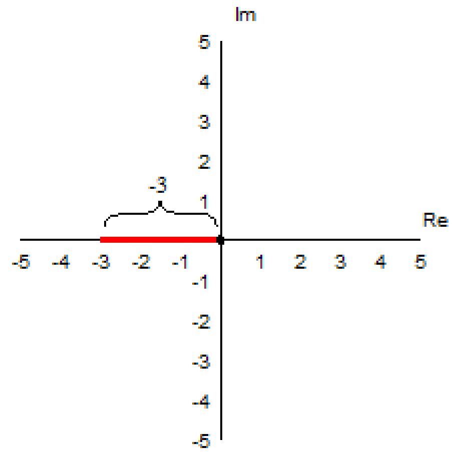
Draw $z_1 = -3+5i$

You can visualize this process as shown in the sequence of pictures below.

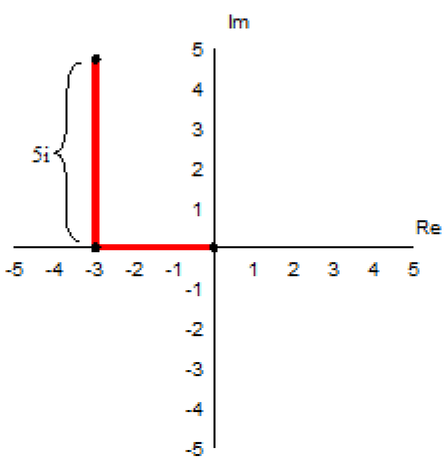
1) Construct a coordinate plane. In this plane the vertical axis is lableld "Im" for imaginary, and the horizontal axis is labeled "Re" for Real.



2) Draw a line from 0 to -3 along the x axis. This is the "real" part of the number.

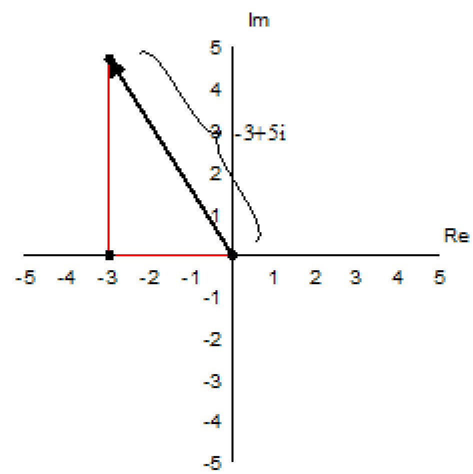


3) Turn and draw a line at a right angle relative to the red line. This is the 5i or imaginary part of the number.



4) Draw $z_1 = -3+5i$

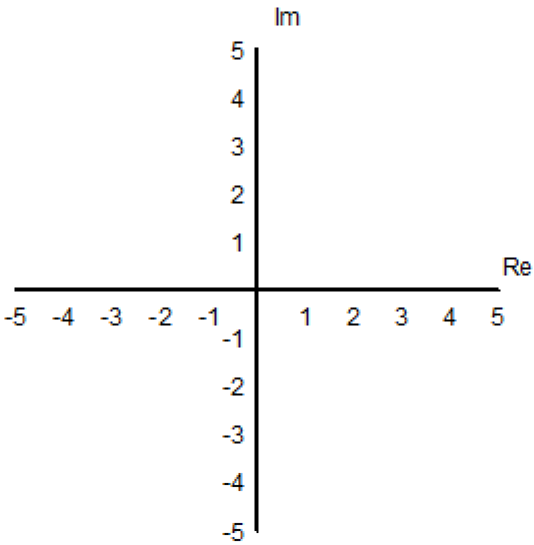
This is the arrow shown.



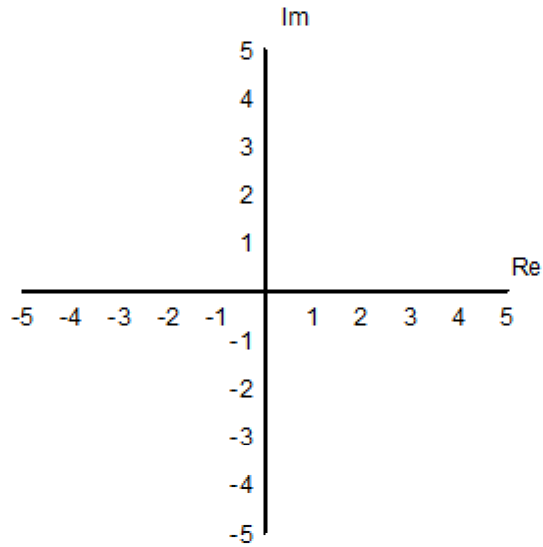
Use this template to practice drawing complex numbers. You can download a PDF version by following the link given at the end of the booklet. Print several copies, and put one number on each

Try these: $z_1=2-i$, $z_1=-2-i$, $z_1=1+i$, $z_1=-\pi-i$, $z_1=-i$, $z_1=2$, $z_1=2+i$

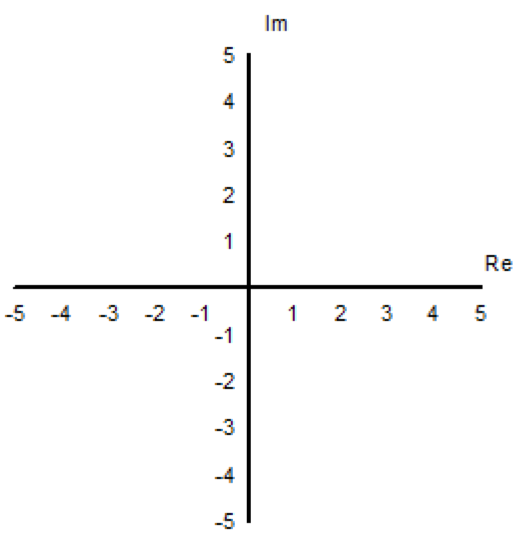
1) Construct a coordinate plane. In this plane the vertical axis is lableld _____ for imaginary, and the horizontal axis is labeled ___ for real.



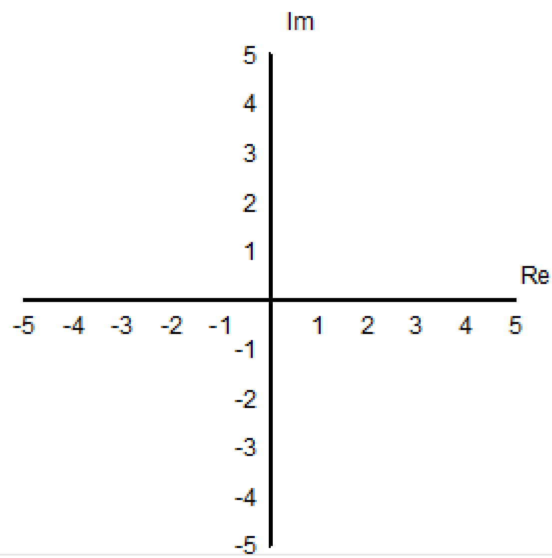
2) Draw a line from 0 to ___ along the x axis. This is the "real" part of the number.



3) Turn and draw a line at a right angle relative to the first line. This is the ___ or imaginary part of the number.



4) Draw z_1=____________ This is the arrow shown.



2) How to add complex numbers symbolically and visually

$z_1=1+4i$

$z_2=2-3i$

$z_3=(1+4i)+(2-3i)$

$=1+4i+2-3i$	Drop the parenthesis
$=1+2+4i-3i$	Regroup
$=3+1i$	Combine like terms
$=3+i$	Rewrite 1i as just i

General Principle:

Given $z_1=a+bi$

$z_2=c+di$

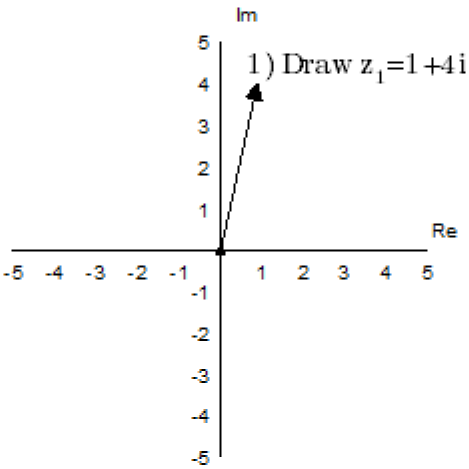
Then the sum is $z_3=z_1+z_2=(a+bi)+(c+di)$

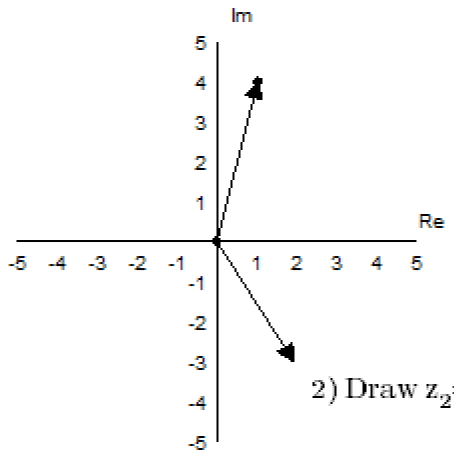
$=a+bi+c+di$

$=a+c+bi+di$

$=(a+c)+(b+d)i$

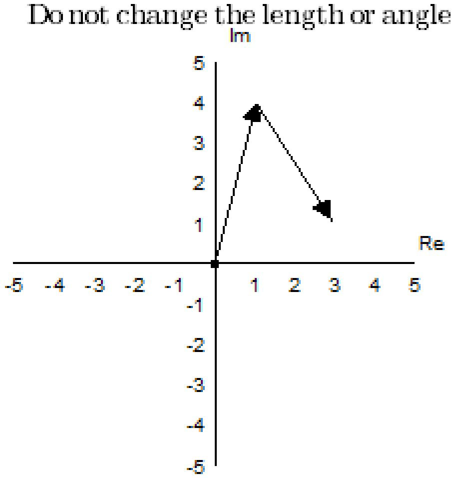
You can visualize this process as shown in the sequence of pictures below.

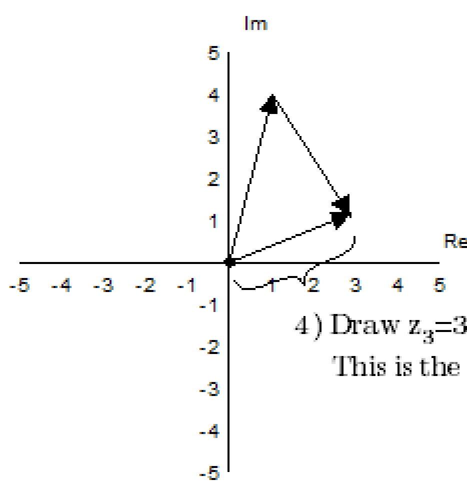




2) Draw $z_2=2-3i$

3) Move z_2 and place it as shown.
Do not change the length or angle.





4) Draw $z_3=3+i$
This is the sum.

Use this template to practice adding complex numbers.

z_1 =__________

z_2 =__________

z_3 =_______________ Write the sum of the two numbers

=_______________ Drop the parenthesis

=_______________ Regroup

=_______________ Combine like terms

=_______________ Rewrite 1i as just i

General Principle:

Given $z_1=a+bi$

$z_2=c+di$

Then the sum is $z_3=z_1+z_2=(a+bi)+(c+di)$

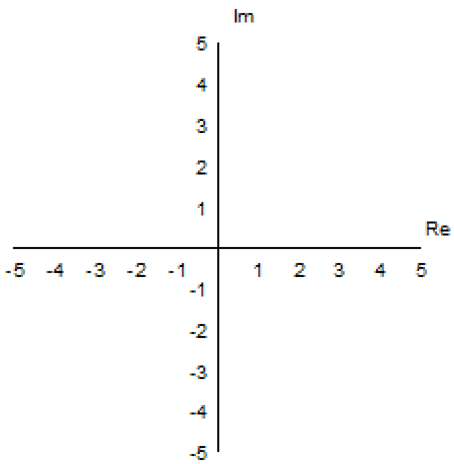
$=a+bi+c+di$

$=a+c+bi+di$

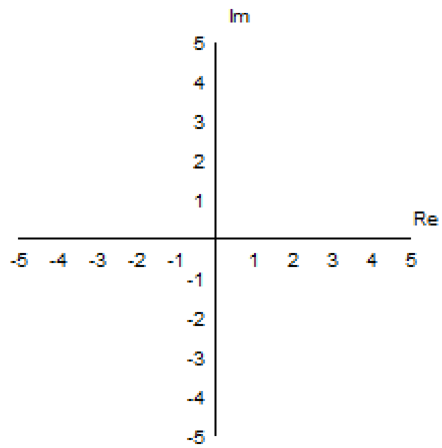
$=(a+c)+(b+d)i$

You can visualize this process as shown in the sequence of pictures below.

1) Draw z_1=_______



2) Draw z_2=_________



Add each pair:

1) 2i and −3i

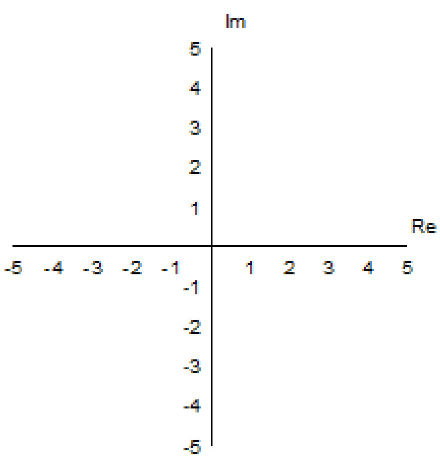
2) 1−2i and 2+3i

3) −2 and 3+4i

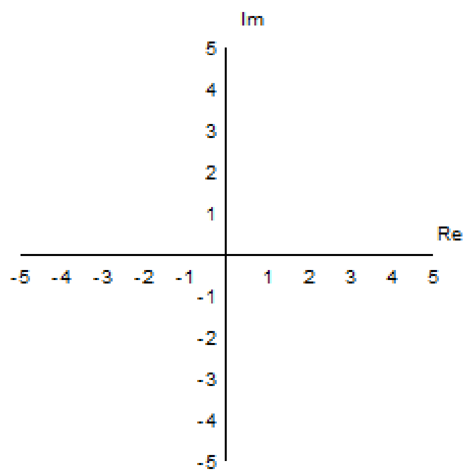
4) 1−i and 1+i

5) −2−i and −1−2i

3) Move z_2 and place it as shown.
Do not change the length or angle.



4) Draw z_3=___________
This is the sum.

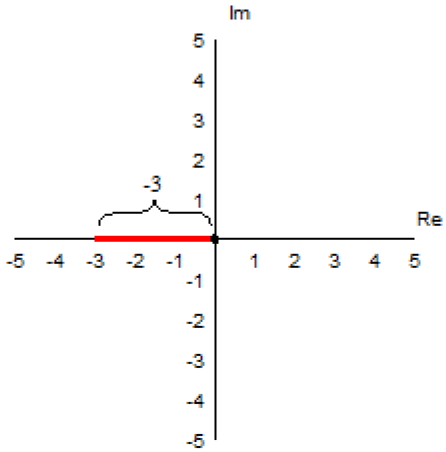


3) How to negate complex numbers

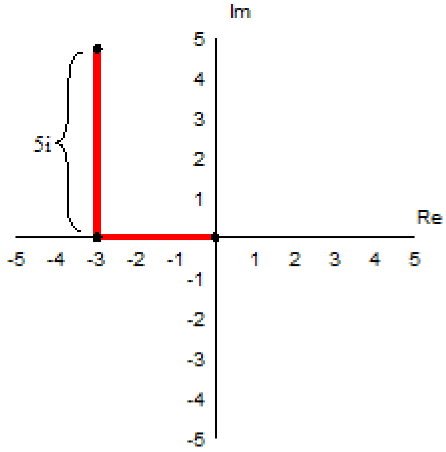
1) Draw $z_1=-1(-3+5i)$

You can visualize this process as shown in the sequence of pictures below.

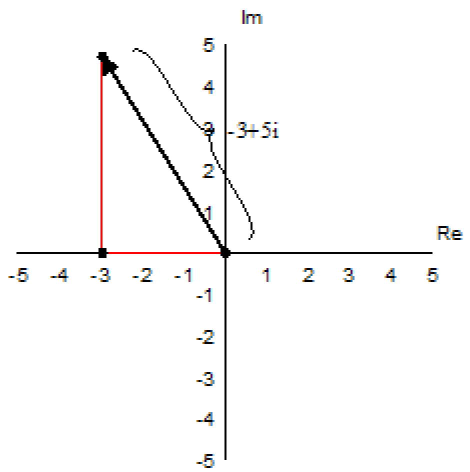
1) Draw a line from 0 to -3 along the x axis.
This is the "real" part of the number.



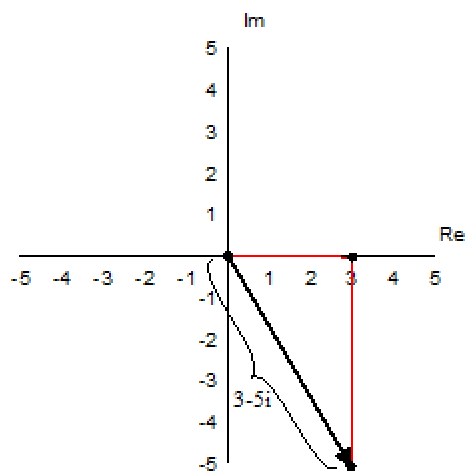
2) Turn and draw a line at a right angle relative to the red line. This is the 5i or imaginary part of the number.



3) Draw $z_1=-3+5i$
This is the arrow shown.



4) Distribute the -1. This amounts to reversing the direction of the number because the sign of the real and imaginary parts is changed.
Draw $z_1=-1(-3+5i)=-1(-1)-1(5i)=3-5i$
This is the arrow shown below.

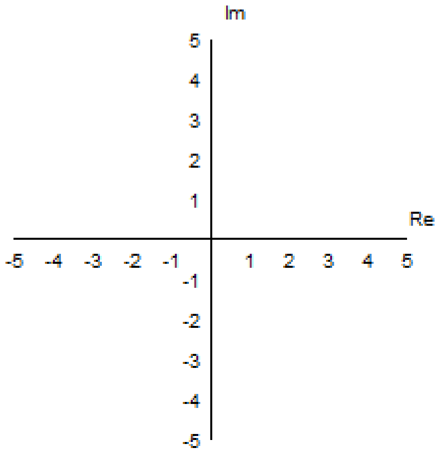


Draw each of $z_1 = -1(2+5i)$, $z_1 = -1(3+2i)$, $z_1 = -1(-1-i)$, $z_1 = -1(1+i)$,

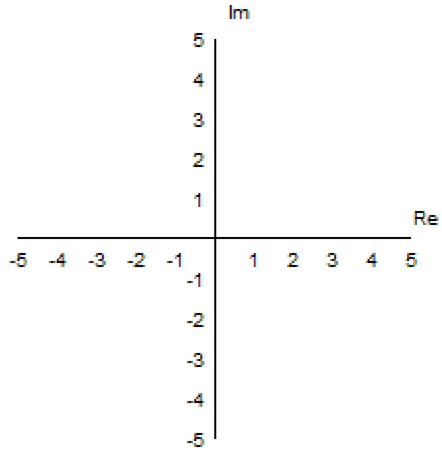
You can visualize this process as shown in the sequence of pictures below.

Print multiple copies of the PDF and draw the numbers one at a time.

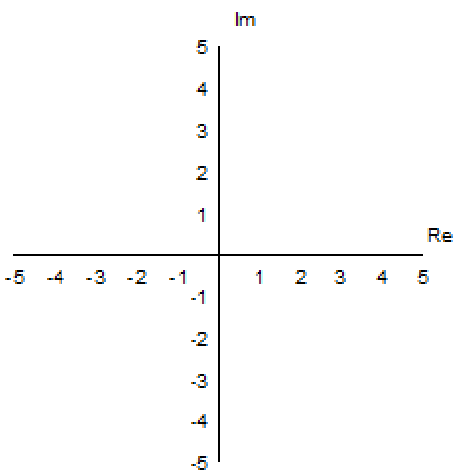
1) Draw a line from 0 to ___ along the x axis.
This is the "real" part of the number.



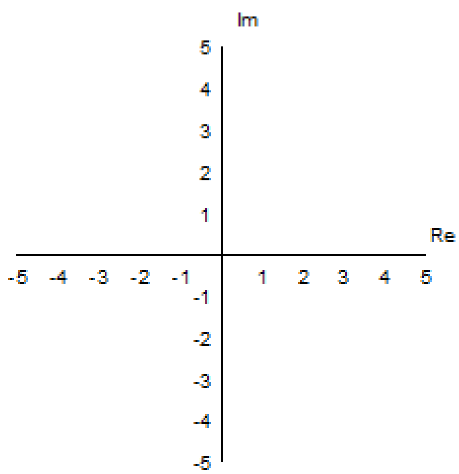
2) Turn and draw a line at a right angle relative to the previous line. This is the __ or imaginary part of the number.



3) Draw z_1=_______
This is the arrow shown.



4) Distribute the -1. This amounts to reversing the direction of the number because the sign of the real and imaginary parts is changed.
Draw $z_1 = -1$(______)$= -1$(___)-1(____)=________
This is the arrow shown below.

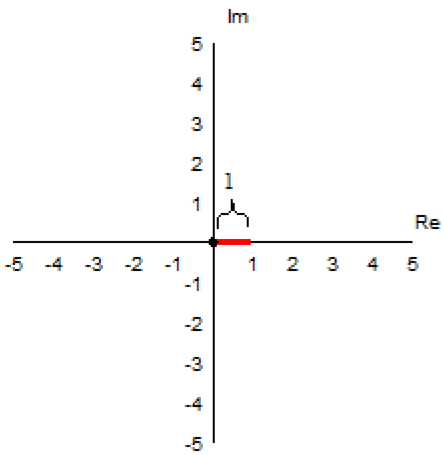


4) How to stretch a complex number

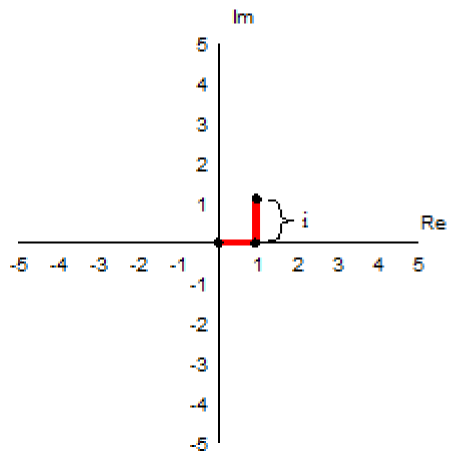
1) Draw $z_1=3(1+i)$

You can visualize this process as shown in the sequence of pictures below.

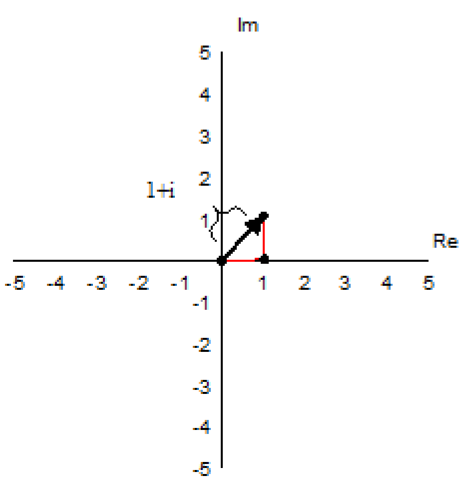
1) Draw a line from 0 to 1 along the x axis.
This is the "real" part of the number.



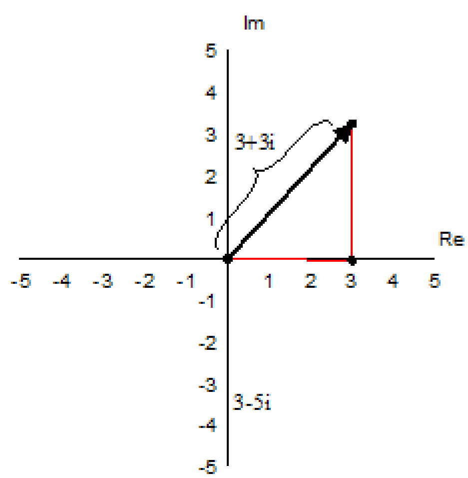
2) Turn and draw a line at a right angle relative to the red line. This is the i or imaginary part of the number.



3) Draw $z_1=1+i$
This is the arrow shown.



4) Distribute the 3.
This amounts to multiplying the real and imaginary parts of the number by 3. This means we have
$3(1+i)=3\cdot1+3\cdot i=3+3i$

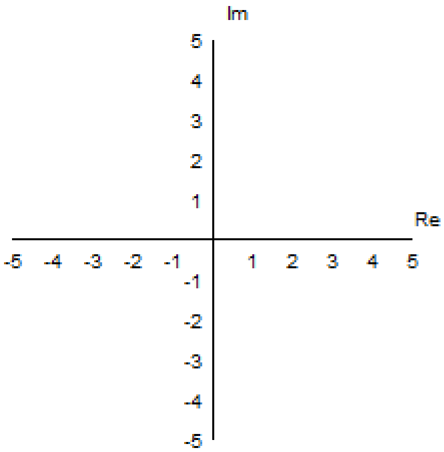


Practice stretching complex numbers

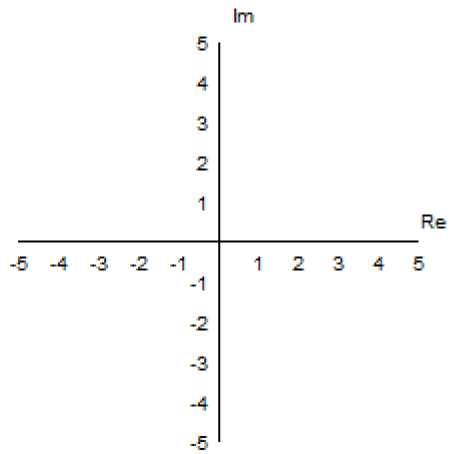
1) Draw each of the following, showing each step with absolut clarity $3(1-i)$, $2(1-2i)$, $-1(2+i)$, $\frac{1}{2}(1-i)$, $\frac{3}{4}(1+i)$

You can visualize this process as shown in the sequence of pictures below.

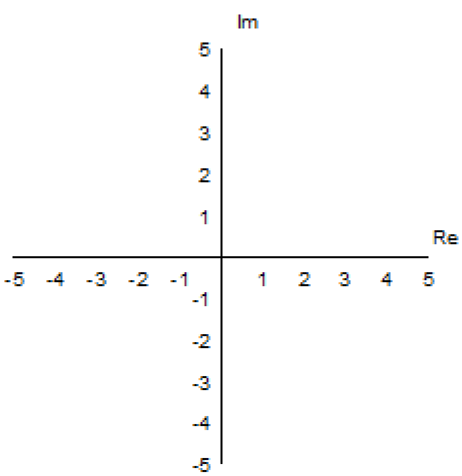
1) Draw a line from 0 to ___ along the x axis.
This is the "real" part of the number.



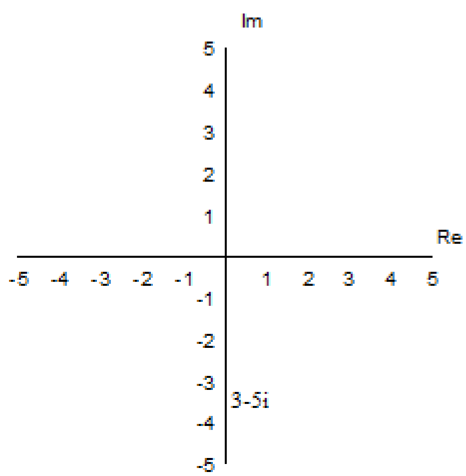
2) Turn and draw a line at a right angle relative to the red line. This is the ___ or imaginary part of the number.



3) Draw z_1=___________
This is the arrow shown.



4) Distribute the ______
This amounts to multiplying the real and imaginary parts of the number by _____. This means we have
____



4) How to rotate with complex numbers   The most important fact is that $i^2 = -1$

$i(2+3i) = 2\cdot i + i\cdot(3i)$ distribute the i

$= 2i + 3(i\cdot i)$

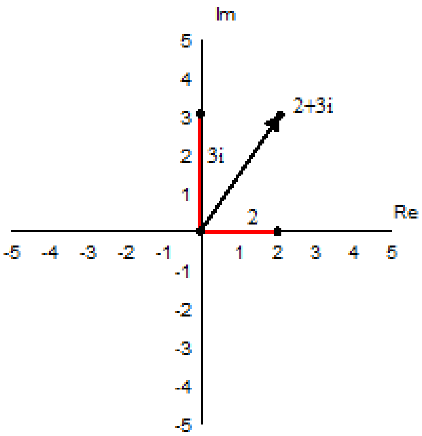
$= 2i + 3i^2$

$= 2i + 3(-1) = 2i - 3 = -3 + 2i$ Replace i^2 with -1

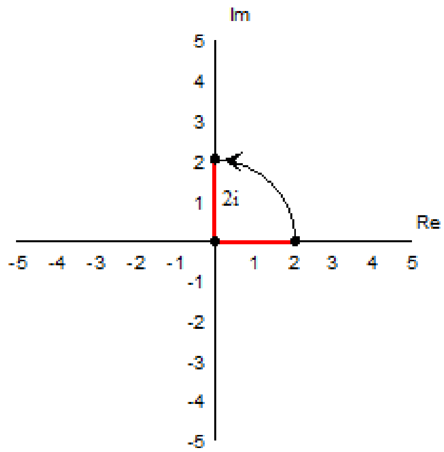
1) Draw $z_1 = i(2+3i)$

You can visualize this process as shown in the sequence of pictures below.

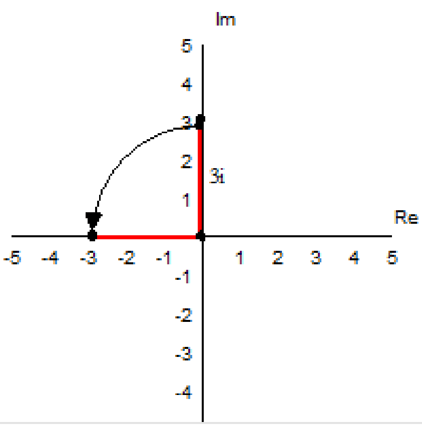
1) Draw $2+3i$ as the real and imaginary parts.



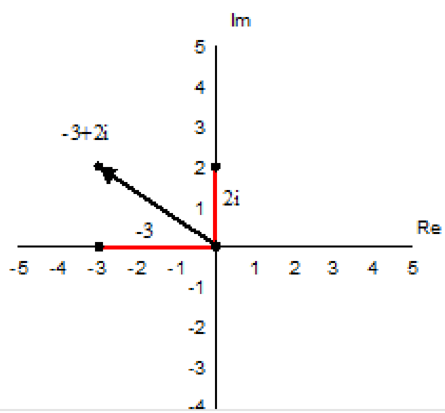
2) Distribute the i into the parenthesis.
Multiplying 2 by i has the effect of giving 2i
This is a 90° rotation.



3) When you multiply 3i by i,
you get the rotation shown below.
$i(3i) = 3(i\cdot i) = 3\cdot i^2 = 3(-1) = -3$
So the imaginary part of $2+3i$ is also rotated by 90°



4) Now that we understand that the rotation is applied to each part, we can draw the final result as shown below. Also add the arrow to represent the number.

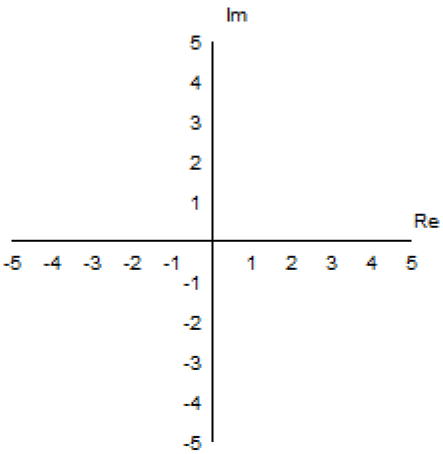


Practice rotating with complex numbers   The most important fact is that $i^2 = -1$

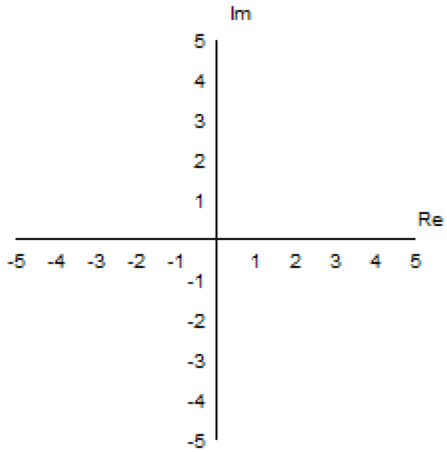
Perform each multiplication both symbolically and visually. Be careful and thoughtful, or you'll miss the journey.

1) $i(2-i)$, $2i(-1+i)$, $-i(1+i)$, $\frac{1}{2}i(2+i)$, $-1(1+i)$, $i \cdot i$, $i(a+bi)$

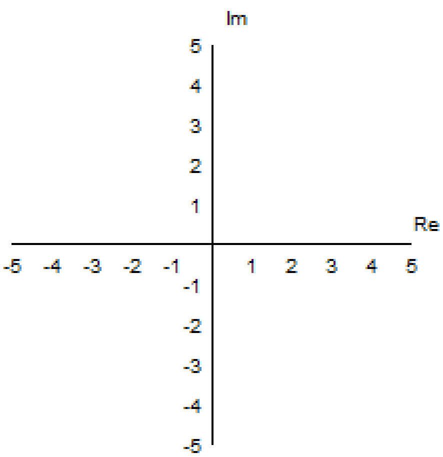
1) Draw ________ as the real
and imaginary parts.



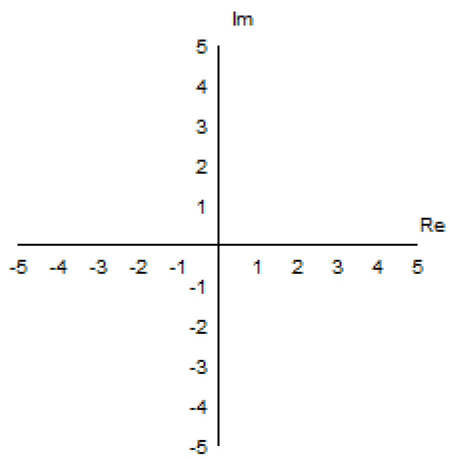
2) Distribute the ___ into the parenthesis.
Multiplying ___ by ___ has the effect of giving ___
This is a ____ rotation.



3) When you multiply ____ by ___,
you get the rotation shown below.
____
So the imaginary part of _______ is also rotated by ____



4) Now that we understand that the rotation
is applied to each part, we can draw the final result
as shown below.
Also add the arrow to represent the number.



How to multiply complex numbers using a shortcut

$z_1=1+2i$

$z_2=2-i$

$z_3=z_1\times z_2=(1+2i)\times(2-i)$

$=1\times2-1\times i+2i\times2-2i\times i$ use Product of Firsts + Product of Outers + Product of Inners + Product of Lasts

$=2-i+4i-2i^2$ 1×2 + $-1\times i$ + $2i\times2$ + $-2i\times i$

$=2-i+4i-2(-1)$ replace i^2 with -1

$=2-i+4i+2$

$=2+3i+2$

$=4+3i$

How to multiply complex numbers using the distributive property (the method above is a short version of this)

$z_3=z_1\times z_2=(1+2i)\times(2-i)$

$=1(2-i)+2i(2-i)$ distribute $2-i$ over the terms inside the first parenthesis

$=1\times2-1\times i+2i\times2-2i\times i$ distribute 1 into $2-i$ and distribute $2i$ into $2-i$ (we have the same thing as above)

$=2-i+4i-2(-1)$ replace i^2 with -1

$=2-i+4i+2$

$=2+3i+2$

$=4+3i$

How to multiply complex numbers using vertical multiplication

$$\begin{array}{r} 1+2i \\ \underline{2-i} \\ -i-2i^2 \\ \underline{2+4i} \\ 2+3i-2(-1)=2+3i+2=4+3i \end{array}$$

setup as shown and multiply as if doing regular multplication

Note that all these methods amount to the same thing: using the distributive property two times.

Practice: multiply using your choice of method

1) i and $2+i$	2) $-i$ and i
3) $3i(i+4)$	4) $2i(-i+2)$
5) $(2+i)(3-i)$	6) $(-2+i)(1+2i)$
7) $(1+i)(2-i)$	8) $(3+2i)(-4+3i)$

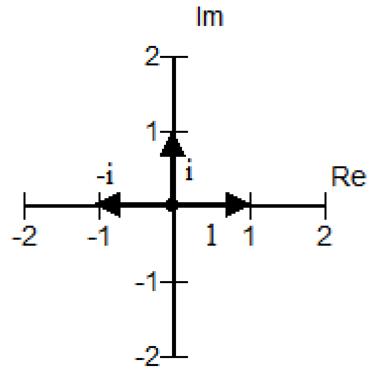
How to divide complex numbers

$\frac{1}{i}=\frac{1}{i}\left(\frac{i}{i}\right)$ $\frac{i}{i}=1$, so this does not change anything

$=\frac{1\times i}{i\times i}$

$=\frac{i}{i^2}$ i times i is i squared

$=\frac{i}{-1}=-i$ replace i^2 with -1



Perform each division below

1) $\frac{1}{2i}$	2) $\frac{3}{2i}$
3) $\frac{-3}{4i}$	4) $\frac{1}{-\pi i}$

You can access the PDF version, and the HD video library below. Use THISISMAGNIFICENT as the password.

http://www.tomsmath.com/complex-numbers.html