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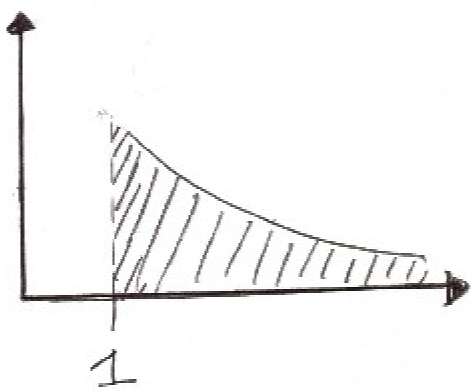
1) $\int_1^\infty \frac{1}{x^2}\,dx = \int_1^\infty x^{-2}\,dx$

$= \lim_{t\to\infty} \int_1^t x^{-2}\,dx = \lim_{t\to\infty} \left.\frac{x^{-2+1}}{-2+1}\right]_1^t$

$= \lim_{t\to\infty} \left.\frac{x^{-1}}{-1}\right]_1^t = \lim_{t\to\infty} (-1)\left.\frac{1}{x}\right]_1^t$

$= (-1)\left[\lim_{t\to\infty} \frac{1}{t} - \frac{1}{1}\right] = (-1)(0-1) = (-1)(-1)$

$= \boxed{1}$ converges



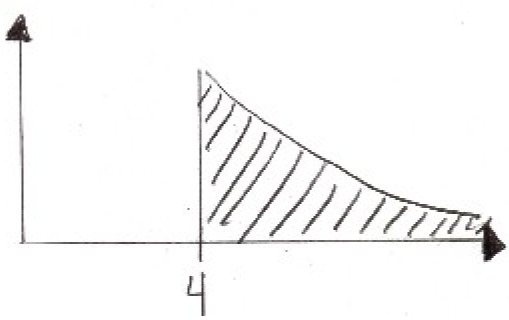
2) $\int_4^\infty \frac{1}{\sqrt{x}}\,dx = \lim_{t\to\infty} \int_4^t x^{-\frac{1}{2}}\,dx$

$= \lim_{t\to\infty} \left.\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\right]_4^t = \lim_{t\to\infty} \left.\frac{x^{-\frac{1}{2}+\frac{2}{2}}}{-\frac{1}{2}+\frac{2}{2}}\right]_4^t$

$= \lim_{t\to\infty} \left.\frac{x^{\frac{1}{2}}}{\frac{1}{2}}\right]_4^t = \lim_{t\to\infty} \left.\frac{2}{1}\cdot x^{\frac{1}{2}}\right]_4^t$

$= 2\lim_{t\to\infty} \left.\sqrt{x}\right]_4^t = 2\left[\lim_{t\to\infty}\sqrt{t} - \sqrt{4}\right]$

$= 2\left[\infty - 2\right] = \infty$ Diverges



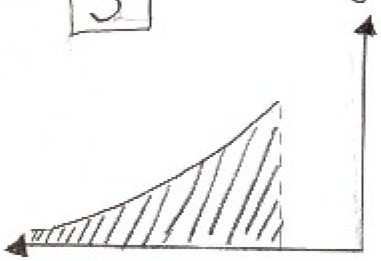
3) $\displaystyle\int_{-\infty}^{-1} \frac{1}{x^4}\,dx = \int_{-\infty}^{-1} x^{-4}\,dx$

$$= \lim_{t\to-\infty} \frac{x^{-4+1}}{-4+1}\Bigg]_t^{-1} = \lim_{t\to-\infty} \frac{x^{-3}}{-3}\Bigg]_t^{-1}$$

$$= \lim_{t\to\infty} -\frac{1}{3}\cdot\frac{1}{x^3}\Bigg]_t^{-1} = -\frac{1}{3}\lim_{t\to-\infty}\frac{1}{x^3}\Bigg]_t^{-1}$$

$$= -\frac{1}{3}\left[\frac{1}{(-1)^3} - \lim_{t\to-\infty}\frac{1}{x^3}\right] = -\frac{1}{3}\left[-1-0\right] = -\frac{1}{3}[-1]$$

$$= \boxed{\frac{1}{3}}$$ Converges

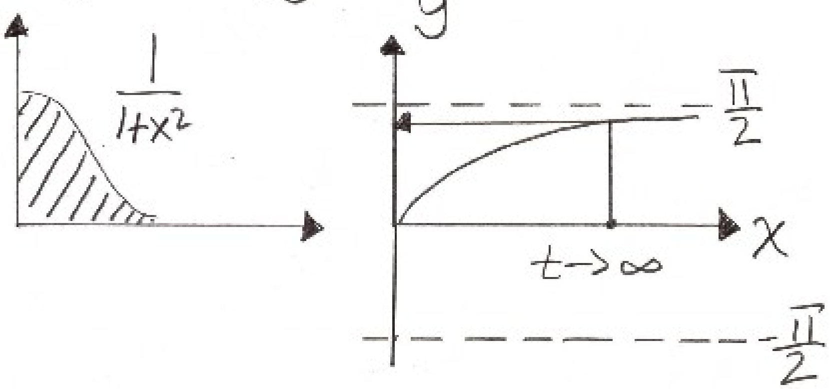


4) $\int_0^\infty \frac{1}{1+x^2}dx = \lim_{t \to \infty} \int_0^t \frac{1}{1+x^2}dx$

$= \lim_{t \to \infty} \tan^{-1}(x)\Big]_0^t$

$= \lim_{t \to \infty} \tan^{-1}(t) - \tan^{-1}(0)$

$= \frac{\pi}{2} - 0 = \frac{\pi}{2}$

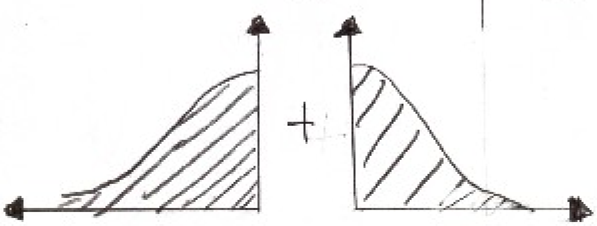


5) $\int_{-\infty}^{\infty}\frac{1}{1+x^2}dx=\int_{-\infty}^{0}\frac{1}{1+x^2}dx+\int_{0}^{\infty}\frac{1}{1+x^2}dx$

$=\lim_{t\to-\infty}\int_{t}^{0}\frac{1}{1+x^2}dx+\lim_{t\to\infty}\int_{0}^{\infty}\frac{1}{1+x^2}dx$

$=\underbrace{\lim_{t\to-\infty}\tan^{-1}(x)\Big]_{t}^{0}}+\underbrace{\lim_{t\to\infty}\tan^{-1}(x)\Big]_{0}^{t}}$

$\tan^{-1}(0)-\lim_{t\to-\infty}\tan^{-1}(t)$	$\lim_{t\to\infty}\tan^{-1}(t)-\tan^{-1}(0)$
$0-\left(-\frac{\pi}{2}\right)=\frac{\pi}{2}$	$\frac{\pi}{2}-0=\frac{\pi}{2}$



$\int_{-\infty}^{0}\frac{1}{1+x^2}dx+\int_{0}^{\infty}\frac{1}{1+x^2}dx=\frac{\pi}{2}+\frac{\pi}{2}=\frac{2\pi}{2}=\pi$

6) $\int_0^\infty 2e^{-2x}\,dx = \lim_{t\to\infty} \int_0^t 2e^{-2x}\,dx$

$u = -2x$

$du = -2\,dx$

$\frac{du}{-2} = \frac{\cancel{-2}\,dx}{\cancel{-2}}$

$-\frac{1}{2}du = dx$

$2\int -\frac{1}{2} \cdot e^u\,du = \frac{2}{-2}\int e^u\,du$

$= -1\int e^u\,du$

$= -e^u$

Replace u

$-e^{-2x}$

$\lim_{t\to\infty} -e^{-2x}\Big]_0^t$

$= \lim_{t\to\infty} -1 \cdot e^{-2x}\Big]_0^t = -1 \cdot \lim_{t\to\infty} e^{-2x}\Big]_0^t$

$= -1\left[\lim_{t\to\infty} e^{-2t} - e^{-2(0)}\right] = -1\left[\lim_{t\to\infty} \frac{1}{e^{2t}} - e^0\right]$

$= -1\left[0 - 1\right] = -1[-1] = \boxed{1}$

7) $\int_{-\infty}^{0} e^{3x}dx = \lim_{t \to -\infty} \int_{t}^{0} e^{3x} dx$

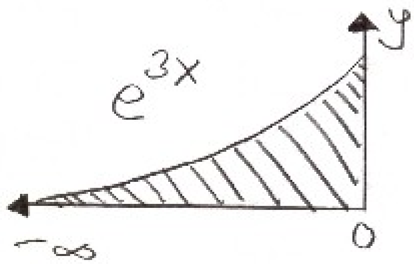
$u = 3x \Rightarrow du = 3dx \Rightarrow \frac{du}{3} = dx$

$\int e^{u} \frac{du}{3} = \int e^{u} \cdot \frac{1}{3} \cdot du = \frac{1}{3} \int e^{u} du = \frac{1}{3} e^{u}$

replace u with 3x: $\frac{1}{3}e^{3x}$

$\lim_{t \to \infty} \frac{1}{3} e^{3x} \Big]_{t}^{0} = \frac{1}{3} e^{3(0)} - \lim_{t \to -\infty} \frac{1}{3} e^{3t}$

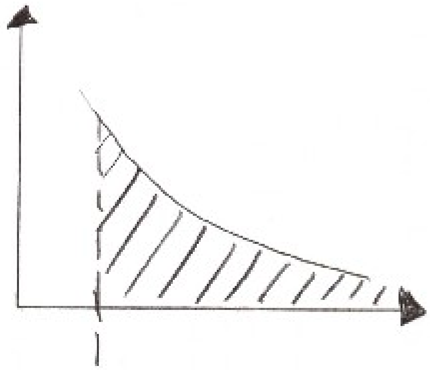
$= \frac{1}{3} e^{0} - \lim_{t \to -\infty} \frac{1}{3} e^{3t} = \frac{1}{3} - 0 = \boxed{\frac{1}{3}}$



8) $\int_1^\infty \frac{1}{x}dx = \lim_{t\to\infty} \int_1^t \frac{1}{x}dx$

$= \lim_{t\to\infty} \ln(x)\Big]_1^t = \lim_{t\to\infty} \ln(t) - \ln(1)$

$= \infty - 0 = \infty$ Diverges



9) $\int_0^\infty \frac{1}{2x+1}\,dx$

$u = 2x+1$

$du = 2dx$

$\frac{du}{2} = dx$

$$\int \frac{1}{u}\cdot\frac{du}{2} = \int \frac{1}{u}\cdot\frac{1}{2}\cdot du = \frac{1}{2}\int\frac{1}{u}\,du = \frac{1}{2}\ln|u|$$

replace u with 2x+1

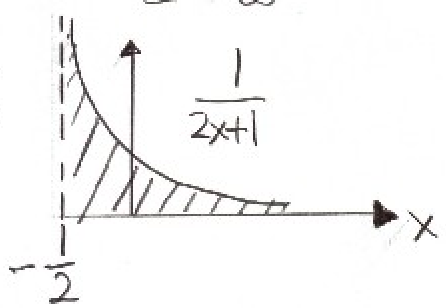
$\frac{1}{2}\ln|2x+1|$

$$\lim_{t\to\infty}\frac{1}{2}\ln|2x+1|\Big]_0^t = \frac{1}{2}\lim_{t\to\infty}\ln|2x+1|\Big]_0^t$$

$$= \frac{1}{2}\left[\lim_{t\to\infty}\ln|2t+1| - \ln|2(0)+1|\right]$$

$$= \frac{1}{2}\left[\infty - \ln(1)\right] = \frac{1}{2}\left[\infty - 0\right] = \frac{1}{2}(\infty) = \boxed{\infty}$$

Diverges

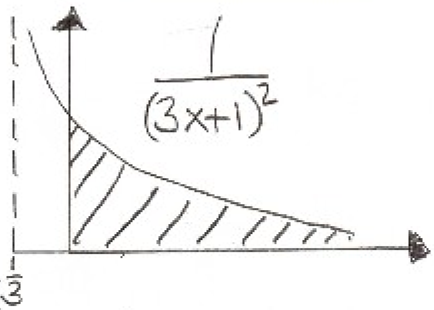


10) $\int_0^\infty \frac{1}{(3x+1)^2}dx$

$u = 3x+1$

$du = 3dx$

$\frac{du}{3} = dx$



$$\int \frac{1}{u^2}\frac{du}{3} = \int \frac{1}{u^2}\cdot\frac{1}{3}\cdot du = \frac{1}{3}\int u^{-2}du$$

$$= \frac{1}{3}\frac{u^{-2+1}}{-2+1} = \frac{1}{3}\frac{u^{-1}}{-1} = -\frac{1}{3}\frac{1}{u} = -\frac{1}{3u}$$

replace u with $3x+1$

$$-\frac{1}{3(3x+1)}$$

$$\lim_{t\to\infty} -\frac{1}{3(3x+1)}\Big]_0^t = \lim_{t\to\infty} -\frac{1}{3(3t+1)} - \left(-\frac{1}{3(3(0)+1)}\right)$$

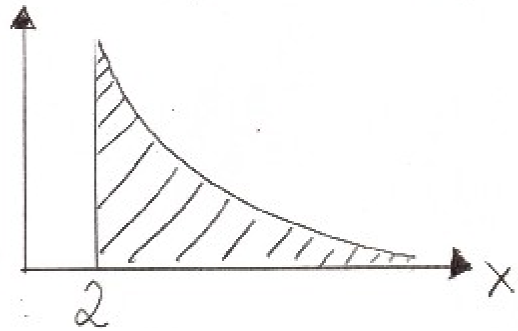
$$= 0 - \left(\frac{-1}{3(0+1)}\right) = \frac{1}{3(1)} = \boxed{\frac{1}{3}}$$ Converges

11) $\int_2^\infty \frac{2x+4}{x^2+4x}\,dx$ $u = x^2+4x$
$du = 2x+4\,dx$

$$\int \frac{du}{u} = \ln|u|$$

$$\lim_{t\to\infty} \ln|x^2+4x|\Big]_2^t = \lim_{t\to\infty} \ln|t^2+4t| - \ln|2^2+4(2)|$$

$$= \infty - \ln|4+8| = \infty - \ln|12| = \infty$$ Diverges



12) $\int_3^\infty \frac{1}{x\ln(x)}\,dx$ $\quad u=\ln(x)$

$du=\frac{1}{x}dx=\frac{dx}{x}$

$$\int_3^\infty \frac{1}{x}\cdot\frac{1}{\ln(x)}\,dx=\int_3^\infty \frac{1}{\ln(x)}\frac{dx}{x}$$

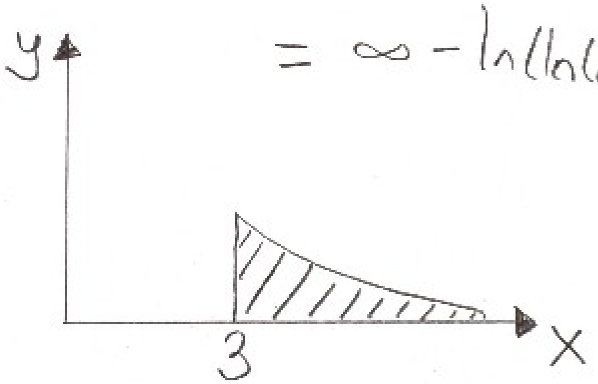
$\int \frac{1}{u}\,du=\ln|u|$ replace u with $\ln(x)$

$\ln|\ln(x)|$

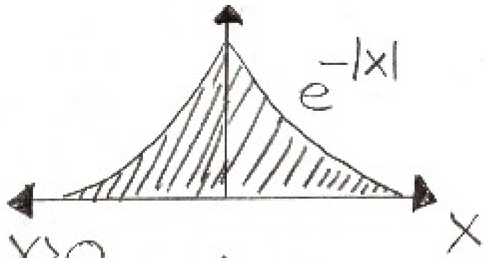
$$\lim_{t\to\infty} \ln|\ln(x)|\Big]_3^t=\lim_{t\to\infty}\ln(\ln(t))-\ln(\ln(3))$$

$$=\infty-\ln(\ln(3))=\boxed{\infty}.$$

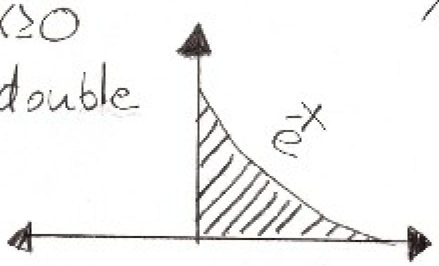
Diverges



(13) $\int_{-\infty}^{\infty} e^{-|x|}\,dx$



Use symmetry for $x \geq 0$
Drop the bars and double



$$2\int_0^{\infty} e^{-x}\,dx$$

$$\lim_{t\to\infty} 2\int_0^t e^{-x}\,dx = 2\lim_{t\to\infty}\int_0^t e^{-x}\,dx$$

$$= 2\lim_{t\to\infty} -e^{-x}\Big]_0^t = 2\lim_{t\to\infty}(-1)e^{-x}\Big]_0^t$$

$$= 2(-1)\lim_{t\to\infty} e^{-x}\Big]_0^t = -2\left[\lim_{t\to\infty} e^{-t} - e^{-0}\right]$$

$$= -2\left[\lim_{t\to\infty}\frac{1}{e^t} - 1\right] = -2[0-1] = -2[-1] = \boxed{2}$$

Converges

14) $\int_0^\infty xe^{-x}dx$ Use Parts

$u=x \qquad dv=e^{-x}dx$

$du=dx \qquad v=-e^{-x}$

$$\int xe^{-x}dx=-xe^{-x}-\int -e^{-x}dx=-xe^{-x}+\int e^{-x}dx$$

$$=-xe^{-x}-e^{-x}$$

$$\lim_{t\to\infty} -xe^{-x}-e^{-x}\Big]_0^t=\lim_{t\to\infty} -\frac{x}{e^x}-\frac{1}{e^x}\Big]_0^t$$

$$=\lim_{t\to\infty}(-1)\left[\frac{x}{e^x}+\frac{1}{e^x}\right]_0^t=-1\cdot\lim_{t\to\infty}\frac{x}{e^x}+\frac{1}{e^x}\Big]_0^t$$

$$=-1\left[\lim_{t\to\infty}\left(\frac{t}{e^t}+\frac{1}{e^t}\right)-\left(\frac{0}{e^0}+\frac{1}{e^0}\right)\right]$$

$$=-1\left[\lim_{t\to\infty}\frac{t}{e^t}+\lim_{t\to\infty}\frac{1}{e^t}-\left(0+\frac{1}{1}\right)\right]$$

$$=-1\left[0+0-0-\frac{1}{1}\right]=-1[-1]=\boxed{1}$$

$$\lim_{t\to\infty}\frac{t}{e^t}\underset{\text{L'Hopital}}{=}\lim_{t\to\infty}\frac{1}{e^t}=0$$

Converges

15) $\int_0^\infty \frac{e^{2x}}{1+e^{4x}}dx = \int_0^\infty \frac{e^{2x}dx}{1+(e^{2x})^2}$

$u = e^{2x}$ $\quad du = 2e^{2x}dx$ $\quad \frac{du}{2} = e^{2x}dx$

$\int \frac{1}{1+u^2}\frac{du}{2} = \int \frac{1}{2}\cdot\frac{1}{1+u^2}\cdot du = \frac{1}{2}\int \frac{1}{1+u^2}du$

$= \frac{1}{2}\tan^{-1}(u)$ replace u with e^{2x}

$\frac{1}{2}\tan^{-1}(e^{2x})$

$\lim_{t\to\infty} \frac{1}{2}\tan^{-1}(e^{2x})\Big]_0^t = \lim_{t\to\infty}\frac{1}{2}\tan^{-1}(e^{2t}) - \frac{1}{2}\tan(e^{2(0)})$

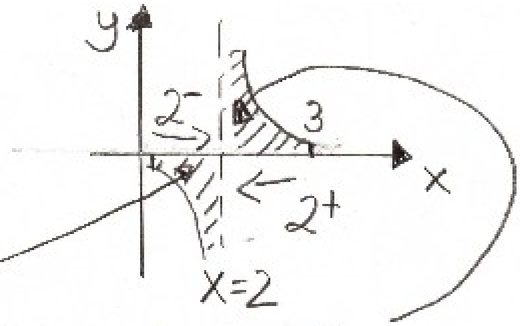
$= \frac{1}{2}\left[\lim_{t\to\infty}\tan^{-1}(e^{2t}) - \tan^{-1}(e^0)\right]$

$= \frac{1}{2}\left[\tan^{-1}\left(\lim_{t\to\infty}e^{2t}\right) - \tan^{-1}(1)\right]$

$= \frac{1}{2}\left[\tan^{-1}(\infty) - \frac{\pi}{4}\right] = \frac{1}{2}\left[\frac{\pi}{2} - \frac{\pi}{4}\right] = \frac{1}{2}\left[\frac{2\pi}{4} - \frac{\pi}{4}\right]$

$= \frac{1}{2}\left[\frac{\pi}{4}\right] = \boxed{\frac{\pi}{8}}$ Converges

16) $\int_1^3 \frac{1}{x-2}\,dx$



$\lim_{t\to 2^-} \int_1^t \frac{1}{x-2}\,dx$

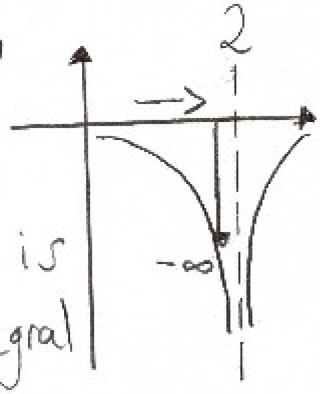
$\lim_{t\to 2^+} \int_t^3 \frac{1}{x-2}\,dx$

$\lim_{t\to 2^-} \ln|x-2| \Big]_1^t$

$\lim_{t\to 2^-} \ln|t-2| - \ln|1-2|$

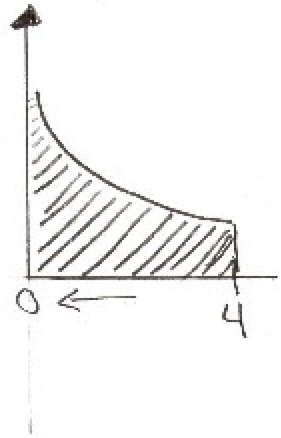
$\lim_{t\to 2^-} \ln|t-2| - \underbrace{\ln|-1|}_{=0}$

$\underbrace{\lim_{t\to 2^-} \ln|t-2|}_{-\infty}$



Because this is $-\infty$, the integral diverges.

17) $\int_0^4 \frac{1}{\sqrt{x}}\,dx = \int_0^4 \frac{1}{x^{1/2}}\,dx = \int_0^4 x^{-\frac{1}{2}}\,dx$



$\lim_{t \to 0^+} \int_t^4 x^{-\frac{1}{2}}\,dx$

$\lim_{t \to 0^+} \left. \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} \right]_t^4$

$\lim_{t \to 0^+} \left. \frac{x^{-\frac{1}{2}+\frac{2}{2}}}{-\frac{1}{2}+\frac{2}{2}} \right]_t^4$

$\lim_{t \to 0^+} \left. \frac{x^{\frac{1}{2}}}{\frac{1}{2}} \right]_t^4$

$\lim_{t \to 0^+} \left. 2\sqrt{x} \right]_t^4$

$2\sqrt{4} - \lim_{t \to 0^+} 2\sqrt{t} = 2 \cdot 2 - 2\sqrt{0} = \boxed{4}$

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