$$a_n = \frac{n}{3n+2}$$ remember that $$\frac{\frac{1}{n}}{\frac{1}{n}} = 1$$

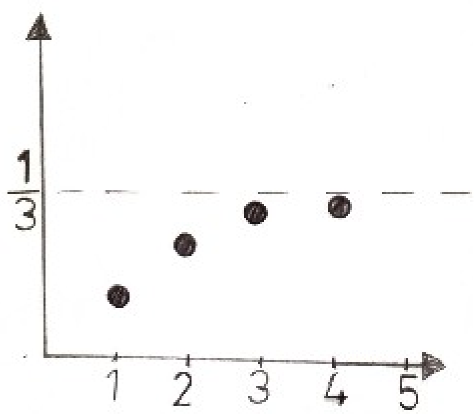
$$\lim_{n\to\infty} \frac{n}{3n+2}$$

$$\lim_{n\to\infty} \frac{\frac{1}{n}n}{\frac{1}{n}(3n+2)} = \lim_{n\to\infty} \frac{1}{\left(\frac{3n}{n} + \frac{2}{n}\right)} = \lim_{n\to\infty} \frac{1}{\left(3 + \frac{2}{n}\right)}$$

$$\frac{\lim_{n\to\infty} 1}{\lim_{n\to\infty} 3 + \lim_{n\to\infty} \frac{2}{n}} = \frac{1}{3+0} = \frac{1}{3}$$ converges

You can achieve the same result to dropping the 2 because it's very small compared to 3n when n is very large.

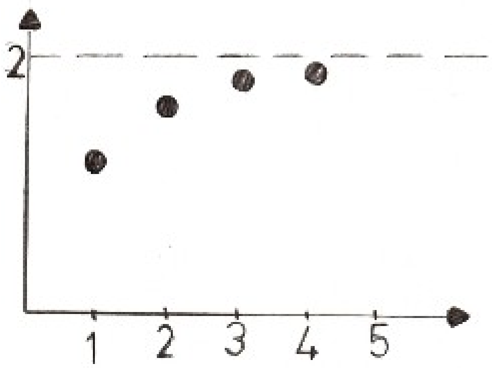
$$\lim_{n\to\infty} \frac{n}{3n+2} = \lim_{n\to\infty} \frac{n}{3n} = \lim_{n\to\infty} \frac{1}{3} = \frac{1}{3}$$



$$a_n = 2 - \frac{1}{n}$$

$$\lim_{n \to \infty} \left(2 - \frac{1}{n}\right) = \lim_{n \to \infty} 2 - \lim_{n \to \infty} \frac{1}{n} = 2 - 0 = 2$$

This means that the sequence converges.



Formal Approach

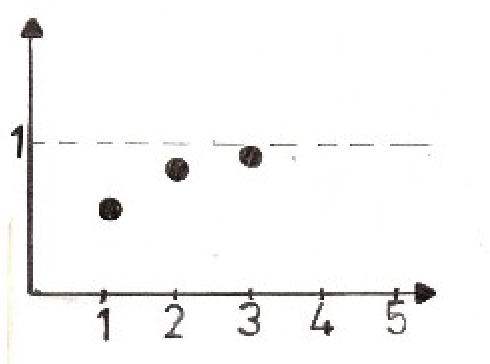
$$a_n = \frac{n^4}{1+n^4} \qquad \text{remember that} \qquad \frac{\frac{1}{n^4}}{\frac{1}{n^4}} = 1$$

$$\lim_{n \to \infty} \frac{\frac{1}{n^4} n^4}{\frac{1}{n^4}\left(1+n^4\right)} = \lim_{n \to \infty} \frac{1}{\left(\frac{1}{n^4} + \frac{n^4}{n^4}\right)} = \lim_{n \to \infty} \frac{1}{\left(\frac{1}{n^4} + 1\right)}$$

$$\frac{\lim_{n \to \infty} 1}{\lim_{n \to \infty} \frac{1}{n^4} + \lim_{n \to \infty} 1} = \frac{1}{0+1} = \frac{1}{1} = 1 \quad \text{converges}$$

You can achieve the same result to dropping the 1 because it's very small compared to terms with n.

$$\lim_{n \to \infty} \frac{n^4}{1+n^4} = \lim_{n \to \infty} \frac{n^4}{n^4} = \lim_{n \to \infty} \frac{1}{1} = 1$$



$$a_n = \cos\left(\frac{n\,\pi}{2}\right)$$

$$a_1 = \cos\left(\frac{1\cdot\pi}{2}\right) = \cos\left(\frac{\pi}{2}\right) = 0$$

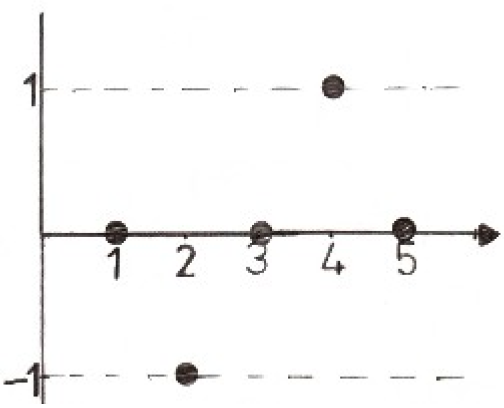
$$a_2 = \cos\left(\frac{2\pi}{2}\right) = \cos(\pi) = -1$$

$$a_3 = \cos\left(\frac{3\pi}{2}\right) = 0$$

$$a_4 = \cos\left(\frac{4\pi}{2}\right) = \cos(2\pi) = 1$$

$$a_5 = \cos\left(\frac{5\pi}{2}\right) = 0$$

You can see that values jump between -1 and 1. This means the limit does not exist.



Formal Approach

$$a_n = \frac{an^2 + bn}{cn^3 + dn} \qquad \text{remember that} \qquad \frac{\frac{1}{n^3}}{\frac{1}{n^3}} = 1$$

$$\lim_{n \to \infty} \frac{\frac{1}{n^3}\left(an^2 + bn\right)}{\frac{1}{n^3}\left(cn^3 + bn\right)} = \lim_{n \to \infty} \left(\frac{\frac{an^2}{n^3} + \frac{bn}{n^3}}{\frac{cn^3}{n^3} + \frac{bn}{n^3}} \right) = \lim_{n \to \infty} \left(\frac{\frac{a}{n} + \frac{b}{n^2}}{c + \frac{b}{n^2}} \right)$$

$$\frac{\lim_{n \to \infty} \frac{a}{n} + \lim_{n \to \infty} \frac{b}{n^2}}{\lim_{n \to \infty} c + \lim_{n \to \infty} \frac{b}{n^2}} = \frac{0 + 0}{c + 0} = \frac{0}{c} = 0$$

This means that this sequence converges. You can also find this same result by dropping bn and dn because these terms are insignificant relative to the higest power of n.

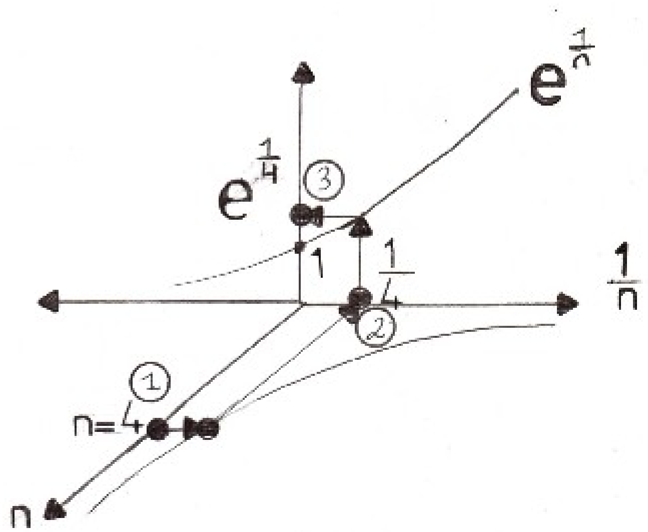
$$\lim_{n \to \infty} \frac{an^2 + bn}{cn^3 + dn} = \lim_{n \to \infty} \frac{an^2}{cn^3} = \lim_{n \to \infty} \frac{a}{cn} = 0$$

$$a_n = e^{\frac{1}{n}}$$

$$\lim_{n \to \infty} e^{\frac{1}{n}} = e^{\lim_{n \to \infty} \frac{1}{n}} = e^0 = 1$$

You can place in the limit in the exponent because the e function is continous. This means that this sequence converges. You can visualize this process in a clever way in the graph shown above. Just follow the arrows to see the process of taking the limit. Look at the picture below.

1) n goes to infinity
2) 1 divide by n goes to zero
3) e to the 1 over n goes to 1

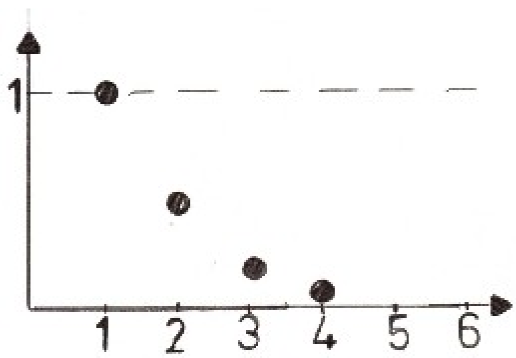


$$a_n = \frac{2^{n+1}}{4^n}$$

$$a_n = \frac{2^n \cdot 2^1}{\left(2^2\right)^n} = \frac{2^n \cdot 2}{\left(2^n\right)^2} = \frac{2^n \cdot 2}{2^n \cdot 2^n} = \frac{2}{2^n}$$

$$\lim_{n \to \infty} \frac{2}{2^n} = \frac{\lim\limits_{n \to \infty} 2}{\lim\limits_{n \to \infty} 2^n} = \frac{2}{\infty} = 0$$

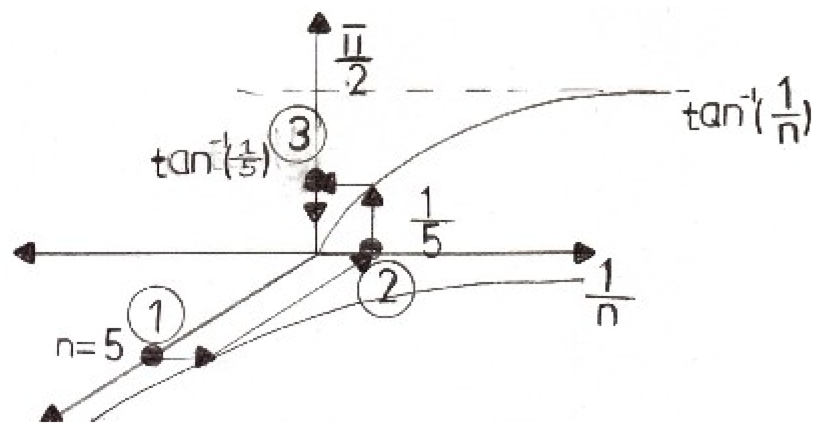
This means the sequence converges.



$$a_n = \tan^{-1}\left(\frac{1}{n}\right)$$

$$\lim_{n \to \infty}\left[\tan^{-1}\left(\frac{1}{n}\right)\right] = \tan^{-1}\left(\lim_{n \to \infty}\frac{1}{n}\right) = \tan^{-1}(0) = 0$$

Because the inverse tangent function is continous, you can move the limit into the function, and then simply take the inverse tangent of the limit. This is illustrated visually below. Just follow the numbers in the picture to see the flow.



$$a_n = \sqrt{\frac{n}{3n+2}}$$

$$\lim_{n \to \infty} \sqrt{\frac{n}{3n+2}} = \sqrt{\lim_{n \to \infty} \frac{n}{3n+2}}$$

$$\sqrt{\lim_{n \to \infty} \frac{\frac{1}{n}n}{\frac{1}{n}(3n+2)}} = \sqrt{\lim_{n \to \infty} \frac{1}{\left(\frac{3n}{n} + \frac{2}{n}\right)}}$$

Remember that $\dfrac{\frac{1}{n}}{\frac{1}{n}} = 1$

$$\sqrt{\frac{\lim_{n \to \infty} 1}{\lim_{n \to \infty} 3 + \lim_{n \to \infty} \frac{2}{n}}} = \sqrt{\frac{1}{3+0}} = \sqrt{\frac{1}{3}} = \frac{\sqrt{1}}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

Because the square root function is continous, you can bring the limit inside the square root function, and then take the square root of the limit.

$$a_n = (-1)^n \cdot \frac{n}{n^2 + 1}$$

Take the absolute value

$$\left|a_n\right| = \left|(-1)^n \cdot \frac{n}{n^2 + 1}\right| = \left|(-1)^n\right| \cdot \left|\frac{n}{n^2 + 1}\right| = \frac{n}{n^2 + 1}$$

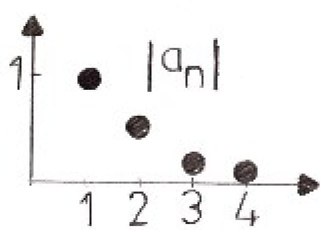
The sequence converges to 0 in absolute value.

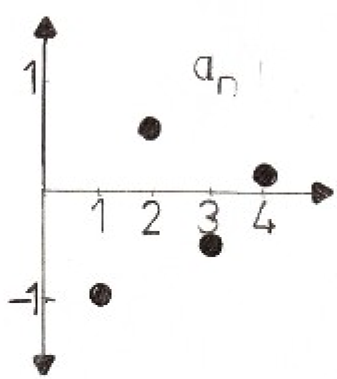
$$\lim_{n \to \infty} \frac{n}{n^2 + 1} = \lim_{n \to \infty} \frac{n}{n^2} = \lim_{n \to \infty} \frac{1}{n} = 0$$

This tells us that the original sequence also converges to the number 0.

$$a_n = \frac{(-1)^n \cdot n^2}{n^2 + 1}$$

The presence of -1 to the n simply changes the sign of the terms, but because the denominator is of bigger degree than the numerator, the sequence converges to 0.





$$a_n = \frac{(-1)^n \cdot n^2}{n^2 + 1}$$

Take the absolute value, as shown below.

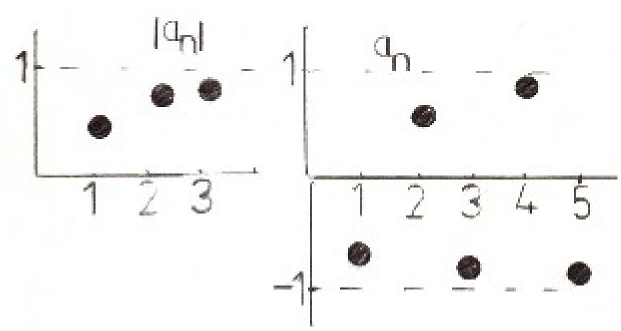
$$\left|\frac{(-1)^n \cdot n^2}{n^2 + 1}\right| = \left|(-1)^n\right| \cdot \left|\frac{n^2}{n^2 + 1}\right| = 1 \cdot \frac{n^2}{n^2 + 1} = \frac{n^2}{n^2 + 1}$$

Now take the limit.

$$\lim_{n \to \infty} \frac{n^2}{n^2 + 1} = \lim_{n \to \infty} \frac{n^2}{n^2} = \lim_{n \to \infty} 1 = 1$$

This sequence will NOT converge because the limit in absolute value is 1, and the presence of

$(-1)^n$ This causes thes values to start bouncing between -1 and 1, as shown below. The left graph shows the absolute value, and the right graph shows the original, which clearly bounces and therefore does not converge.

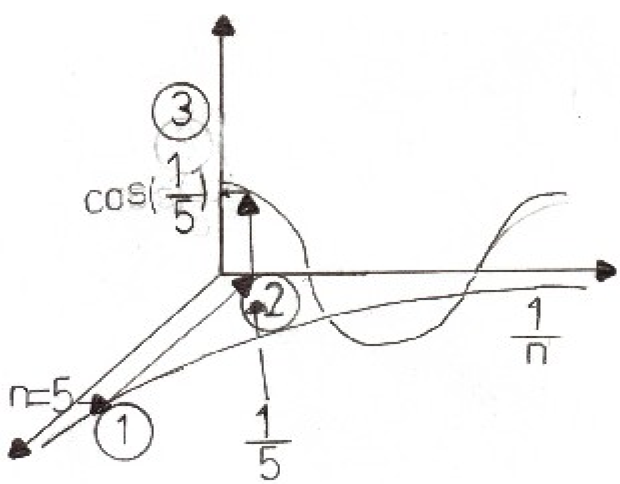


$$a_n = \cos\left(\frac{1}{n}\right)$$

You can bring the limit inside because the cosine function is cont.

$$\lim_{n \to \infty} \cos\left(\frac{1}{n}\right) = \cos\left(\lim_{n \to \infty} \frac{1}{n}\right) = \cos(0) = 1$$

This result tells us that the sequence converges.



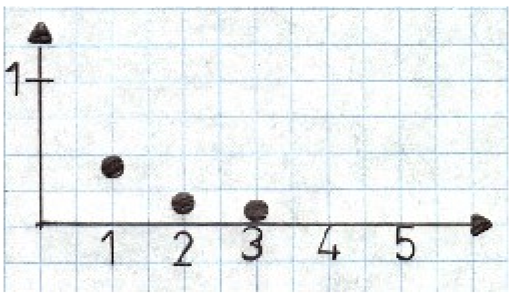
$$a_n = \frac{(2n)!}{(2n+1)!}$$

First we simplify the expression using a basic property of factorials.

$$a_n = \frac{(2 \cdot n)!}{(2n+1)\cdot(2n+1-1)!} = \frac{(2n)!}{(2n+1)(2n)!} = \frac{1}{2n+1}$$

Now we can take the limit easily.

$$\lim_{n \to \infty} \frac{1}{2n+1} = \lim_{n \to \infty} \frac{1}{2n} = \frac{1}{2} \cdot \lim_{n \to \infty} \frac{1}{n} = \frac{1}{2}(0) = 0$$

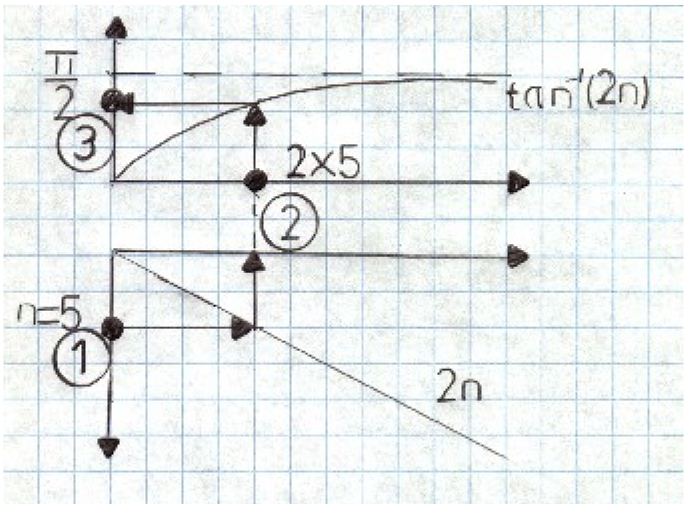


$a_n = \tan^{-1}(2n)$

Because the inverse tangent function is continous, you can bring the limit inside, and just take the inverse tangent of infinity.

$$\lim_{n \to \infty} \left[\tan^{-1}(2n)\right] = \tan^{-1}\left[\lim_{n \to \infty} (2n)\right] = \tan^{-1}(\infty) = \frac{\pi}{2}$$

This means that the sequence converges to $\frac{\pi}{2}$



$$a_n = \frac{\ln(n)}{\ln(10n)}$$

First rewrite the expression using a basic law of logs.

$$a_n = \frac{\ln(n)}{\ln(10n)} = \frac{\ln(n)}{\ln(10) + \ln(n)}$$

remember that $\dfrac{\dfrac{1}{\ln(n)}}{\dfrac{1}{\ln(n)}} = 1$

Now take the limit.

$$\lim_{n \to \infty} \frac{\ln(n)}{\ln(10) + \ln(n)} = \lim_{n \to \infty} \left[\frac{\dfrac{1}{\ln(n)}}{\dfrac{1}{\ln(n)}} \frac{(\ln(n))}{\ln(10) + \ln(n)} \right]$$

$$\lim_{n \to \infty} \left(\frac{\dfrac{1}{\ln(n)} \cdot \ln(n)}{\dfrac{\ln(10)}{\ln(n)} + \dfrac{\ln(n)}{\ln(n)}} \right) = \lim_{n \to \infty} \frac{1}{\dfrac{\ln(10)}{\ln(n)} + 1} = \frac{\displaystyle\lim_{n \to \infty} 1}{\displaystyle\lim_{n \to \infty} \frac{\ln(10)}{\ln(n)} + \lim_{n \to \infty} 1} = \frac{1}{0 + 1} = 1$$

$$a_n = n^{-1} \cdot e^n$$

First rewrite, as shown below.

$$a_n = \frac{e^n}{n}$$

Because this is of the form $\frac{\infty}{\infty}$ you can treat this as a function.

You can use the rule of L'Hopital to transition from step 1 to step 2 below.

$$\lim_{x \to \infty} \frac{e^x}{x} = \lim_{x \to \infty} \frac{e^x}{1} = \propto$$

Therefore we can conclude that this sequence diverges.

$$a_n = n \cdot \sin\left(\frac{n\pi}{2}\right)$$

$$a_1 = 1 \cdot \sin\left(\frac{1 \cdot \pi}{2}\right) = \sin\left(\frac{\pi}{2}\right) = 1$$

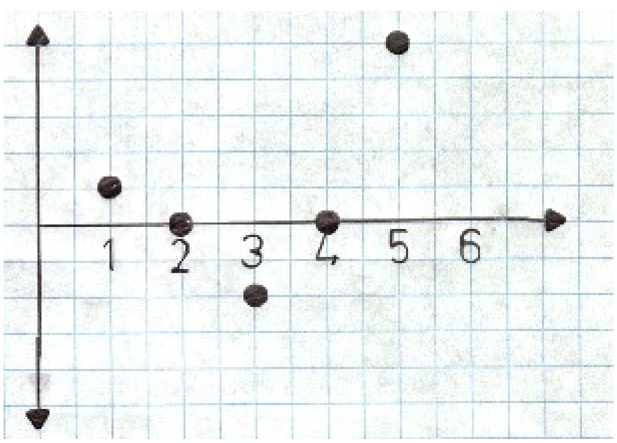
$$a_2 = 2 \cdot \sin\left(\frac{2\pi}{2}\right) = 2 \cdot \sin(\pi) = 0$$

$$a_3 = 3 \cdot \sin\left(\frac{3 \cdot \pi}{2}\right) = 3(-1) = -1$$

$$a_4 = 4 \sin\left(\frac{4 \cdot \pi}{2}\right) = 4 \cdot \sin(2\pi) = 4 \cdot 0 = 0$$

$$a_5 = 5 \sin\left(\frac{5\pi}{2}\right) = 5 \cdot 1 = 5$$

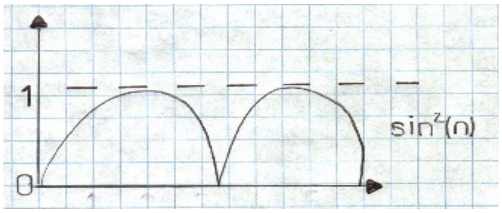
As you can see from the pattern above, the values change in sign, and become either big and positive, or very negative. The change in sign comes from the presence of sine and the growth towards positve or negative infinity comes from the presence of the n in front of the sine. Therefore, this sequence diverges.



$$a_n = \frac{\sin^2(n)}{2^n}$$

Begin with the basic fact that the square of sine is always between 0 and 1, as shown below.

$$0 \le \sin^2(n) \le 1$$



Now divide everthing you see by 2^n

$$\frac{0}{2^n} \le \frac{\sin^2(n)}{2^n} \le \frac{1}{2^n}$$

$$0 \le \frac{\sin^2(n)}{2^n} \le \frac{1}{2^n}$$

Now take the limit of both sides. This is an application of the squeeze theorem.

$$\lim_{n \to \infty} 0 = 0$$

$$\lim_{n \to \infty} \frac{1}{2^n} = 0$$

Because the square of the sine lies between these, it's limit is also 0.

$a_n = \ln(n+2) - \ln(2)$

First rewrite using a basic law of logs.

$$a_n = \ln\left(\frac{n+2}{n}\right) = \ln\left(\frac{n}{n} + \frac{2}{n}\right) = \ln\left(1 + \frac{2}{n}\right)$$

Because the ln functions is continous, bring the limit inside when takin it.

$$\lim_{n \to \infty} \ln\left(1 + \frac{2}{n}\right) = \ln\left[\lim_{n \to \infty} \left(1 + \frac{2}{n}\right)\right] = \ln\left(\lim_{n \to \infty} 1 + \lim_{n \to \infty} \frac{2}{n}\right)$$

$$\ln(1+0) = \ln(1) = 0$$

Therefore we can conclude that this sequence is convergent.

$$a_n = \frac{\cos(2n)}{1+n}$$

Begin with the basic fact that cos(2n) is always between -1 and 1.

$$-1 \le \cos(2n) \le 1$$

Now divide every term by 1+n, as shown below.

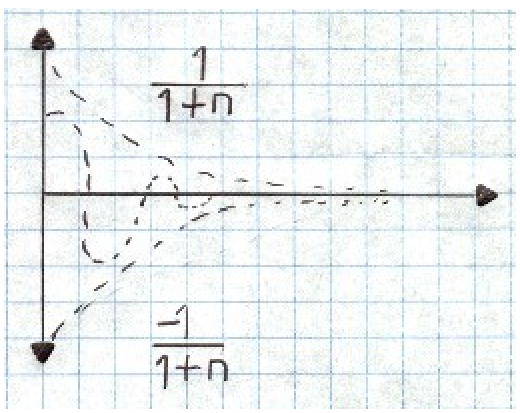
$$\frac{-1}{1+n} \le \frac{\cos(2n)}{1+n} \le \frac{1}{1+n}$$

Now we apply the squeeze theorem.

$$\lim_{n \to \infty} \frac{-1}{1+n} = 0$$

$$\lim_{n \to \infty} \frac{1}{1+n} = 0$$

Because the sequence we're given lies between these two, it also converges to 0.



$$a_n = \sqrt[n]{2^{1+5n}}$$

First rewrite, as shown below.

$$a_n = \sqrt[n]{2^1 \cdot 2^{5n}} = \left(2^1 \cdot 2^{5n}\right)^{\left(\frac{1}{n}\right)} = 2^{\frac{1}{n}} \cdot 2^{5n \cdot \frac{1}{n}} = 2^{\frac{1}{n}} \cdot 2^5 = 32 \cdot 2^{\frac{1}{n}}$$

Now you can take the limit. Here, you can put the limit in the exponent because the exponential function shown is continous.

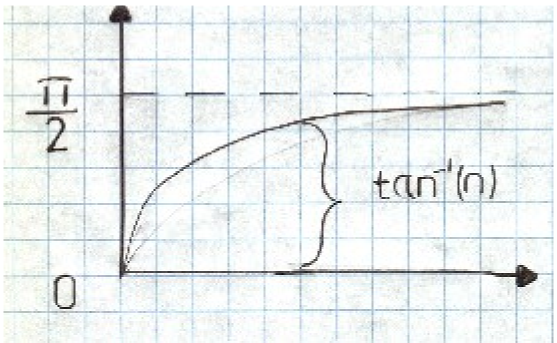
$$\lim_{n \to \infty} \left(32 \cdot 2^{\frac{1}{n}}\right) = 32 \cdot \lim_{n \to \infty} 2^{\frac{1}{n}} = 32 \cdot 2^{\lim_{n \to \infty} \frac{1}{n}} = 32 \cdot 2^0 = 32 \cdot 1 = 32$$

Therefore we can conclude that this sequence converges.

$$a_n = \frac{\tan^{-1}(n)}{n}$$

Begin with the basic fact that when n is zero or more, the inverse tangent function lies between the limits shown below.

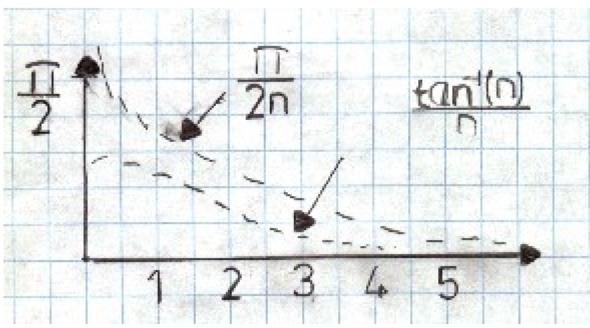
$$0 \le \tan^{-1}(n) \le \frac{\pi}{2}$$



Now divide everything you see by n. Remember, n is positive, so you keep the inequality signs you see. There is no need to change the direction.

$$\frac{0}{n} \le \frac{\tan^{-1}(n)}{n} \le \frac{\frac{\pi}{2}}{n}$$

$$0 \le \frac{\tan^{-1}(n)}{n} \le \frac{\pi}{2n}$$



Now we apply the squeeze theorem, getting the limits shown below. Because ours is between these, it also converges to 0.

$$\lim_{0 \to \infty} 0 = 0$$

$$\lim_{n \to \infty} \frac{\pi}{2n} = 0$$

$a_n = n^{\frac{1}{n}}$

Rewrite as a function. $y = x^{\frac{1}{x}}$

Take the natural log of both sides $\ln(y) = \ln\left(x^{\frac{1}{x}}\right)$

Bring the $\frac{1}{x}$ down $\ln(y) = \frac{1}{x}\ln(x)$

Now take the limit of both sides. $\lim_{n \to \infty} \ln(y) = \lim_{n \to \infty} \left(\frac{\ln(x)}{x}\right)$

On the right side we have infinity
over infinity, so strike hard with L'Hopital $\lim_{n \to \infty} \ln(y) = \lim_{n \to \infty} \frac{\frac{1}{x}}{1} = 0$

Now exponentiate both sides. $e^{\lim_{n \to \infty} \ln(y)} = e^{0}$

Because ln is contious, bring the limit inside $e^{\ln\left(\lim_{n \to \infty} y\right)} = 1$

e and ln are inverses so they cancel $\lim_{n \to \infty} y = 1$

Replace y with its definiotin in terms of x

$$\lim_{n \to \infty} x^{\frac{1}{x}} = 1$$

This means our sequence also converges to 1.

$$a_n = 3^{1-\frac{1}{n}}$$

The following sequence of steps is not magical. It's just using the continuaity of the function.

$$\lim_{n \to \infty} 3^{1-\frac{1}{n}} = 3^{\lim_{n \to \infty}\left(1-\frac{1}{n}\right)} = 3^{\lim_{n \to \infty} 1 - \lim_{n \to \infty}\frac{1}{n}} = 3^{1-0} = 3$$

This means our sequence converges to 3.

$$a_n = e^{\cos\left(\frac{1}{n}\right)}$$

$$\lim_{n \to \infty} e^{\cos\left(\frac{1}{n}\right)} = e^{\lim_{n \to \infty} \cos\left(\frac{1}{n}\right)} = e^{\cos\left(\lim_{n \to \infty} \frac{1}{n}\right)} = e^{\cos(0)} = e^{1} = e$$

Because the natural exponential function, and cosien are both continous, you can move the limit into the cosine function, and work as shown above.

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