Chain Rule:

Imagine z depends on t through x and y.

That is, $z(x(t), y(t))$. This means we input t, get x and y and then those are inputs into z.

Example:

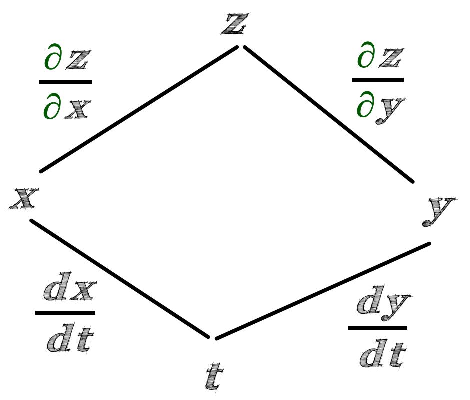
$x = 2t, y = t^2$

$z = x^3 + 2y$

Then $z(2t, t^2) = (2t)^3 + 2t^2 = 8t^3 + 2t^2 = z(t)$

$z(1) = 8 \cdot 1^3 + 2 \cdot 1^2 = 8 + 2 = 10$

To find the rate at which z changes with respect to t, create a dependence tree.



Since z depends on x and y first, we get partials along those branches. Since x and y each depends on t only, we get regular derivatives along those branches.

Thus, we get $\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$

Multiply the derivatives down the branches and add the products across the branches.

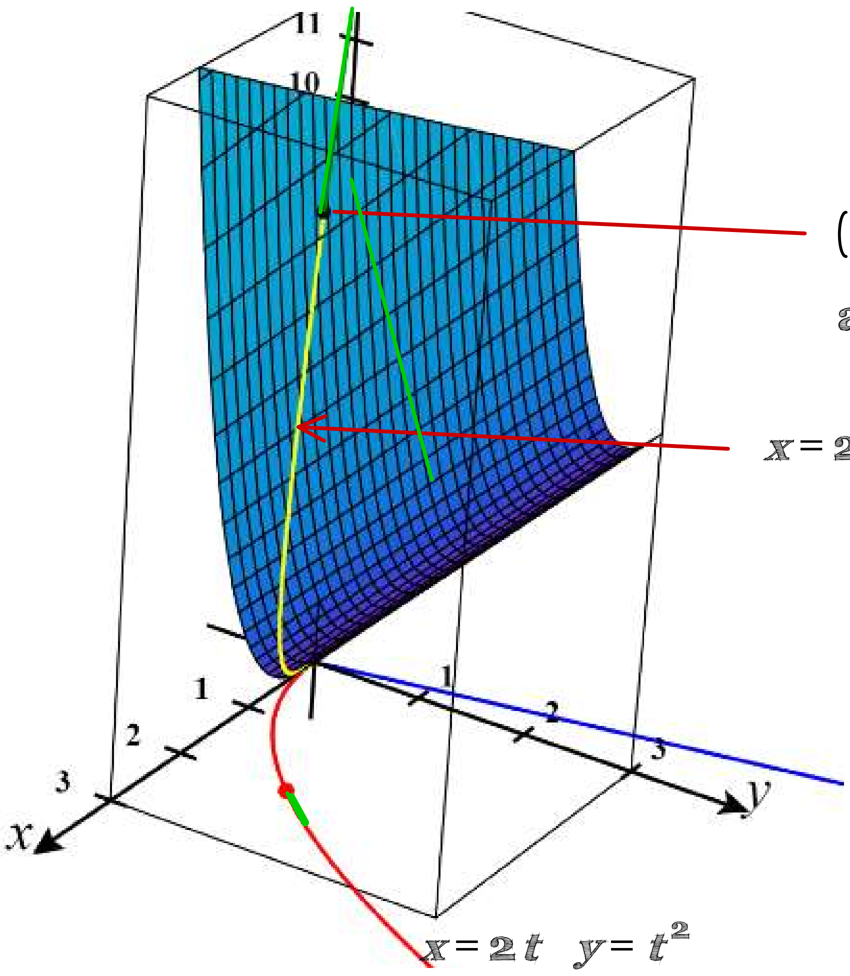
Thus, for the example above, $x = 2t, y = t^2, z = x^3 + 2y$, we get

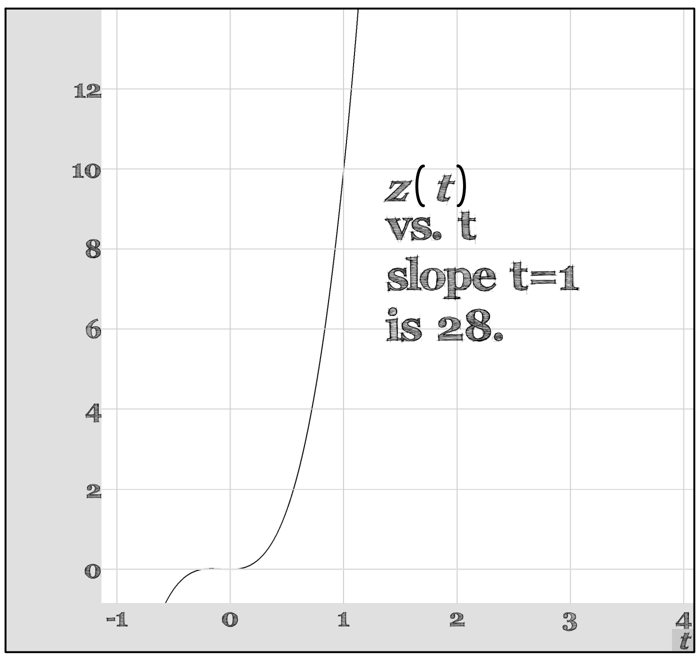
$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt} = 3x^2 \cdot 2 + 2 \cdot 2t = 6x^2 + 4t \quad \left.\frac{dz}{dt}\right|_{t=1} = 6(2 \cdot 1)^2 + 4 \cdot 1 = 6 \cdot 2^2 + 4 = 6 \cdot 4 + 4 = 24 + 4 = 28$$

$x = 2(1)$
from x=2t

or $\frac{dz}{dt} = 24t^2 + 4t \Rightarrow t = 1 \Rightarrow 24 + 4 = 28$

Since at t=1, we have x=2· 1=2, y=1^2 =1 , 28 is the rate of change on the surface x^3 +2y at t=1 along the curve $x = 2t, y = t^2$.





estimate of slope

$$\frac{8(1.001)^3 + 2(1.001)^2 - [8(1)^3 + 2 \cdot 1^2]}{1.001 - 1} \approx 28.026$$

$z = x^2 + y^2$

along $x = cos(t), y = sin(t) \quad 0 \le t \le 2\pi$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt} = 2x \cdot (-sin(t)) + cos(t) \cdot 2y = -2x\,sin(t) + 2y\,cos(t)$$

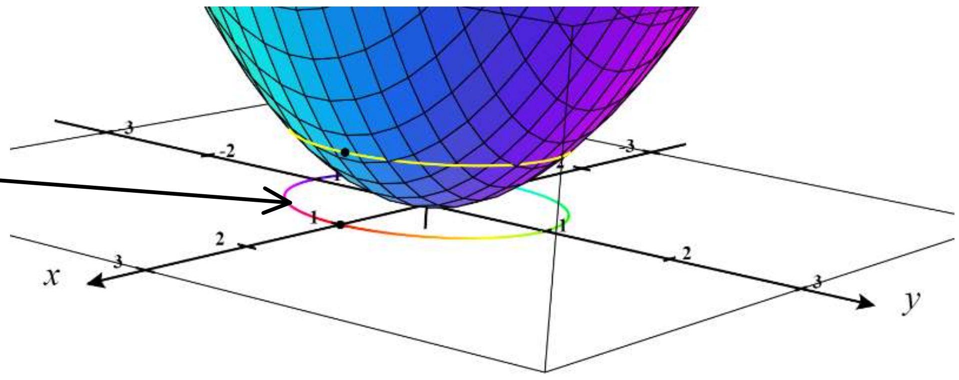
when $t = \pi / 2$:

$x(\pi/2) = cos(\pi/2) = 0$

$y(\pi/2) = sin(\pi/2) = 1$

$x = cost$

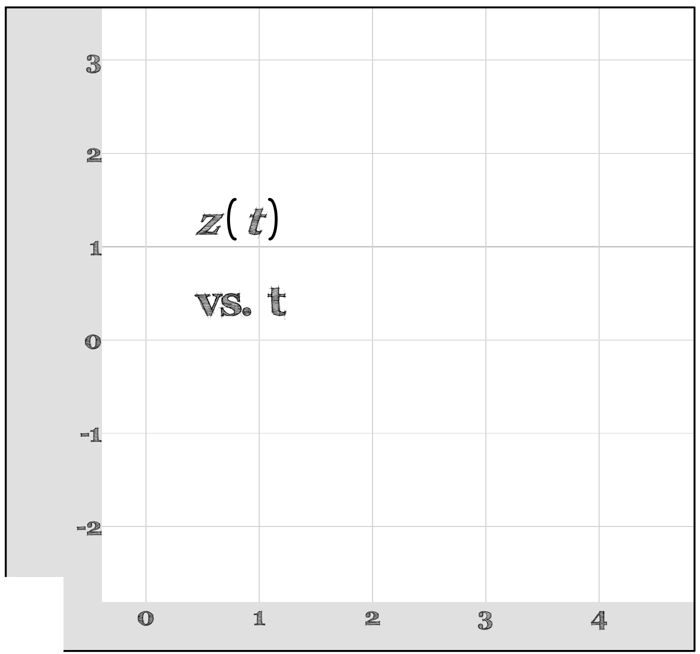
$y = sint$

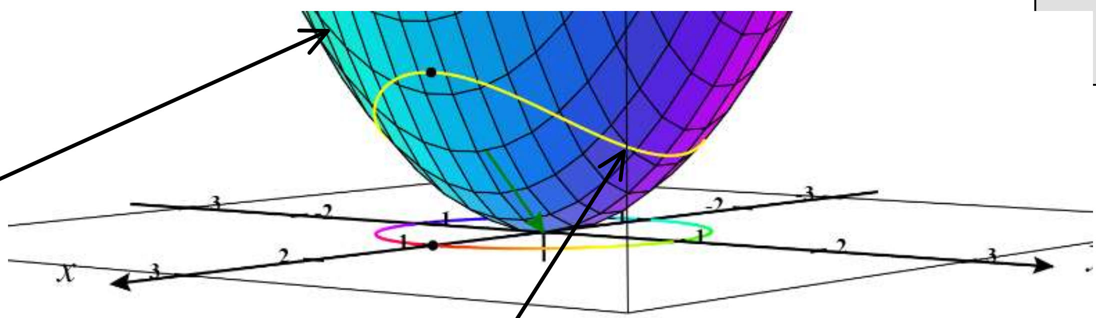


$$\left.\frac{dz}{dt}\right|_{t=\pi/2} = 2(0)[-sin(\pi/2)] + 2(1)\,cos(\pi/2) = 0$$

Or convert to t only: $z(cost, sint) = z(t) = cos^2 t + sin^2 t = 1$

Since z(t)=1, the derivative $\frac{dz}{dt} = 0$ always.





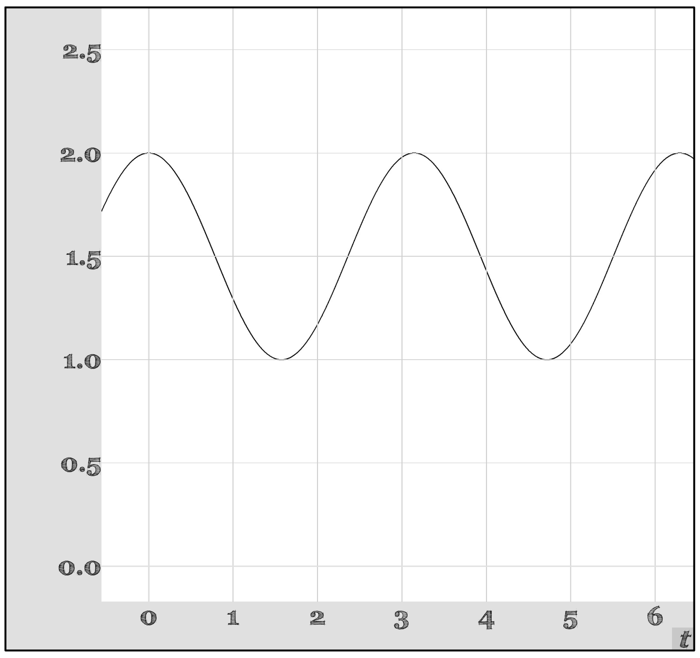
$z = 2x^2 + y^2$

$x = cos(t)$

$y = sin(t)$

$z(cos(2t), sin(2t)) = 2cos^2 t + sin^2 t$

$$\frac{dz}{dt} = 4cos(t)[-sin(t)] + 2sin(t)cos(t) = -4cos(t)sin(t) + 2sin(t)cos(t) = -2sin(t)cos(t) = -2sin(2t)$$



$z(t) = 2cos^2 t + sin^2 t$

vs. t

$$\left.\frac{dz}{dt}\right|_{t=\pi/4} = -2sin\left(2 \cdot \frac{\pi}{4}\right) = -2sin\left(\frac{\pi}{2}\right) = -2$$