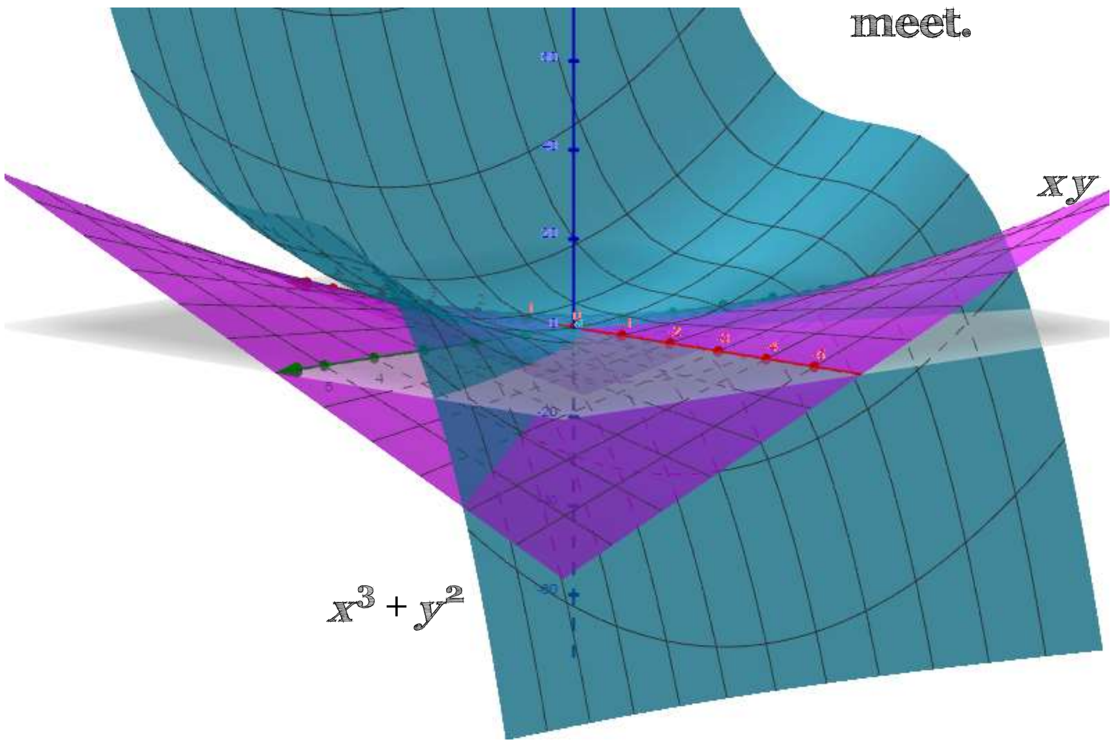
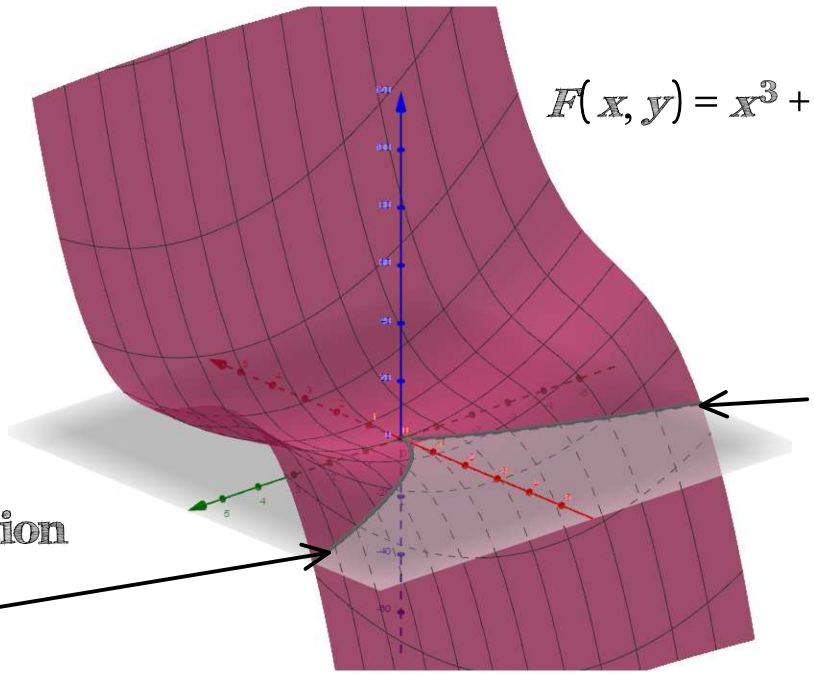
Implicit Differentiation:

For the case of $x^3 + y^2 = xy$, we can imagine the solution to this equation as the curve where the surfa meet.



We can create a function that says
$F(x, y) = x^3 + y^2 - xy = 0$.
This just creates a new surface and we are interested in seeing where this surface has the value z=0.



Along the curve $x^3 + y^2 - xy = 0$, we get y as a function of x. By the chain rule, we get

$F(x, y) = 0$

$F(x, f(x)) = 0$ since y=f(x)

$$\frac{\partial F}{\partial x}\frac{dx}{dx} + \frac{\partial F}{\partial y}\frac{dy}{dx} = 0$$

$$\frac{\partial x}{\partial x} = 1$$

$$\frac{\partial F}{\partial x}\cdot 1 + \frac{\partial F}{\partial y}\frac{dy}{dx} = 0$$

Now solve for $\frac{dy}{dx}$:

$$\frac{\partial F}{\partial y}\frac{dy}{dx} = -\frac{\partial F}{\partial x}$$

$$\frac{dy}{dx} = \frac{-F_x}{F_y}$$

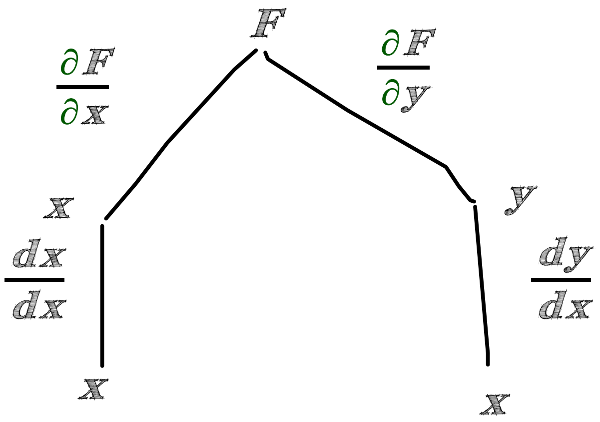
So we get the following:

$F(x,y) = x^3 + y^2 - 2xy = 0$

$F_x = 3x^2 - 2y$

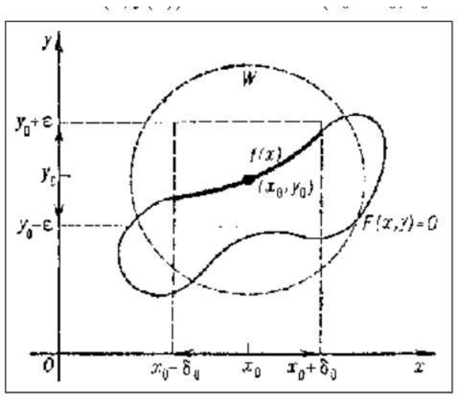
$F_y = 2y - 2x$

Thus, $\frac{dy}{dx} = \frac{-F_x}{F_y} = \frac{-(3x^2 - 2y)}{2y - 2x}$



This is in agreement with the principles of implicit derivatives learned in Calculus I.

The concepts illustrated above are based on the details of the implicit function theorem.
This states that if F is defined on a disk containing point (a,b), where F(a,b)=0, F_y (a,b) $\neq 0$ and F_x and F_y are continous on the disk, then the equation F(x,y)=0 defines y as a function of x near the point (a,b) and the derivative is $\frac{dy}{dx} = \frac{-F_x}{F_y}$.



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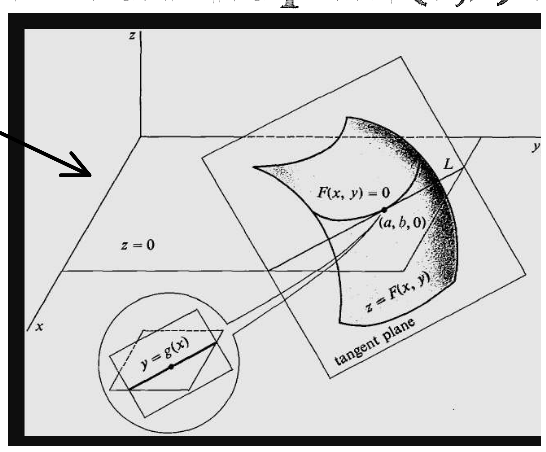

$$y\cos(x) = x^2 + y^2$$

$$F(x, y) = y\cos(x) - x^2 - y^2 = 0$$

$$\frac{dy}{dx} = \frac{-F_x}{F_y} = \frac{-[-y\sin(x) - 2x]}{\cos(x) - 2y} = \frac{y\sin(x) + 2x}{\cos(x) - 2y}$$