Symmetry makes sketching graphs go faster.
Here is a property that has been found to be useful:
$f(-x) = f(x)$

exponent review

$(ab)^p = a^p b^p$

$3^2 = 3 \cdot 3 = 9$

$(2x)^3 = 2^3 x^3 = 8x^3$ $3^2 \neq 3 \cdot 2$

y coordinate when -x goes in is the same as the y coordinate when +x goes into the function.

$f(x) = x^2$

$f(1) = 1, f(-1) = (-1)^2 = 1 \Leftarrow$ both outputs are the same, but the inputs are negated.

$f(2) = 4, f(-2) = (-2)^2 = 4 \Leftarrow$ both outputs are the same, but the inputs are negated.

$f(x) = x^2$ (this gives a point of the form (x ,x^2), (input, output)

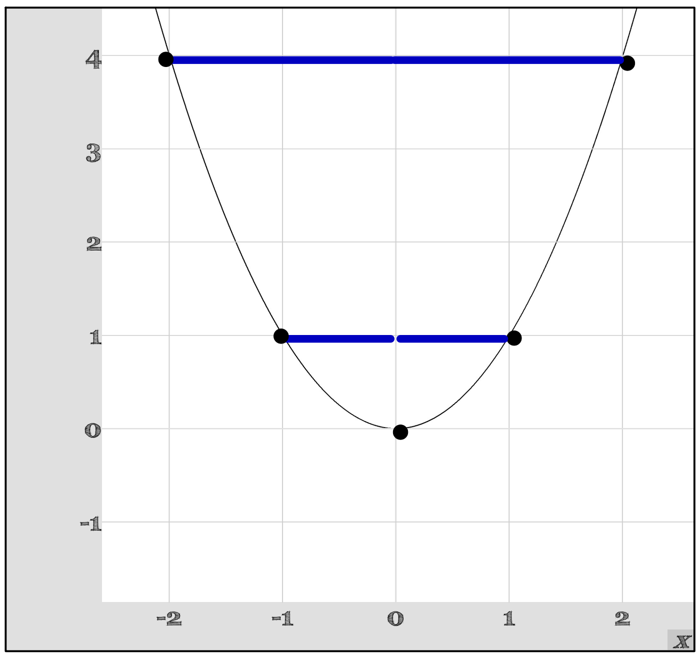
$f(-x) = (-x)^2 = (-1 \cdot x)^2 = (-1)^2 (x^2) = (-1)(-1) x^2 = 1x^2 = x^2$ (-x, x^2) (input, output)

When a function has the property or characteristic that $f(-x) = f(x)$, we call the function an even function. This kind of function is said to have y-axis symmetry.

x	y	(x,y)
-2	4	$(-2, 4)$
-1	1	$(-1, 1)$
0	0	$(0, 0)$
1	$1^2 = 1$	$(1, 1)$
2	$2^2 = 4$	$(2, 4)$

Since $f(x) = x^2$ is even
, once we calculate f(1)=1,
we know that f(-1)=1 *also*.

$f(x) = x^2$, $f(-x) = x^2$



Half on the right is the same as the half on the left.

Another Kind of Symmetry:

$f(x) = x^3$

$f(1) = 1^3 = 1$ point $(1, 1)$

f(-1)= $(-1)^3 = (-1)(-1)(-1) = 1(-1) = -1$ point (-1,-1)

when 1 goes in , 1 comes out.
when -1 goes in , -1 comes out.
outputs are different in the sign.

$f(2) = 2^3 = 8$ gives the point $(2, 8)$

$f(-2) = (-2)^3 = -8$ gives the point $(-2, -8)$

the output is the in absolute value but the signs are different.

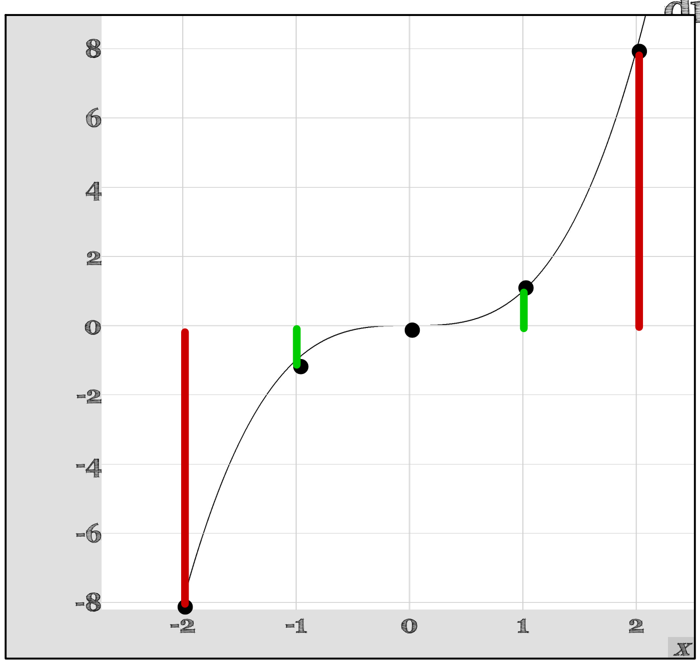
So for $f(x) = x^3$, negating the input also negates the output.

More generally, $f(x) = x^3$ gives the point (x, x^3).
$f(-x) = -x^3$ so the point is $(-x, -x^3)$

signs of inputs differe and signs of outputs differ

$f(3) = 27, f(-3) = -27$

$f(x = 0) = 0$



A function having the property that $f(-x) = -f(x)$ is called an odd function.

(our example: $f(-x) = -x^3 = -f(x)$

It's said to have origin-symmetry.
In terms of points, this means that if a point (x,y) is on the graph, so is the point (-x,-y).

$f(x) = x^2 + x^4$ original definition

Both exponents are even.

$f(1) = 1^2 + 1^4 = 1 + 1 = 2$

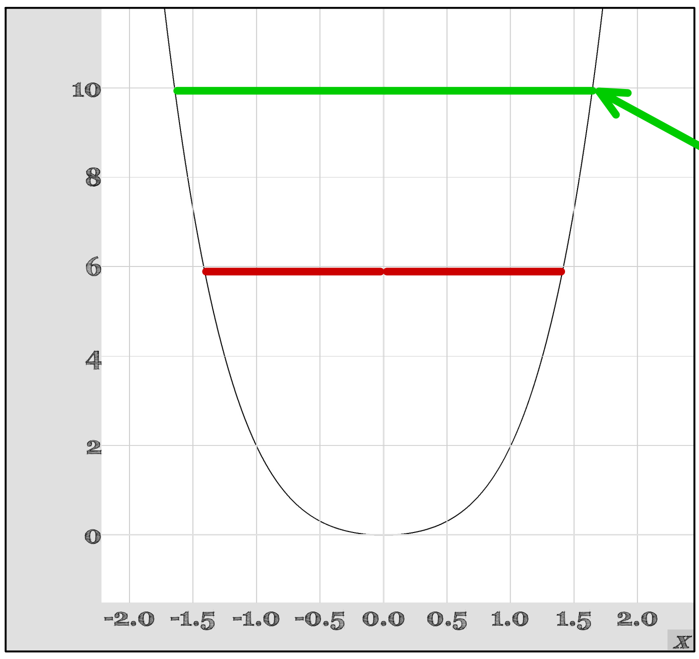
$f(-1) = (-1)^2 + (-1)^4 = 1 + 1 = 2$

$f(-x) = (-x)^2 + (-x)^4 = x^2 + x^4 = f(x)$ $\Rightarrow f(-x) = f(x)$

$(-x)^2 = (-x)(-x) = x^2$

$(-x)^4 = (-x)(-x)(-x)(-x) = x^4$

We can say that the function $f(x) = x^2 + x^4$ is even. In other words, it has y-axis symmetry.



We see the y-coordinates are the same, but the x values are negatives of each other.

When a function has terms with even exponents, the function is even b/c even exponents get rid of negatives.

$f(x) = x^6 + x^8 + x^{10}$, (6, 8, 10 each is an even number.)

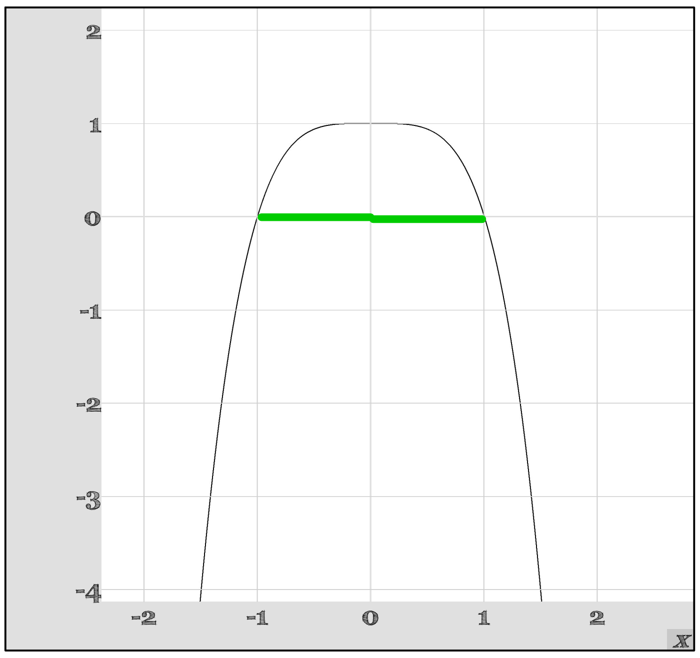
$f(-x) = x^6 + x^8 + x^{10} \Leftarrow$ Same expression produced by $f(-x)$ because each even exponent gets rids of negatives)

$f(x) = 1 - x^4$

Even or odd ?

$f(-x) = 1 - (-x)^4 = 1 - x^4 = f(x)$

$(-x)^4 = x^4$



$f(-x) = f(x)$

The graph and the check with -x indicate that f(x)= $1 - x^4$ is an even function.

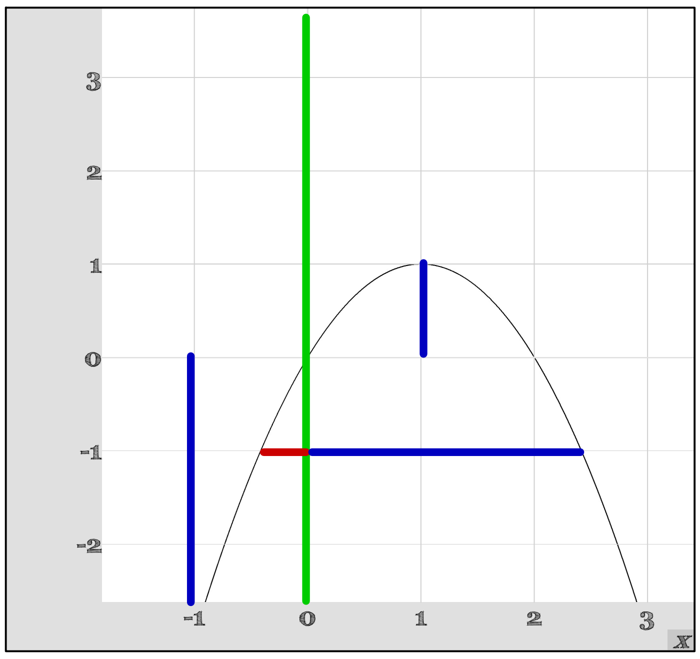
$h(x) = 2x^1 - x^2$

1. what are the exponents? 2,1

1 is an odd number, 2 is an even number.

So we have a mixture of different types of numbers in the exponents.

Is this even? no because both exponents are not even.



Def. not symmetric about the y axis because the red segment is of different length compared to the horizontal blue segment.

Does it have origin symmetry?

There is no origin symmetry.

For example, f(1)=2(1)-1^2 = $2 - 1 = 1$ so the point is $(1, 1)$

$f(-1) = 2(-1) - (-1)^2 = -2 - (1) = -3$ point is $(-1, -3)$

Notice, that when the input is negated, the output is not just -1, -3, so this function is not odd. In other words, it doesn't have origin symmetry. This function has no symmetry.