Partial Derivatives:
Partial derivatives measure rates of change with respect to one variable, while another variable(or other variables) are held constant.

notation:

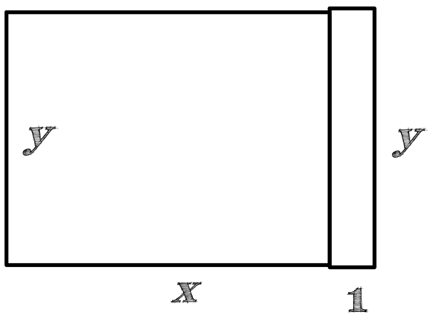
$D_x f = \frac{\partial f}{\partial x} = f_x \leftarrow$ each represents the partial of f with respect to x

$D_y f = \frac{\partial f}{\partial y} = f_y \leftarrow$ each represents the partial of f with respect to y

Defintion: $f_x(x,y) = \lim_{h\to 0} \frac{f(x+h,y)-f(x,y)}{h}$ (this says hold y constant and take limit with respect to x)

Example: $f(x,y) = xy$
find $f_x(x,y)$ by the limit:

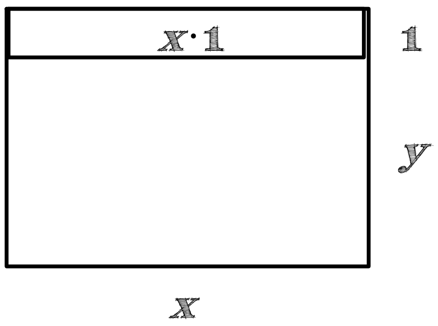
$f_x(x,y) = \lim_{h\to 0} \frac{(x+h)y - xy}{h} = \lim_{h\to 0} \frac{xy+hy-xy}{h} = \lim_{h\to 0} \frac{hy}{y} = y$ That is, $f_x(x,y) = y$



In this case, we can visualize $f_x(x,y) = y$ as saying that when we have an area f(x,y)=xy, when x increases by 1, the area increases by y.

$f_y(x,y) = \lim_{h\to 0} \frac{f(x,y+h)-f(x,y)}{h} = \frac{x(y+h)-xy}{h} = \frac{xy+xh-xy}{h} = \frac{xh}{h} = x$ so $f_y(x,y) = x$

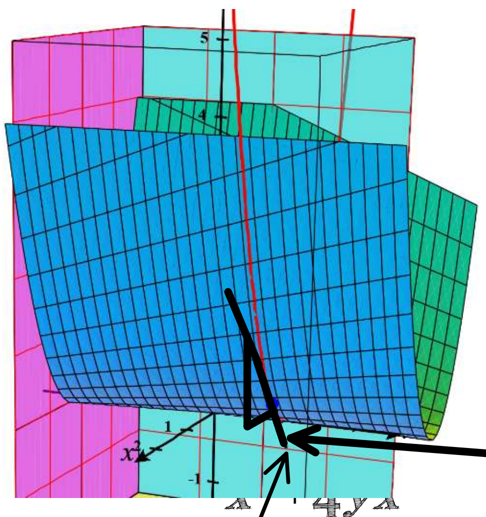
This can be interpreted to mean that when y increaes by 1, the area increases by x.

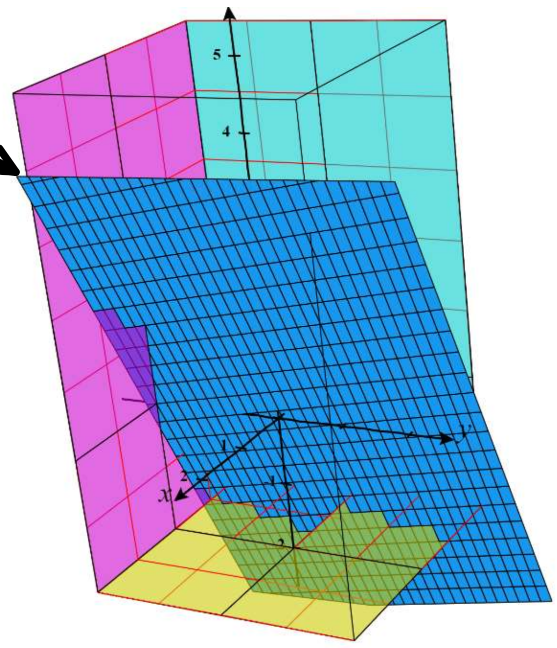


In general, when differentiating with respect to x, treat y like a constant.
Examples:

f(x,y)=x$^2 + \frac{1}{5}yx \Rightarrow \frac{\partial(x^2+4yx)}{\partial x} = \frac{\partial x^2}{\partial x} + \frac{1}{5}y\frac{\partial x}{\partial x} = 2x + \frac{y}{5}$

Just like f(x,y) is a function, f_x (x,y) is just a function.





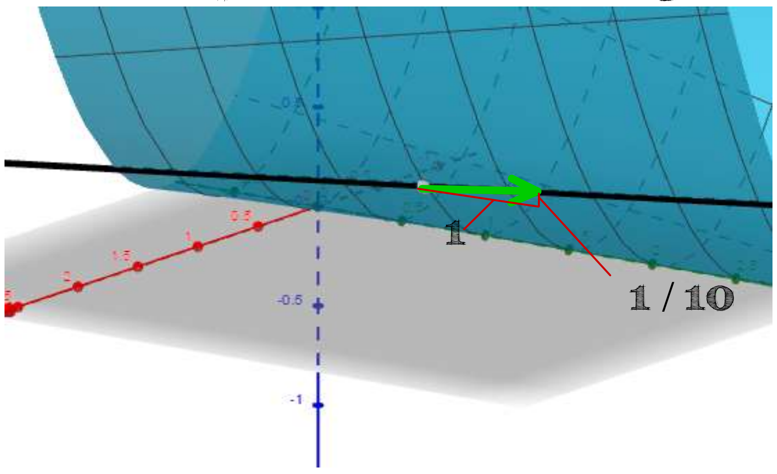
$f_x(0.5,1) =$ tells you the rate of change of f ,with respect to x, at the point (0.5,1).
It's shown as a small tangent line .
Equation of tangent line can be built as
r(t)=r_0 +tv, where v= $\langle 1, 0, 1.2\rangle$ (since $1.2/1 = 1.2$)
so we get $r(t) = \langle 0.5, 1, 0.35\rangle + t\langle 1, 0, 1.2\rangle = \langle 0.5+t, 1, 0.35+1.2t\rangle$

$f(1/2, 1) = (0.5)^2 + \frac{1}{5}\cdot\frac{1}{2}\cdot 1 = 0.35$

$f_x = 2x + \frac{y}{5}$
$f_x(0.5, 1)$ corresponds to finding the rate of change of f, with respect to x, at the point (.5,1).
f_x (.5,1)= $2\cdot\left(\frac{1}{2}\right) + \frac{1}{5}\cdot 1 = 1 + \frac{1}{5} = 1.2$

Using f(x,y)= x^2+4yz, we get $f_y(x,y)=\frac{\partial}{\partial y}\left(x^2+\frac{1}{5}yx\right)=\frac{\partial}{\partial y}x^2+\frac{1}{5}x\frac{\partial}{\partial y}y=0+\frac{1}{5}x(1)=\frac{1}{5}x$

For example, the rate of change of f, with respect to y, at the point (.5,1) is f_y (.5,1)= $\frac{1}{5}\left(\frac{1}{2}\right)=\frac{1}{10}$



0 for x since x doesn't change

tangent line is $r(t)=\langle 0.5, 1, 0.35\rangle+t\langle 0, 1, 1/10\rangle$

$=\langle 0.5, 1+1t, 0.35+1/10\,t\rangle$

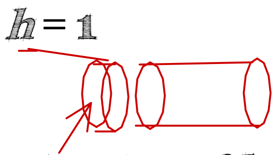
More Examples of Partial Derivatives:

$f(x,y)=x\cos(y)$

$f_x=\frac{\partial}{\partial x}x\cos(y)=\cos(y)\frac{\partial}{\partial x}x=\cos(y)\cdot 1=\cos(y)$

$f_y=\frac{\partial}{\partial y}x\cos(y)=x\frac{\partial}{\partial y}\cos(y)=x[-\sin(y)]=-x\sin(y)$

$V(r,h)=\pi r^2 h$ (volume of cylinder)



$V_r=\frac{\partial}{\partial r}(\pi r^2 h)=\pi h\frac{\partial}{\partial r}r^2=\pi h(2r)=2\pi rh$

$V_h=\frac{\partial}{\partial h}(\pi r^2 h)=\pi r^2\frac{\partial}{\partial h}h=\pi r^2(1)=\pi r^2$ (imagine volume growing by $\pi r^2\cdot 1$, *a cylinder* of height 1)

area of triangle: $A(b,h)=\frac{1}{2}bh$

$A_b=\frac{\partial}{\partial b}\left(\frac{1}{2}bh\right)=\frac{1}{2}h\frac{\partial}{\partial b}b=\frac{1}{2}h$

$A_h=\frac{\partial}{\partial h}\left(\frac{1}{2}bh\right)=\frac{1}{2}b\frac{\partial}{\partial h}h=\frac{1}{2}b$

$f(x,y)=x\sin(y)+xy^2+y\ln(x)$

$f_x(x,y)=\frac{\partial}{\partial x}[x\sin(y)+xy^2+y\ln(x)]$

$=\sin(y)\frac{\partial}{\partial x}x+y^2\frac{\partial}{\partial x}x+y\frac{\partial}{\partial x}\ln(x)$

$=\sin(y)\cdot 1+y^2\cdot 1+y\left(\frac{1}{x}\right)$

$=\sin(y)+y^2+\frac{y}{x}$

$f_y(x,y)=\frac{\partial}{\partial y}[x\sin(y)+xy^2+y\ln(x)]$

$=x\frac{\partial}{\partial y}\sin(y)+x\frac{\partial}{\partial y}y^2+\ln(x)\frac{\partial}{\partial y}y$

$=x\cos(y)+2xy+\ln(x)$

$h(x,y)=e^{xy+2x}$

$h_x=\frac{\partial}{\partial x}e^{xy+2x}=e^{xy+2x}\frac{\partial}{\partial x}[xy+2x]$

$=e^{xy+2x}\frac{\partial}{\partial x}xy+\frac{\partial}{\partial x}2x$

$=e^{xy+2x}\left(y\frac{\partial}{\partial x}x+2\frac{\partial}{\partial x}x\right)$

$=e^{xy+2x}[y\cdot 1+2\cdot 1]$

$=e^{xy+2x}[y+2]$