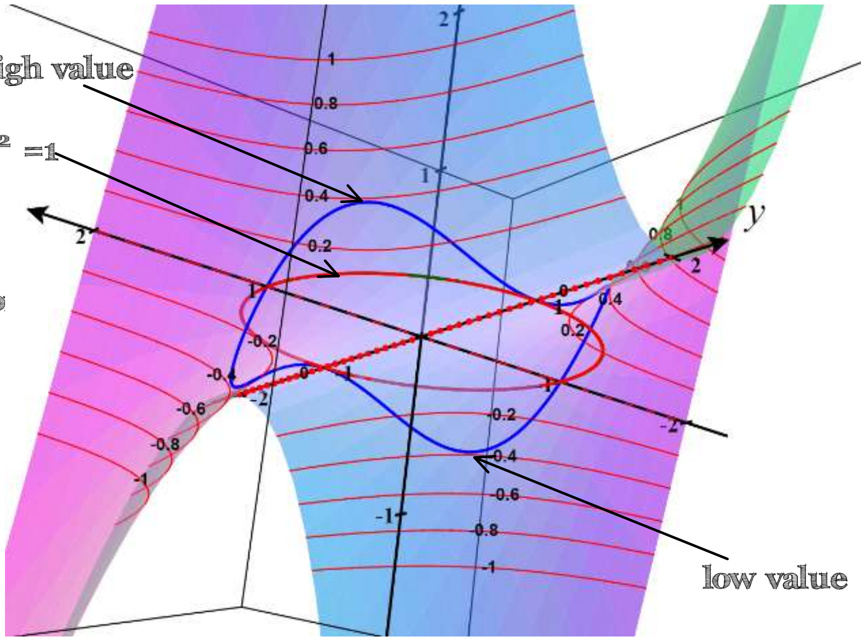
$f(x,y) = x^2 y$

$x^2 + y^2 = 1$

$g(x,y) = x^2 + y^2 - 1 = 0$

$\nabla f = \langle 2xy, x^2 \rangle, \nabla g = \langle 2x, 2y \rangle$

$\left.\begin{array}{l} 2xy = \lambda 2x \\ x^2 = \lambda 2y \end{array}\right\}$ divide these two to get rid o



$\frac{2y}{x} = \frac{x}{y} \Rightarrow 2y^2 = x^2$

$2y^2 + y^2 = 1$

$3y^2 = 1$

$y^2 = \frac{1}{3} \Rightarrow y = \pm \frac{1}{\sqrt{3}}$

$x^2 = 2 \cdot \frac{1}{3} \Rightarrow x^2 = \frac{2}{3} \Rightarrow x = \pm \sqrt{\frac{2}{3}}$

$f\left(\pm \sqrt{\frac{2}{3}}, \pm \frac{1}{\sqrt{3}}\right) = \left(\pm \sqrt{\frac{2}{3}}\right)^2 \left(\pm \frac{1}{\sqrt{3}}\right) = \frac{\pm 2}{3\sqrt{3}}$ So max is $\frac{2}{3\sqrt{3}}$ and min is $\frac{-2}{3\sqrt{3}}$

$f(x,y) = \frac{1}{x} + \frac{1}{y} \Rightarrow \nabla f = \left\langle \frac{-1}{x^2}, \frac{-1}{y^2} \right\rangle$

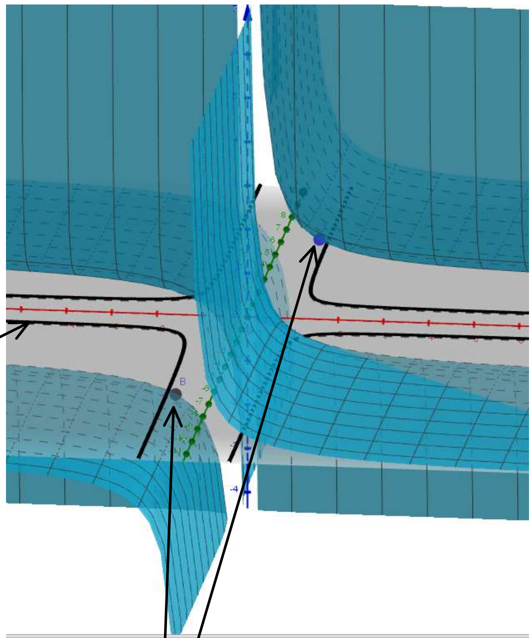
$\frac{1}{x^2} + \frac{1}{y^2} = 1 \Rightarrow g(x,y) = \frac{1}{x^2} + \frac{1}{y^2} - 1 \Rightarrow \nabla g = \left\langle \frac{-2}{x^3}, \frac{-2}{y^3} \right\rangle$

$\frac{-1}{x^2} = \lambda\left(\frac{-2}{x^3}\right), \quad \frac{-1}{y^2} = \lambda\left(\frac{-2}{y^3}\right)$

$\frac{1}{x^2} = \frac{2\lambda}{x^3}, \frac{1}{y^2} = \frac{2\lambda}{y^3}$

$1 = \frac{2\lambda}{x}, \quad 1 = \frac{2\lambda}{y}$

$\frac{1}{x^2} + \frac{1}{y^2} = 1$



$x = 2\lambda, y = 2\lambda$

so $x = y$. $\Rightarrow \frac{1}{x^2} + \frac{1}{x^2} = 1 \Rightarrow \frac{2}{x^2} = 1 \Rightarrow 2 = x^2 \Rightarrow \pm\sqrt{2} = x$

$\Rightarrow$ since y=x, we get $y = \pm\sqrt{2}$

So we get $f(\pm\sqrt{2}, \pm\sqrt{2}) = \pm \frac{1}{\sqrt{2}} \pm \frac{1}{\sqrt{2}}$

$max: \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}}$

$min: \frac{-1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = -\frac{2}{\sqrt{2}}$