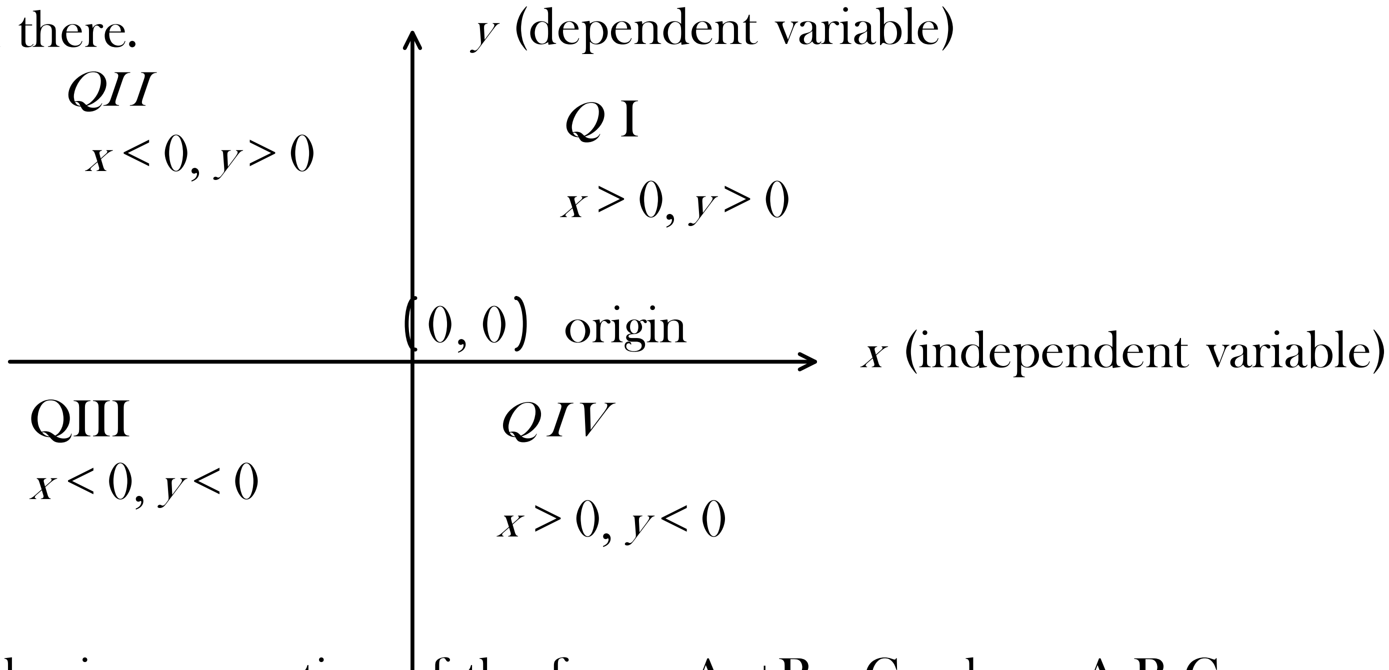
Instructor: Tom

Welcome to Class!
Section 1.1 in Finite Math Book
Read the syllabus in Canvas.
Make an account in MyOpenMath according to the syllabus.
You will do all your homework there.

basics of graphing;

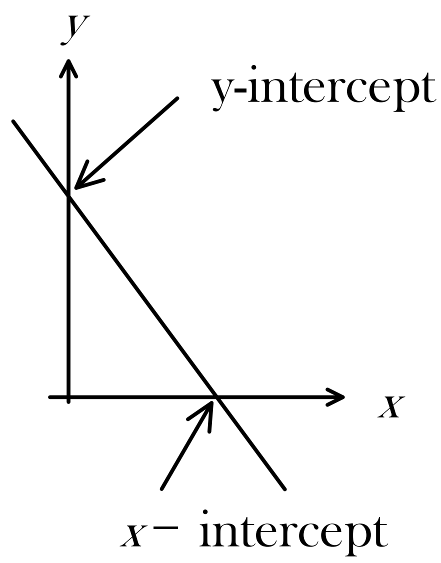


A linear equation in two variables is an equation of the form: Ax+By=C, where A,B,C are real numbers and A and B are not both zero.

examples: $3x-5y-6=0 \Rightarrow$ could write this as $3x-5y=6,\ A=3,\ B=-5,\ C=6$

$-3x=2y-1$ could transform into $-3x-2y=-1$, A=-3, B=-2, C=-1

$y=\frac{3}{4}x-5 \Rightarrow$ can make into $\frac{-3}{4}x+1y=-5,\ A=\frac{-3}{4},\ B=1,\ C=-5$



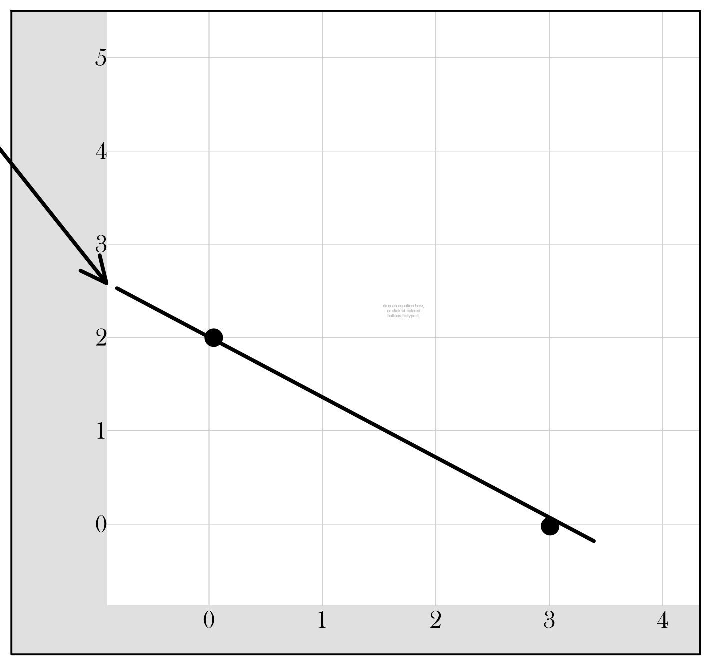
$x=4$ could be written as $1x+0y=4$, A=1,B=0,C=4

Intercepts:
Where a graph crosses the x-axis we get x-intercepts.
Where a graph crosses the y-axis we get y-intercepts.
Begin with Ax+By=C

x intercept set y=0: Ax+B(0)=C $\rightarrow Ax=C \rightarrow x=\frac{C}{A}$

y intercept set x=0: A(0)+By=C $\rightarrow By=C \rightarrow y=\frac{C}{B}$

specific example:
2x+3y=6
x=0 for y intercept: $2(0)+3(y)=6 \rightarrow 3y=6 \rightarrow y=2$
$y=0$ for x intercept: 2 x+3(0)=6 $\rightarrow 2x=6 \rightarrow x=3$



example 2: $y=2x+5$

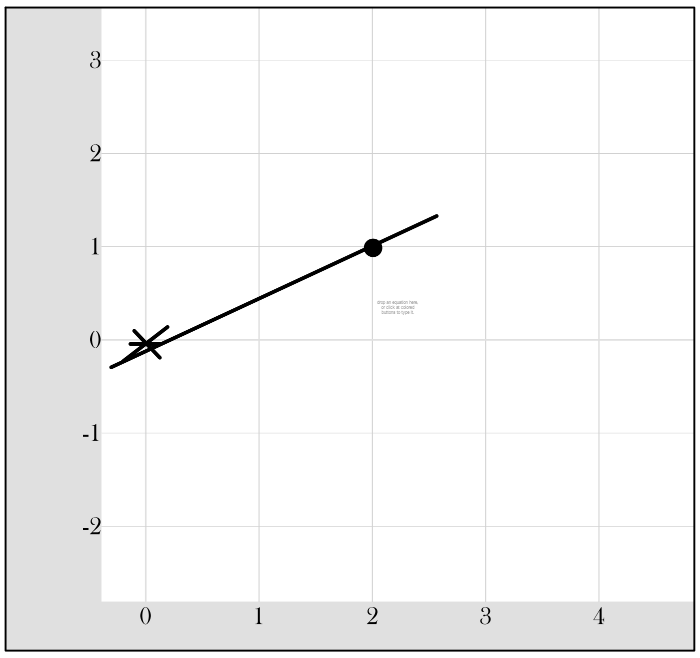
graph by intercepts:
$-2x+y=5$ subtract 2x from both sides
y=0: $-2x+0=5 \rightarrow -2x=5 \rightarrow x=-5/2$
$x=0: -2(0)+y=5 \rightarrow y=5$

$-x+2y=0$

graph this:

$y=0: -x+2(0)=0 \rightarrow -x=0 \rightarrow$ divide by -1 $\rightarrow x=0$

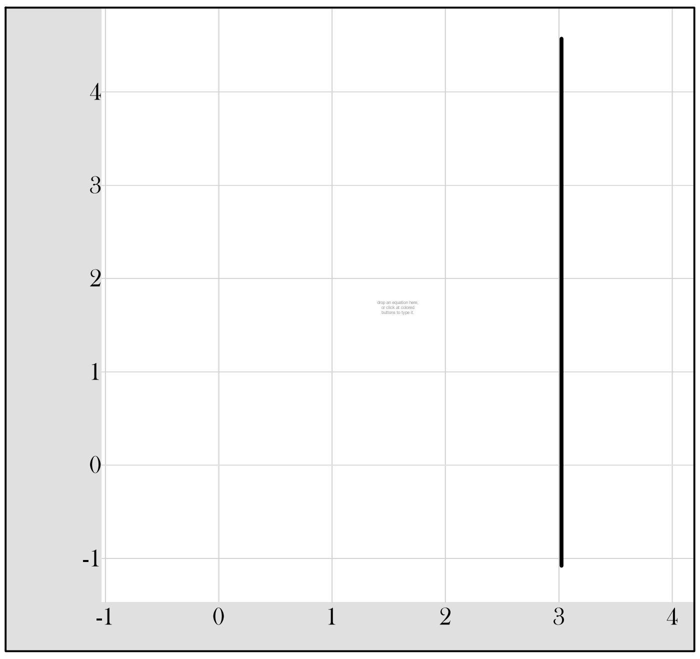
$x=0: -0+2y=0 \rightarrow 2y=0 \rightarrow y=0$



choose another value of x:
x=1:
$-1+2y=0$ could work but would be messy
solve for y
$x=2$:
$-2+2y=0$
solve for y: $2y=2$
divide by 2: $y=1$

graphing a vertical line:
$x=3$:
here y is not present, so its value can be anything.
x=3 is a line and not a dot.
(3, -1), (3,0), (3,2) same x, but the y changes.

$x=$ number is a vertical line b/c
y is not given, it can be anything.



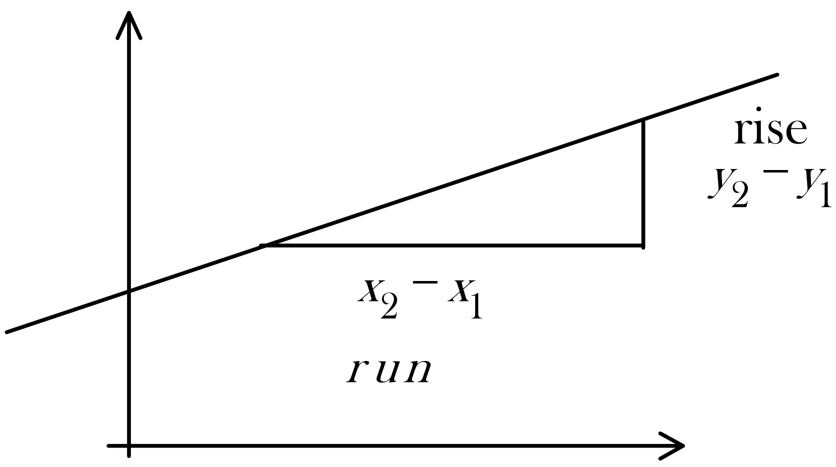
Find equation for the vertical line containing
the point (-1,6): x=-1

Finding the Slope of a Line and Interpreting It:

Slope measures steepness of lines.
Big slope means very steep , and small slope means not very steep.
Slope can be <0, =0 or >0.
Important Formula for Slope:
Given two points:(x_1, y_1), (x_2, y_2)

Slope formula= $\frac{y_2-y_1}{x_2-x_1}$, $x_2 \neq x_1$ because then we would have 0 in the bottom, which is not allowed.

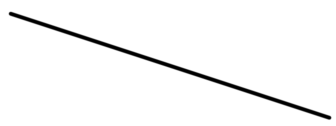
Notice if $y_2=y_1$, then the top is 0, so we have a horizontal line.



slope= $\frac{\text{change in y}}{\text{change in x}} = \frac{\Delta y}{\Delta x}$

Δx read as delta x
it's not $\Delta \cdot x$, it's one symbol Δx

rise can also mean fall
since the line can do this:



For slope, we use m= $\frac{\Delta y}{\Delta x}$ = in applications this is called the "average rate of change"

it's the average rate of change of y with respect to x.
Reason is that as x changes, y changes in some fashion.

Example:

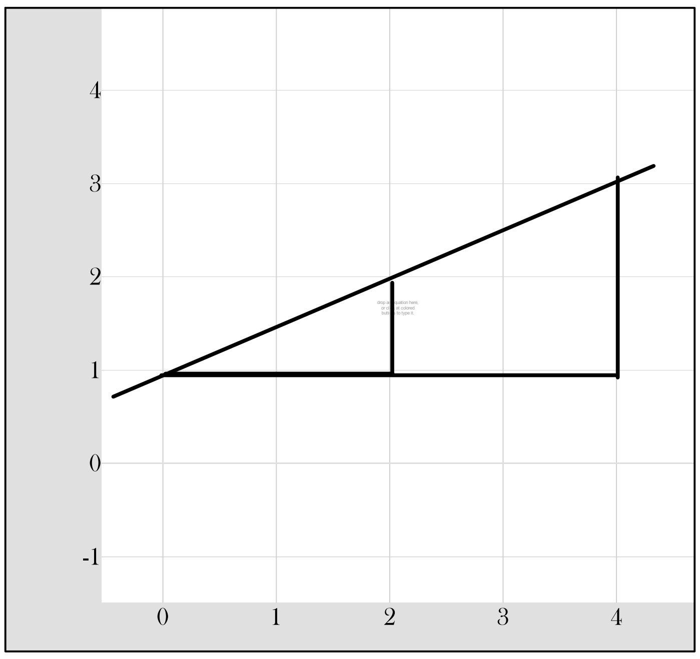
$x^{super\ script}_{subscript}$

Find the slope of the line with points (1,5) and (3,-2):

apply the formula: $m = \frac{\Delta y}{\Delta x} = \frac{5-(-2)}{1-3} = \frac{5+2}{-2} = \frac{7}{-2} = -\frac{7}{2}$ don't do this: $-\frac{5}{2} = \frac{-5}{-2}$ not allowed

$m = \frac{-2-5}{3-1} = \frac{-7}{2}$ notice it's the same

Slope is a ratio, so it can be found using any two points and the value should be the same.

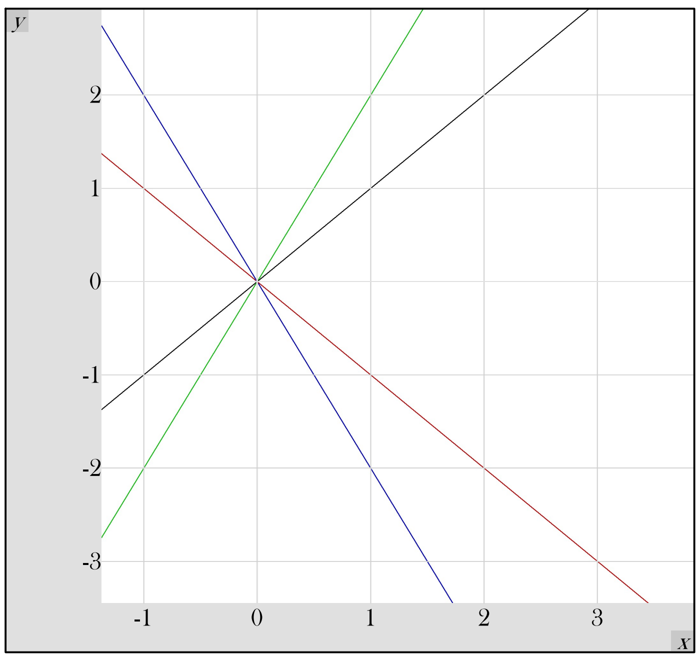


divide $\frac{2}{4} = \frac{1}{2}$

line rises from left to right, so m>0

$m = 0$ since line stays horizontal

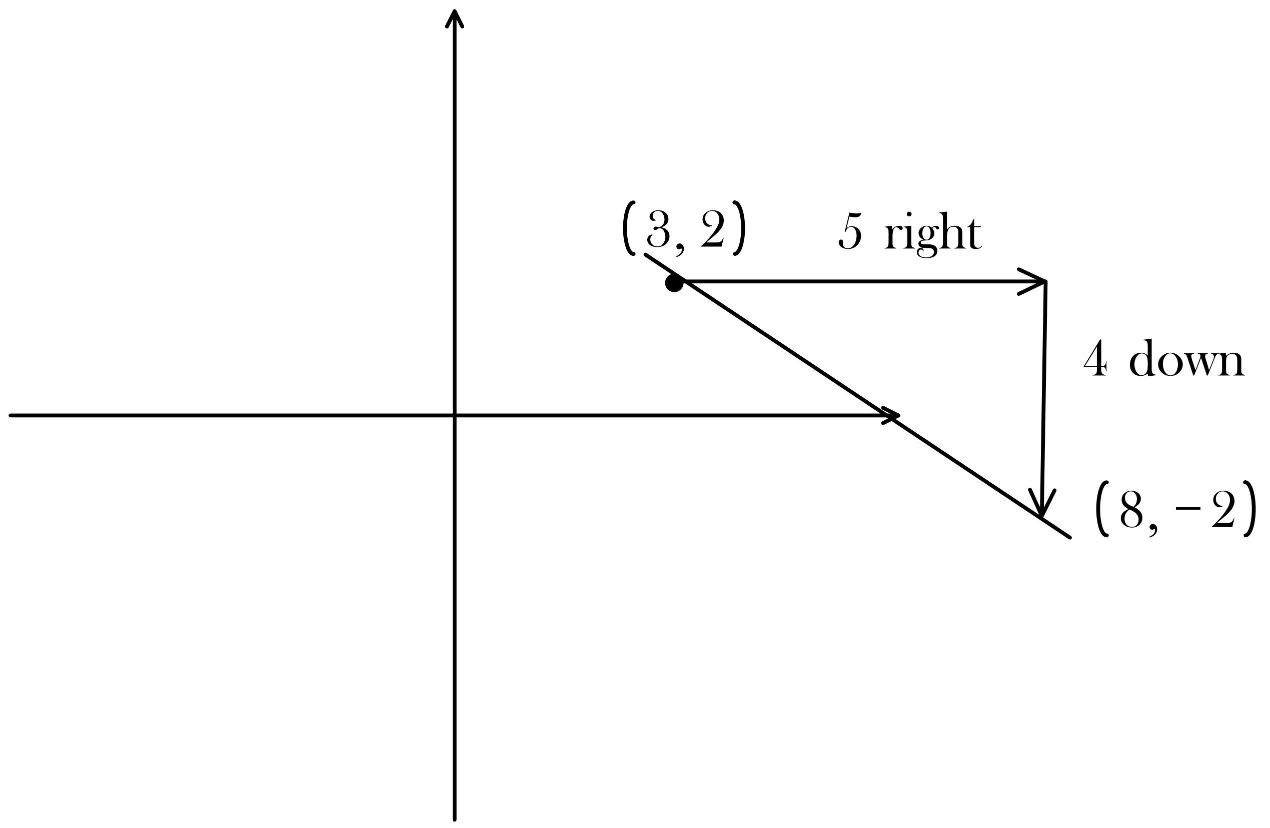
line falls from left to right, so m<0



impact of changing m in y=mx
m>0 and as m increases, the steepness increases.
m=0 line is horizontal
m<0, and as m gets more negative, the line falls ever more quickly.

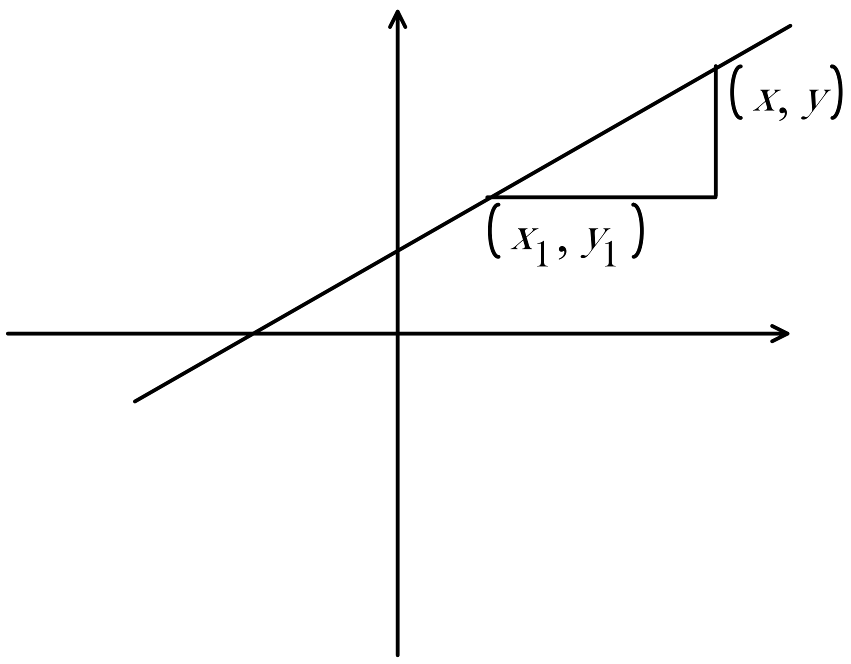
draw a graph of the line with (3,2) with slope -4/5:

begin at the point (3,2) and follow the slope: $\frac{-4}{5} = \frac{4\,down}{5 \text{ to the right}}$



notice we get a second point on the line:
$3+5=8$
$2+(-4)=-2$

Point-Slope Form of a Line:



$m = \frac{y-y_1}{x-x_1}$

multiply by $x-x_1$

$m(x-x_1) = \frac{(y-y_1)}{\cancel{x-x_1}}\cancel{(x-x_1)}$

$m(x-x_1) = y-y_1$

point slope form since it needs m, the slope, and (x_1, y_1) , a specific point.

A line has slope 4 and contains the point (1,2). Write the equation of the line as Ax+By=C.

$y-y_1 = m(x-x_1)$

$x_1 = 1,\ \ y_1 = 2, m = 4$

plug into formula:

$y-2 = 4(x-1)$

now transform using basic operations into the form Ax+By=C

$y-2 = 4x-4$ distribute the 4

$-4x+y-2 = -4$ move 4x to the left with -

$-4x+y = -4+2$ add 2

-4x+y=-2 often here people will divide a negative out

$\frac{-4x}{-1} + \frac{y}{-1} = \frac{-2}{-1}$

$4x-y=2$ general equation $Ax+By=C$ form

Find an equation of the line containing the points (2,3) and (-4,5):
We don't have the slope, but we have two points, so slope can be found.

$$m=\frac{5-3}{-4-2}=\frac{2}{-6}=\frac{1}{-3}=-\frac{1}{3}$$

equation of line can be found now from slope and one of the points (use (2, 3))

$y-3=\frac{-1}{3}(x-2)$ sometimes people take this form

Let's transform into Ax+By=C form:

clear the fraction: multiply by 3: $3(y-3)=3\left(\frac{-1}{3}\right)(x-2)$

$3y-9=-1(x-2)$

distribute the -1, changing signs inside

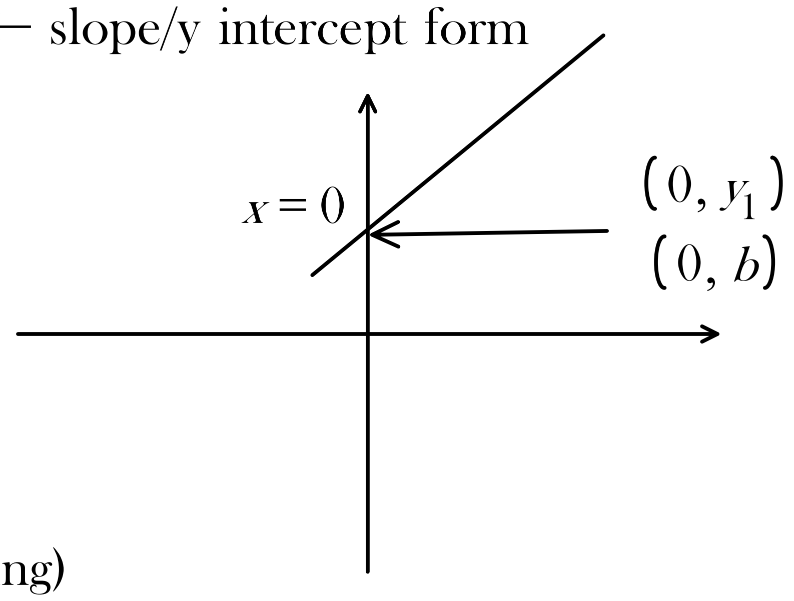
$3y-9=-x+2$

$x+3y-9=2$ move x to the left with addition

$x+3y=2+9$ add 9

x+3y=11 this is the Ax+By=C form

Making the form y-y_1 =m(x-x_1) into the form y=mx+b← slope/y intercept form



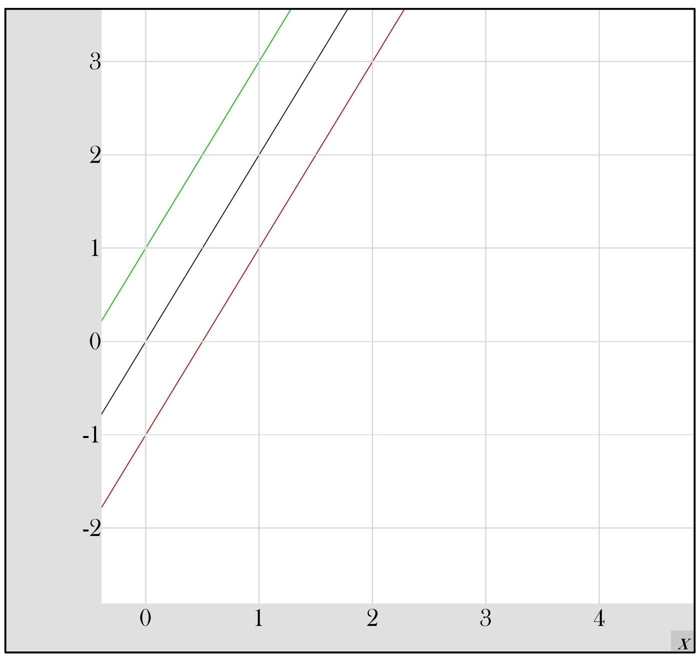
$x=0$ and plug it in:

$y-b=m(x-0)$

$y-b=mx$

$y=mx+b$

let's study the impact of changing the value of b:
y=2x+b, notice m=2 (made up to be 2, but b is changing)



y=2x+b, changing b moves the line up or down along the y-axis.

doens't matter whether we write
y=mx+b
or y=b+mx

Find the slope m and y-intercept (0,b) of the line
2x+4y=8

$2x+4y=8$

we can begin by dividing out 2 because 2 is a common factor of every constant. $\frac{2x}{2}+\frac{4y}{2}=\frac{8}{2}$

$x+2y=4$

$2y=4-x$ subtract x

$y=\frac{4-x}{2}$ divide by 2

y= $\frac{4}{2}-\frac{x}{2}$ split fraction b/c it's only one number in the bottom

$y = 2 - \frac{1}{2}x,\ m = \frac{-1}{2},\ b = 2$

another way:

$2x + 4y = 8$

we always want 1y:

divide by 4:

$\frac{2x}{4} + \frac{4y}{4} = \frac{8}{4}$

$\frac{1}{2}x + y = 2$

move 1/2x over: $y = 2 - \frac{1}{2}x$ same result as above