Math 116 Notes 9/22/2021
questions from co-req 3

$$
\left(-u^{2}+5 u-6\right)+\left(4 u^{2}+2 u+7\right)
$$

Rewrite without-parentheses.

$$
\underbrace{}_{-5 w^{6}\left(2 w^{3}+6 w^{2}-7 w\right)}
$$

combine like terms, like terms have the
same variable part, we're only, so drop parenthesis
$-u^{2}+5 u-6+4 u^{2}+2 u+7$
regroup: $-1 u^{2}+4 u^{2}+5 u+2 u-6+7$
combine: $(-1+4) u^{2}+(5+2) u+1$
simplify: $3 u^{2}+7 u+1$ answer
fast way: add up like terms mentally

$$
\begin{aligned}
\text { distributes }-5 w^{6}: & -5 w^{6} \cdot 2 w^{3}-5 w^{6} \cdot 6 w^{2}-5 w^{6} \cdot\left(-7{ }_{W}\right) \\
\text { regroups: } & -5 \cdot 2 w^{6} w^{3}-5 \cdot 6 w^{6} w^{2}-5(-7) w^{6}{ }_{W} \\
& -10 w^{9}-30 w^{8}+35 w^{7} \text { answer }
\end{aligned}
$$

Multiply.

$$
(w+2)(w-4)
$$

Simplify your answer.
$(w+2)(w-4)$
first distribute $(w-4): w(w-4)+2(w-4)$ distribute $w, 2: w^{\cdot} w^{-}-4 \cdot w^{+}+2 \cdot w^{+} 2(-4)$ multiply: $w^{2}-4 w^{+}+2 w^{-8}$
combine middle terms: $W^{2}-2 W^{-}-8$
Usually we use FOIL b/c it's faster. $\mathrm{F}=$ product of first terms: $\sqrt{x}+a)(\boxed{x}-b): x_{x}=x^{2}$ $O=$ product of outer terms: $(\sqrt[x]{ }+a)(x \boxed{-b}):-b x$ $I=$ product of inner terms: (x+a) ( ( ) b): $a x$ $L=$ product of last terms: $(x+a)(x \boxed{b})=-a b$ $(\underset{x+a) \cdot(x-b}{x})=x^{2}-b x^{+} a x^{-} a b$
$\left(w^{+2)\left(w^{-}-4\right.}\right)=\mathrm{FOIL}=\mathrm{w}^{2}-4 w^{+}+2 w^{-8}$ $=W^{2}-2 W^{-} 8$ same as above
$(x+a)$ is called a
binomial $\mathrm{b} / \mathrm{c}$ it has two terms terms are expressions combined through addition or subtraction
notice parenthesis are gone no like terms are present $\mathrm{b} / \mathrm{c}$ $w^{9}, w^{8}, w^{7}$ are all different

FOIL: $4 x \cdot x+4 x(7 y)-5 y(x)-5 y(7 y)$ multiply out: $4 x^{2}+28 x y-5 x y-35 y^{2}$
combine middle terms : $\left.\begin{array}{l}4 x^{2}+23 x y^{-}-35 y^{2} \\ \text { answer }\end{array}\right]$
b/c each one has $x y$
note : $\mathrm{xy}=\mathrm{yx}$
$2 \cdot 3=3 \cdot 2=6$
product of two binomials,so use FOIL to multiply out

Multiply.
$(3 y-1)(7 y+3)$
Simplify your answer.
$\rightarrow$ multiply out
$=21 y^{2}+9 y-7 y-3$
$\rightarrow$ multiply out
$=21 y^{2}+9 y^{-} 7 y^{-} 3$
product of two binomials
so FOIL applies

$$
(3 y-1)(7 y+3) \rightarrow \text { FOIL }
$$

$$
\begin{equation*}
=3 y \cdot 7 y+3 y(3)+(-1)(7 y)+(-1)(3) \tag{3}
\end{equation*}
$$

$\rightarrow$ combine middle terms $\mathrm{b} / \mathrm{c}$ y is same
$=21 v^{2}+2 y-3$ answer trinomial
Multiply.

$$
(u-3)(u+3)
$$

Simplify your answer.
product of two binomials,so
FOIL applies
(u-3) is one binomial,
$2(\mathrm{u}+3)$ is the other binomial
$21 y^{2}+2 y-3$ answer
$3) \rightarrow$ FOIL $\rightarrow u^{2}+3^{2 .} u^{3 .} 3^{4}-3^{4}(3)$
$\rightarrow$ multiply $\rightarrow u^{2}+\not \mathscr{u}-3 /\langle-9$
cancell $3 u$ and $-3 u\left\{\begin{array}{l}u^{2}-9 \\ \text { answer }\end{array}\right.$
here, we have a binomial as the product $\mathrm{b} / \mathrm{c}$ the middle terms cancel each other

Multiply.

$$
(5 z+2 y)(5 z-2 y)
$$

Simplify your answer.
1 notice 2 y and -2 y are the same except for the sign
so middle terms will cancel
2. product of two binomials so FOIL applies
3. binomial $1=5 \mathrm{z}+2 \mathrm{y}$, binomial $2=5 \mathrm{z}-2 \mathrm{y}$
now FOIL out:
$(5 z+2 y)(5 z-2 y)$ note: $2 y(5 z)=2 \cdot 5 \cdot y \cdot z=10 y z=10 z y$
$=5 z \cdot 5 z+5 z(-2 y)+2 y(5 z)+2 y(-2 y)$
$\begin{array}{lllll}F & O & I & L\end{array}$
$=25 z^{2}-10 z y+10 \angle y-4 y^{2}$
$=25 z^{2}-4 y^{2}$
answer
notice answer is a binomial $\mathrm{b} / \mathrm{c}$ middle terms cancel
Notes should have the following: (and so on for each week) week 1: objective 1 notes, co-req 1 notes , tutoring 1 week 2: objective 2 notes, co-req 2 notes, tutoring 2 week 3: objective 3 notes, co-req 3 notes, tutoring 3

## q11

Find the greatest common factor of $11 \mathrm{~m}^{4}$ and $5 \mathrm{~m}^{2}$.
factor $11 \mathrm{~m}^{4}=H \cdot m \cdot m \cdot m i \cdot m T \quad$ based on these,
factor $\left.5 \mathrm{~m}^{2}=5 \cdot \mathrm{~m} \cdot \mathrm{~m} \quad\right\}$ the $\mathrm{GC} \mathrm{F}=\mathrm{m} \cdot \mathrm{m}=\mathrm{m}^{2}$
Find the greatest common factor of $15 n^{3}$ and $10 n^{4}$.
factor $15 \mathrm{n}^{3}=8 \cdot 5 \cdot n \square \square$

now highlight common pieces
$\mathrm{GCF}=5 \cdot \mathrm{n} \cdot \mathrm{n} \cdot \mathrm{n}=5 n^{3}$ answer
take each matched factor once only

## Factor.

$$
3 w+15
$$

factoring: $6=2 \cdot 3$
factor means write as a product
$2 x=2 \cdot x$ product
$10 x=2 \cdot 5 \cdot x$ product
$4 w^{2}=2 \cdot 2 w^{\cdot} w$ product
$2(x-3)=$ distribute $2=2 x-6$
imagine we want to reverse the distributve property
$4 w^{+} 8=4 . w^{+} 4$
put 4 outside
$=4(\sqrt{w+2})$
$=4\left(w^{+}+2\right)$ answer
$3 w^{+} 15=3 \cdot w^{+} 5 \cdot 3$

$$
=3\left(w^{+} 5\right) \text { answer }
$$

notice it shows 3 times ( $\mathrm{w}+5$ )

$$
\text { Factor } 15 y^{2}-12 y^{3}
$$

identify the GCF
$3 \cdot 5 \cdot \sqrt{y}-4 \cdot \sqrt{y} \cdot y \cdot y \quad$ factor each
put these outside parenthesis
$3 y y(5-4 y)$
$5-4 y$ is not in the red boxes
$3 y^{2}(5-4 y)$ answer
notice it's $3 y^{2}$ times (5-4y)

Factor $6 c^{2}+4 c$. check: $2 c\left(3 \underset{\text { distribution }}{c+2)=6 c^{2}}+4 c\right.$ factoring $2 \cdot 3 \cdot c \cdot c+2 \cdot 2 \cdot c$ factor each expression first so the GCF is 2 c
$6 c^{2}+4 c=2 \cdot 3 \cdot c \cdot c+2 \cdot 2 \cdot c=3 c(2 c)+2(2 c)=2 c(3 c+2)$ answer

Factoring is the reverse of the distributive property. Or the distributive property is the reverse of factoring.

$$
x^{2}+x+2 x+2 \text { given this has } 4 \text { terms }
$$

factor this into $(x+a)(x+b)$ expression of this form product of two binomials
$\left(x^{2}+x\right)+(2 x+2) \quad$ what's the GCF of $\mathrm{x}^{2}+\mathrm{x} \cdot 1$ ? it's x , so we get $x \cdot x$
$\left(x^{\cdot} \cdot x+x \cdot 1\right)+(2 \cdot x+2 \cdot 1)$
$x(x+1)+2(x+1)$
$(x+1)(x+2)$ or $(x+2)(x+1)$ either one is the answer
example 2: $4 x^{2}+4 x+8 x+8$
$\left(4 x^{2}+4 x\right)+(8 x+8)$
$(4 \cdot x \cdot x+4 x \cdot 1)+(8 \cdot x+8 \cdot 1)$
$4 x(x+1)+8(x+1)$
$(4 x+8)(x+1)$
$(4 \cdot x+4 \cdot 2)(x+1) \quad$ write 4 x as $4 \cdot \mathrm{x}, 8$ as $4 \cdot 2$
$4(x+2)(x+1)$
answer
group $4 x^{2}+4 x$, group $8 x+8 \mathrm{~b} / \mathrm{c}$ they have common factors factor each term inside each parenthesis
pull 4 x out, pull 8 out
notice $4 x+8$ have a common factor of 4
since 4 is common, outside parenthesis
$\mathrm{x}+2$ no common factors, $\mathrm{x}+1$ no common factors, so stop

Factor by grouping.

$$
\begin{aligned}
& 5 x^{3}+4 x^{2}+25 x+20=\left(5 x^{3}+4 x^{2}\right)+(25 x+20) \quad \text { grouped } \\
&=\left(5 \cdot \sqrt[x]{x} \cdot x x^{+2} \cdot 2 \cdot x \cdot x\right)+(5 \cdot 5 x+5 \cdot 4) \\
&=\left(5 x^{2} x^{2}+4 \cdot x^{2}\right)+(5 \cdot 5 x+5 \cdot 4) \\
&=x^{2}\left(5 x^{2}+4\right)+5(5 x+4) \quad \text { pull } x^{2} \text { out, pull } 5 \text { out } \\
&=\left(x^{2}+5\right)(5 x+4) \quad \begin{array}{l}
\text { pull }(5 x+4) \text { out } \\
\text { the answer }
\end{array} \\
& \begin{array}{l}
\text { notice } x^{2}+5 \text { no common factor left } \\
5 x+4 \text { no common factor left } \\
\text { so stop }
\end{array} \\
& \hline
\end{aligned}
$$

Factor by grouping.

$$
\begin{array}{rlr}
4 y^{3}+7 y^{2}+20 y+35 & =\left(4 y^{3}+7 y^{2}\right)+(20 y+35) \quad \text { group } \\
& =(4 \cdot y \cdot \sqrt{y \cdot y}+7 \boxed{y y})+(5 \cdot 4 y+5 \cdot 7) \\
& =(4,7 \text { no common factor } \\
& =y^{2}\left(4 y^{2}+7 y^{2}\right)+\left(5 \cdot 4 y^{+5 \cdot 7)+5(4 y+7) \quad \text { regrouped yyy as } \mathrm{y} \cdot \mathrm{y}^{2}} \quad \text { pull }{ }^{2} \text { out, pull } 5\right. \text { out } \\
& =\left(\mathrm{y}^{2}+5\right)(4 y+7) \text { answer } \quad \text { pull }(4 \mathrm{y}+7) \text { out }
\end{array}
$$

separate example: (in case you have trouble understanding how to factor a binomial out) $y(x+5)+6(x+5)$, set $\mathrm{a}=\mathrm{x}+5 \mathrm{~b} / \mathrm{c}$ it's the same in both terms ya +6 a now we have an expression with a only, might be easier to see how to pull a out $(y+6)$ a pull a out
remember $\mathrm{a}^{=} \mathrm{x}+5$, so replace back a back with $(\mathrm{x}+5):(y+6)(x+5)$ answer

