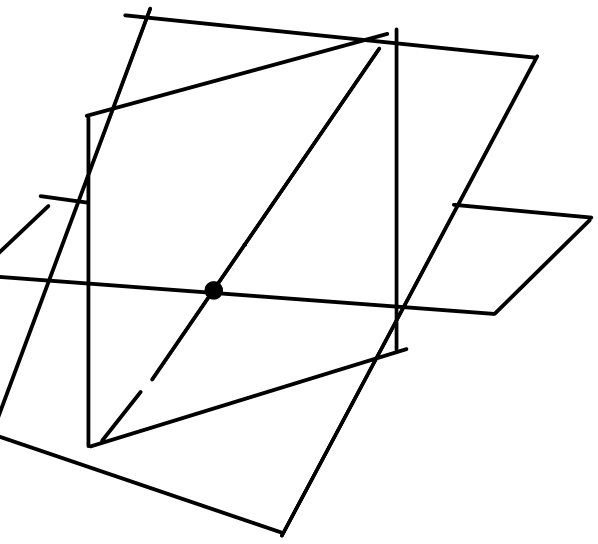
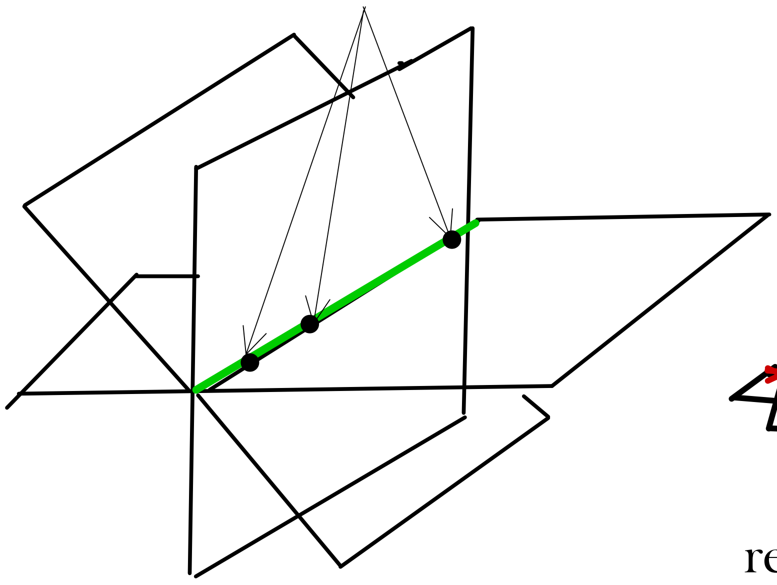
Solving Systems of Three Linear Equations:
9/21/2021 (Part of section 2.1 in Sullivan's book)
When we have a system of three linear equations, each equation is usually a plane.

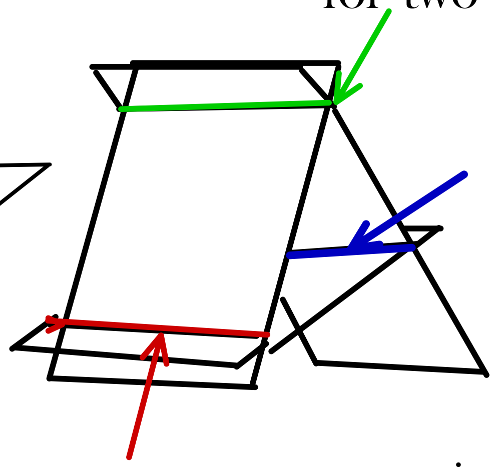


three possible points



this line works
for two of the planes

these work
for two
but again
not the third



red line works
as a solution for
two of the equations
but not the third

inconsistent
system,
no solution

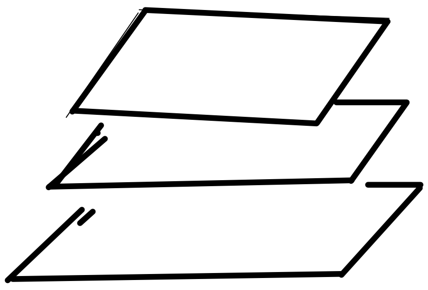
a system of three equations
has exactly one solution point
this is called a consistent
system

computer drawn pictures
are on page 59 of sullivan

infinite number of
solutions b/c the
intersection line between
the planes is infinite

the three planes
are really the same
plane

you can also have three parallel
planes, so no solution again



Solving a system of three equation in three unknowns:

$$\begin{cases} x+y-z=-1 \\ 4x-3y+2z=16 \\ 2x-2y-3z=5 \end{cases}$$

geometrically, this is a system of three equations, each of which represents a plane

How do we find the point (x,y,z) that solves the system?
To solve the system means to make each equation true with the point (x,y,z).

multiply the top by -4 and add to the second equation to get a NEW second equation.

$-4(x+y-z=-1) \Rightarrow \cancel{-4x}-4y+4z=4$

middle equatio: $\cancel{4x}-3y+2z=16$ add down

$-7y+6z=20$ this is the new middle equation

equivalent system:

$$\begin{cases} x+y-z=-1 \\ -7y+6z=20 \\ 2x-2y-3z=5 \end{cases}$$

multiply top equation by -2 and add that to the bottom equation:

$-2(x+y-z=-1) \Rightarrow \cancel{-2x}-2y+2z=2$

bottom equation: $\cancel{2x}-2y-3z=5$

$-4y-z=7$ this is the new bottom equation

new equivalnet system:

$$\begin{cases} x+y-z=-1 \\ -7y+6z=20 \\ -4y-z=7 \end{cases}$$

Notice that the bottom two equations form their own little subsystem:

$$-7y+6z=20$$
$$-4y-z=7$$

Note for this subsystem we can find y and z b/c we have two equations and two variables.

$-7y+6z=20$ say we want to get rid of z

$-4y-z=7 \leftarrow$ multiply this by 6:

$6(-4y-z=7)$

$-24y-6z=42$

top stays as $-7y+\cancel{6z}=20$

$-24y-\cancel{6z}=42$

now add the two

$-31y=62$

final equivalent system:

$$\begin{cases} x+y-z=-1 \\ -7y+6z=20 \\ -31y=62 \end{cases}$$

from bottom equation of final system:

$-31y=62$

$y=-2$

from middle equation of final system, we get $-7(-2)+6z=20$

$14+6z=20$

$6z=20-14$

$6z=6$

$z=1$

now that we know y=-2, z=1, plug into top equation of final system:

$x-2-1=-1$

$x-3=-1$

$x=-1+3=2$

So the solution point is the point $(2,-2,1)$ This makes the top equation, middle equation and bottom eqution all true.

Solve the system $\begin{cases} -x-3y-2z=22 \\ -3x-3y-2z=30 \\ -2x+y-z=7 \end{cases}$

Our goal is to produce equivalent systems along the way. $\begin{cases} \text{top equation with three variables} \\ \text{middle with two variables} \\ \text{bottom with one variable} \end{cases}$

1. we want to get rid of -3x in middle equation: so multiply top equation by -3 and add to middle equation: $-3(-x-3y-2z=22) \Rightarrow \cancel{3x}+9y+6z=-66$

copy the middle equation: $-\cancel{3x}-3y-2z=30$

$6y+4z=-36$ new middle equation

new equivalent system:

$$\begin{cases} -x-3y-2z=22 \\ 6y+4z=-36 \\ -2x+y-z=7 \end{cases}$$

2. Let's eliminate -2x from bottom equation: multiply top equation by -2 and add to bottom equation: $-2(-x-3y-2z=22) \Rightarrow \cancel{2x}+6y+4z=-44$

add the bottom equation: $-\cancel{2x}+y-z=7$

$7y+3z=-37$

new equivlanet system:

$$\begin{cases} -x-3y-2z=22 \\ 6y+4z=-36 \\ 7y+3z=-37 \end{cases}$$

3. Now solve the system formed by the middle and bottom equations:

$6y+4z=-36$ get rid of z: LCM of 4 and 3 is 12

$7y+3z=-37$ we have 4 and 3 as coefficients on z

one has to be positive and one negative

multiply top equation by -3: $-3(6y+4z=-36) \Rightarrow -18y-\cancel{12z}=108$

multiply bottom equation by 4: $4(7y+3z=-37) \Rightarrow \underline{28y+\cancel{12z}=-148}$ add down

$10y=-40$

We have the final equivalnet system:

$$\begin{cases} -x-3y-2z=22 \\ 6y+4z=-36 \\ 10y \quad =-40 \end{cases}$$

From here, $10y=-40$ gives y=-4

Replace y with -4 in middle: $6(-4)+4z=-36$

$-24+4z=-36$

$4z=-36+24$

$4z=-12$

$z=-3$

In top equation , replace y with -4, z with -3 and solve for x:

$-x-3(-4)-2(-3)=22$

$-x+12+6=22$

$-x+18=22$

$-x=22-18$

$-x=4$

$x=-4$

So the solution point is the point $(-4,-4,-3)$

Supply and Demand Example:

For a particular product, the supply equation is y=9x+520 (y is the number of items, x is price)

and the demand equation is

$y=-7x+920$

What is the point of intersection:

Since each is already solved for y, equate the expressions with x:

$9x+520=-7x+920$

$9x+7x=920-520$

$16x=400$

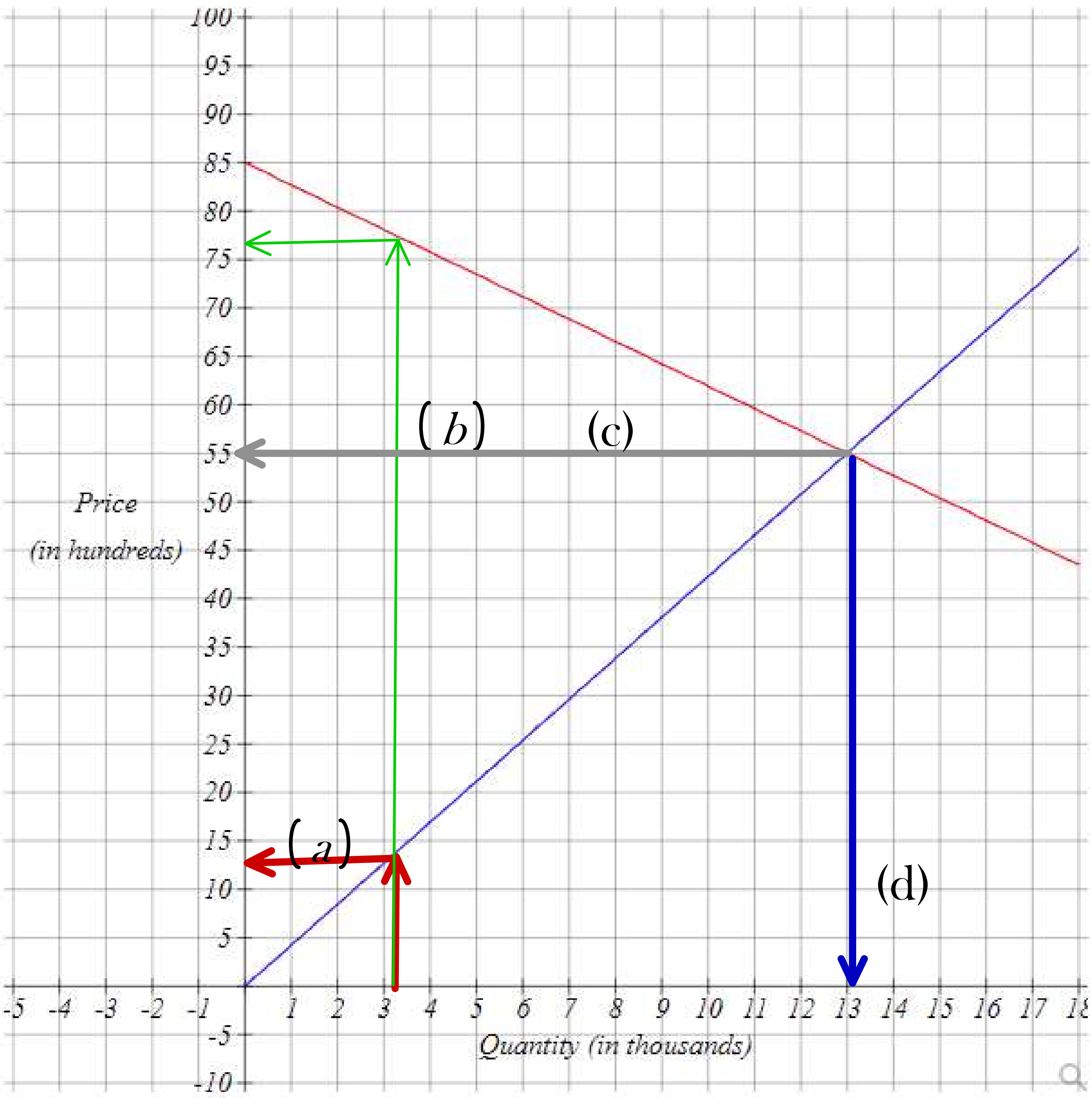
$x=\frac{400}{16}=25$

Now that x=25, find y= $9\cdot 25+520=745$ So the system is solved by $(25,745)$

Now interpret: What is the selling price when supply and demand are in balance? 25

What is the number of items in the market when in balance? 745

The monthly supply and demand for a particular bike is represented in the accompanying graph. The quantity is in thousands and the price is in hundreds of dollars. Use the graph to answer the following. The demand curve is red and the the supply curve is blue.



(a) Approximately at what price is 3200 bikes supplied? $ 1400 (Round to the nearest $50.)

(b) Approximately at what price is 3200 bikes demanded? $ 7700 (Round to the nearest $50.)

(c) Estimate the equilibrium price. $ 5500

(d) Estimate the equilibrium quantity. 13000