1.

$$f(x) = (2x - 1)^3$$

$$\text{inner} = u = 2x - 1$$

$$\text{outer} = f(u) = u^3$$

2.

$$f(x) = \frac{1}{\sqrt{2x + 1}}$$

$$\text{inner} = u = 2x + 1$$

$$\text{outer} = f(u) = \frac{1}{\sqrt{u}}$$

3.

$$f(x) = \sqrt{x^3 + x}$$

$$\text{inner} = u = x^3 + x$$

$$\text{outer} = f(u) = \sqrt{u}$$

4.

$$f(x) = \cos\left(\pi^2 x\right)$$

$$\text{inner} = u = \pi^2 x$$

$$\text{outer} = f(u) = \cos(u)$$

5.

$$f(x) = \sin^2(x)$$

Rewrite as $f(x) = (\sin(x))^2$

$$\text{inner} = u = \sin(x)$$

$$\text{outer} = f(u) = u^2$$

6.

$$f(x) = \tan\left(\frac{3}{2}x\right)$$

$$\text{inner} = u = \frac{3}{2}x$$

$$\text{outer} = f(u) = \tan(u)$$

7.

$$f(x) = \sqrt{\sec(x)}$$

$$\text{inner} = u = \sec(x)$$

$$\text{outer} = f(u) = \sqrt{u}$$

8.

$$f(x) = \ln\left(x^2\right)$$

$$\text{inner} = u = x^2$$

$$\text{outer} = \ln(u)$$

9.

$$f(x) = 2\ln\left(x^2 - 4\right)$$

$$\text{inner} = u = x^2 - 4$$

$$\text{outer} = f(u) = 2\ln(u)$$

10.

$$f(x) = e^{2x}$$

$$\text{inner} = u = 2x$$

$$\text{outer} = f(u) = e^{u}$$

11.

$$f(x) = e^{x^2 - 1}$$

$$\text{inner} = u = x^2 - 1$$

$$\text{outer} = f(u) = e^{u}$$

12.

$$f(x) = e^{\ln(x)}$$

$$\text{inner} = u = \ln(x)$$

$$\text{outer} = f(u) = e^{u}$$

13.

$$f(x) = 4^{2x}$$

$$\text{inner} = u = 2x$$

$$\text{outer} = f(u) = 4^{u}$$

14.

$$f(x) = e^{\sin(x)}$$

$$\text{inner} = u = \sin(x)$$

$$\text{outer} = f(u) = e^{u}$$

15.

$$f(x) = (2x)^{4}$$

$$\text{inner} = u = 2x$$

$$\text{outer} = f(u) = u^{4}$$

16.

The rule is f'(g(x))g'(x)

In words, this means differentiate the outside, copy the inside and then multiply by the derivative of the inside.

$$f(x) = (2x + 1)^2$$

$$\text{inside} = u = 2x + 1$$

$$\text{outside} = f(u) = u^2$$

derivatie of outside= $2u$

derivative of inside= 2

Multiply the derivatives together $2u \cdot 2$

Replace u with 2x+1

$$2(2x + 1) \cdot 2$$

Simplify

$$\frac{d}{dx} f(x) = 2(2x + 1) \cdot 2 = 4(2x + 1) = 8x + 4$$

17.

The rule is f'(g(x))g'(x)
In words, this means differentiate the outside, copy the inside and then multiply by the derivative of the inside.

$$f(x) = 3(-2x - 1)^2$$

$$\text{inside} = u = -2x - 1$$

$$\text{outside} = f(u) = 3 \cdot u^2$$

derivatie of outside= $3 \cdot 2u^1 = 6u$
derivative of inside= -2

Multiply the derivatives together $6u \cdot (-2)$
Replace u with -2x-1

$6(-2x - 1) \cdot (-2)$

Simplify

$$\frac{d}{dx} f(x) = 6(-2x - 1) \cdot (-2) = -12(-2x - 1) = 24x + 12$$

18.

The rule is f'(g(x))g'(x)

In words, this means differentiate the outside, copy the inside and then multiply by the derivative of the inside.

$$f(x) = (mx + b)^2$$

$$\text{inside} = u = mx + b$$

$$\text{outside} = f(u) = u^2$$

derivatie of outside= $2u^1 = 2u$

derivative of inside= m

Multiply the derivatives together $2u \cdot m$

Replace u with mx+b

$$2(mx + b) \cdot m$$

Simplify

$$\frac{d}{dx} f(x) = 2(mx + b) \cdot m = 2\,m(mx + b) = 2m^2x + 2b$$

19.

The rule is f'(g(x))g'(x)

In words, this means differentiate the outside, copy the inside and then multiply by the derivative of the inside.

$$f(x) = (2x+1)^{\frac{2}{3}}$$

inside = $u = 2x + 1$ outside = $f(u) = u^{\frac{2}{3}}$

derivative of outside= $\frac{2}{3}u^{\frac{2}{3}-1} = \frac{2}{3}u^{\frac{2}{3}-\frac{3}{3}} = \frac{2}{3}u^{\frac{-1}{3}}$

derivative of inside= 2

Multiply the derivatives together $\frac{2}{3}u^{\frac{-1}{3}}\cdot 2$

Replace u with 2x+1

$$\frac{2}{3}(2x+1)^{\frac{-1}{3}}\cdot 2$$

Simplify

$$\frac{d}{dx}f(x) = \frac{2}{3}(2x+1)^{\frac{-1}{3}}\cdot 2 = \frac{4}{3}\cdot(2x+1)^{\frac{-1}{3}} = \frac{4}{3}\cdot\frac{1}{(2x+1)^{\frac{1}{3}}} = \frac{4}{3}\cdot\frac{1}{\sqrt[3]{2x+1}} = \frac{4}{3\cdot\sqrt[3]{2x+1}}$$

20.

The rule is f'(g(x))g'(x)

In words, this means differentiate the outside, copy the inside and then multiply by the derivative of the inside.

$$f(x) = (1-x)^{\frac{1}{2}}$$

inside $= u = 1 - x$ outside $= f(u) = u^{\frac{1}{2}}$

derivative of outside= $\frac{1}{2}(u)^{\frac{1}{2}-1} = \frac{1}{2}u^{\frac{1}{2}-\frac{2}{2}} = \frac{1}{2}u^{\frac{-1}{2}}$

derivative of inside= -1

Multiply the derivatives together $\frac{1}{2}u^{\frac{-1}{2}}\cdot(-1)$

Replace u with 1-x

$$\frac{1}{2}(1-x)^{\frac{-1}{2}}\cdot(-1)$$

Simplify

$$\frac{d}{dx}f(x) = \frac{1}{2}(1-x)^{\frac{-1}{2}}\cdot(-1) = \frac{-1}{2}\cdot(1-x)^{\frac{-1}{2}} = \frac{-1}{2}\frac{1}{(1-x)^{\frac{1}{2}}} = \frac{-1}{2}\cdot\frac{1}{\sqrt{1-x}} = \frac{-1}{2\sqrt{1-x}}$$

21.

The rule is f'(g(x))g'(x)

In words, this means differentiate the outside, copy the inside and then multiply by the derivative of the inside.

$f(x) = \sqrt{4-2x}$

inside $= u = 4 - 2x$ outside $= f(u) = \sqrt{u} = u^{\frac{1}{2}}$

derivative of outside= $\frac{1}{2}(u)^{\frac{1}{2}-1} = \frac{1}{2}u^{\frac{1}{2}-\frac{2}{2}} = \frac{1}{2}u^{\frac{-1}{2}}$

derivative of inside= -2

Multiply the derivatives together $\frac{1}{2}u^{\frac{-1}{2}}\cdot(-2)$

Replace u with 4-2x

$\frac{1}{2}(4-2x)^{\frac{-1}{2}}\cdot(-2)$

Simplify

$$\frac{d}{dx}f(x) = \frac{1}{2}(4-2x)^{\frac{-1}{2}}\cdot(-2) = \frac{-2}{2}\cdot(4-2x)^{\frac{-1}{2}} = -1\,\frac{1}{(4-2x)^{\frac{1}{2}}} = -1\cdot\frac{1}{\sqrt{4-2x}} = \frac{-1}{\sqrt{4-2x}}$$

22.
The rule is f'(g(x))g'(x)
In words, this means differentiate the outside, copy the inside and then multiply by the derivative of the inside.

$$f(x) = \sqrt{x^2 - 2x}$$

inside $= u = x^2 - 2x$ outside $= f(u) = \sqrt{u} = u^{\frac{1}{2}}$

derivative of outside= $\frac{1}{2}(u)^{\frac{1}{2}-1} = \frac{1}{2}u^{\frac{1}{2}-\frac{2}{2}} = \frac{1}{2}u^{\frac{-1}{2}}$

derivative of inside= $2x - 2$

Multiply the derivatives together $\frac{1}{2}u^{\frac{-1}{2}}\cdot(2x-2)$

Replace u with $x^2 - 2x$

$$\frac{1}{2}\left(x^2 - 2x\right)^{\frac{-1}{2}}\cdot(2x-2)$$

Simplify

$$\frac{d}{dx}f(x) = \frac{1}{2}\left(x^2 - 2x\right)^{\frac{-1}{2}}\cdot(2x-2) = \frac{1}{2}\cdot\frac{1}{\sqrt{x^2-2x}}\cdot 2(x-1) = \frac{2}{2}\cdot\frac{1}{\sqrt{x^2-2x}}\cdot(x-1) = \frac{x-1}{\sqrt{x^2-2x}}$$

23.

$f(x) = 2\sqrt{x^2 - b}$

inside $= u = x^2 - b$ outside $= f(u) = 2\sqrt{u} = 2 \cdot u^{\frac{1}{2}}$

derivative of outside= $2 \cdot \frac{1}{2}(u)^{\frac{1}{2}-1} = \frac{2}{2}u^{\frac{1}{2}-\frac{2}{2}} = 1u^{\frac{-1}{2}} = \frac{1}{\sqrt{u}}$

derivative of inside= $2x$

Multiply the derivatives together $\frac{1}{\sqrt{u}} \cdot 2x$

Replace u with $x^2 - b$

$$\frac{1}{\sqrt{x^2 - b}} \cdot 2x$$

Simplify

$$\frac{d}{dx} f(x) = \frac{1}{\sqrt{x^2 - b}} \cdot 2x = \frac{2x}{\sqrt{x^2 - b}}$$

24.

$f(x) = \sqrt[3]{mx^2 + b}$

$\text{inside} = u = mx^2 + b \qquad \text{outside} = f(u) = \sqrt[3]{u} = u^{\frac{1}{3}}$

derivative of outside= $\frac{1}{3}(u)^{\frac{1}{3}-1} = \frac{1}{3}u^{\frac{1}{3}-\frac{3}{3}} = \frac{1}{3}u^{\frac{-2}{3}} = \frac{1}{3}\cdot\frac{1}{u^{\frac{2}{3}}} = \frac{1}{3}\cdot\frac{1}{\sqrt[3]{u^2}}$

derivative of inside= $2mx$ b goes away because it's a constant

Multiply the derivatives together $\frac{1}{3}\cdot\frac{1}{\sqrt[3]{u^2}}\cdot(2mx)$

Replace u with $mx^2 - b$

$\frac{1}{3}\cdot\frac{1}{\sqrt[3]{\left(mx^2 + b\right)^2}}\cdot 2mx$

Simplify

$$\frac{d}{dx}f(x) = \frac{1}{3}\cdot\frac{1}{\sqrt[3]{\left(mx^2 + b\right)^2}}\cdot 2mx = \frac{2mx}{3\sqrt[3]{\left(mx^2 + b\right)^2}}$$

25.

$f(x) = (2x - 1)^{-1}$

inside $= u = 2x - 1$ outside $= f(u) = u^{-1}$

derivative of outside= $-1(u)^{-1-1} = -1u^{-2} = \frac{-1}{u^2}$

derivative of inside= 2

Multiply the derivatives together $\frac{-1}{u^2} \cdot 2$

Replace u with $2x - 1$

$$\frac{-1}{(2x-1)^2} \cdot 2$$

Simplify

$$\frac{d}{dx} f(x) = \frac{-1}{(2x-1)^2} \cdot 2 = \frac{-2}{(2x-1)^2}$$

26.

$f(x) = (mx + b)^{a}$

$\text{inside} = u = mx + b$ $\quad$ $\text{outside} = f(u) = u^{a}$

derivative of outside= $a(u)^{a-1}$

derivative of inside= m b is treated as a constant, so it goes away

Multiply the derivatives together $a(u)^{a-1} \cdot m$

Replace u with $mx + b$

$a(mx + b)^{a-1} \cdot m$

Simplify

$$\frac{d}{dx} f(x) = a(mx + b)^{a-1} \cdot m = ma(mx + b)^{a-1}$$

27.

$$f(x) = \frac{1}{mx + b}$$

inside $= u = mx + b$ outside $= f(u) = \frac{1}{u} = u^{-1}$

derivative of outside= $-1u^{-1-1} = -1u^{-2} = \frac{-1}{u^2}$

derivative of inside= m b is treated as a constant, so it goes away

Multiply the derivatives together $\frac{-1}{u^2} \cdot m$

Replace u with $mx + b$

$$\frac{-1}{(mx + b)^2} \cdot m$$

Simplify

$$\frac{d}{dx} f(x) = \frac{-1}{(mx + b)^2} \cdot m = \frac{-m}{(mx + b)^2}$$

29.

$$f(x) = \frac{1}{mx^2 + bx + c}$$

$\text{inside} = u = mx^2 + bx + c$ $\quad$ $\text{outside} = f(u) = \frac{1}{u} = u^{-1}$

derivative of outside= $-1u^{-1-1} = -1u^{-2} = \frac{-1}{u^2}$

derivative of inside= $2mx + b$ c is treated as a constant, so it goes away

Multiply the derivatives together $\frac{-1}{u^2}\cdot(2mx + b)$

Replace u with $mx^2 + bx + c$

$$\frac{-1}{\left(mx^2 + bx + c\right)}\cdot(2mx + b)$$

Simplify

$$\frac{d}{dx}f(x) = \frac{-1}{\left(mx^2 + bx + c\right)^2}\cdot(2mx + b) = \frac{-1(2mx + b)}{\left(mx^2 + bx + c\right)^2} = \frac{-2mx - b}{\left(mx^2 + bx + c\right)^2}$$

30.

$f(x) = \ln(\sin(x))$

$\text{inside} = u = \sin(x)$ $\text{outside} = \ln(u)$

derivative of outside= $\frac{1}{u}$

derivative of inside= $\cos(x)$

Multiply the derivatives together $\frac{1}{u} \cdot \cos(x)$

Replace u with $\sin(x)$

$\frac{1}{\sin(x)} \cdot \cos(x)$

Simplify

$\frac{d}{dx} f(x) = \frac{1}{\sin(x)} \cdot \cos(x) = \frac{\cos(x)}{\sin(x)} = \cot(x)$

32.

$f(x) = \sin(\ln(x))$

$inside = u = \ln(x)$ $\qquad$ $outside = \sin(u)$

derivative of outside= $\cos(u)$

derivative of inside= $\frac{1}{x}$

Multiply the derivatives together $\cos(u)\cdot\frac{1}{x}$

Replace u with $\ln(x)$

$\cos(\ln(x))\cdot\frac{1}{x}$

Simplify

$$\frac{d}{dx}f(x) = \cos(\ln(x))\cdot\frac{1}{x} = \frac{\cos(\ln(x))}{x}$$

33.

$f(x) = \cos(2x + 4)$

inside $= u = 2x + 4$ outside $= \cos(u)$

derivative of outside= $-\sin(u)$

derivative of inside= 2

Multiply the derivatives together $-\sin(u) \cdot 2$

Replace u with $2x + 4$

$-\sin(2x + 4) \cdot 2$

Simplify

$$\frac{d}{dx} f(x) = -\sin(2x + 4) \cdot 2 = -2 \sin(2x + 4)$$

34.

$f(x) = e^{\sin(x)}$

inside $= u = \sin(x)$ outside $= e^{u}$

derivative of outside= e^{u}

derivative of inside= $\cos(x)$

Multiply the derivatives together $e^{u}\cdot\cos(x)$

Replace u with $\sin(x)$

$e^{\sin(x)}\cdot\cos(x)$

Simplify

$$\frac{d}{dx}f(x) = \cos(x)\,e^{\sin(x)}$$

35.

$f(x) = e^{x^2-1}$

inside = $u = x^2 - 1$ outside = e^u

derivative of outside= e^u

derivative of inside= $2x$

Multiply the derivatives together $e^u \cdot 2x$

Replace u with $x^2 - 1$

$e^{x^2-1} \cdot 2x$

Simplify

$$\frac{d}{dx} f(x) = e^{x^2-1} \cdot 2x = 2x \cdot e^{x^2-1}$$

36.

$f(x) = e^{\ln(x)}$

inside $= u = \ln(x)$ outside $= e^{u}$

derivative of outside= e^{u}

derivative of inside= $\frac{1}{x}$

Multiply the derivatives together $e^{u} \cdot \frac{1}{x}$

Replace u with $\ln(x)$

$e^{\ln(x)} \cdot \frac{1}{x}$

Simplify

$$\frac{d}{dx} f(x) = e^{\ln(x)} \cdot \frac{1}{x} = \frac{e^{\ln(x)}}{x}$$

37.

$f(x) = \ln(x^2 + 4)$

inside $= u = x^2$ outside $= \ln(u)$

derivative of outside= $\frac{1}{u}$

derivative of inside= $2x$

Multiply the derivatives together $\frac{1}{u} \cdot 2x$

Replace u with $x^2 + 4$

$$\frac{1}{x^2 + 4} \cdot 2x$$

Simplify

$$\frac{d}{dx} f(x) = \frac{1}{x^2 + 4} \cdot 2x = \frac{2x}{x^2 + 4}$$

38.

$f(x) = 2^{x^2}$

inside $= u = x^2$ outside $= 2^u$

derivative of outside= $\ln(2)\, 2^u$

derivative of inside= $2x$

Multiply the derivatives together $\ln(2)\, 2^u \cdot 2x$

Replace u with x^2

$\ln(2)\, 2^{x^2} \cdot 2x$

Simplify

$$\frac{d}{dx} f(x) = \ln(2)\, 2^{x^2} \cdot 2x = 2x \cdot \ln(2) \cdot 2^{x^2}$$

39.

$f(x) = 4^{x^2 - 1}$

inside $= u = x^2 - 1$ outside $= 4^u$

derivative of outside= $\ln(4) \cdot 4^u$

derivative of inside= $2x$

Multiply the derivatives together $\ln(4)\, 4^u \cdot 2x$

Replace u with $x^2 - 1$

$\ln(4)\, 4^{x^2 - 1} \cdot 2x$

Simplify

$$\frac{d}{dx} f(x) = \ln(4)\, 4^{x^2 - 1} \cdot 2x = 2x \cdot \ln(4) \cdot 4^{x^2 - 1}$$

40.

$$f(x) = \tan^{-1}(2x)$$

inside $= u = 2x$ outside $= \tan^{-1}(u)$

derivative of outside= $\dfrac{1}{1+u^2}$

derivative of inside= 2

Multiply the derivatives together $\dfrac{1}{1+u^2}\cdot 2$

Replace u with $2x$

$$\frac{1}{1+(2x)^2}\cdot 2$$

Simplify

$$\frac{d}{dx}f(x) = \frac{1}{1+(2x)^2}\cdot 2 = \frac{1}{1+2^2x^2}\cdot 2 = \frac{1}{1+4x^2}\cdot 2 = \frac{2}{1+4x^2}$$

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