Math 111 Notes/Section 1.5 Complex Numbers
Make sure to load your class notes with your PDF homework solutions.
For the midterm, make sure to have your computer. You'll get one try on each question and put the work on paper for proof. I'll review the tests to make sure the work backs up any correct answers and award credit for any partial work on incorrect answers.
Any answer without backing work will be a 0. So if you study hard, get your answers right and the right work is shown, you'll know your grade right away:)
Section 1.5/Complex numbers:

$\sqrt{-1}$ $\quad \sqrt{4} = 2$ b/c $2 \cdot 2=4$

$\sqrt{-1} = -1$ not good b/c -1(-1)=1 (not -1)

$\sqrt{-1} = 1$ not good b/c $1 \cdot 1=1$ and not -1

Deeper reason …why do we need i or $\sqrt{-1}$? Based on some equations somebody tried to solve in the 1670 or 1650s.

$\sqrt{-1} = i$ (just call this i)
square both sides:
$(\sqrt{-1})^2 = i^2$
$-1 = i^2 \Leftarrow$ This is used often. Any time we encounter i^2 we replace it with -1.

$$-5+\sqrt{-9} \xrightarrow{\text{separate off -1}} -5+\sqrt{-1\cdot 9} \xrightarrow{\text{apply } \sqrt{ab}=\sqrt{a}\sqrt{b}} -5+\sqrt{-1}\sqrt{9} \longrightarrow -5+i(3) = -5+3i$$

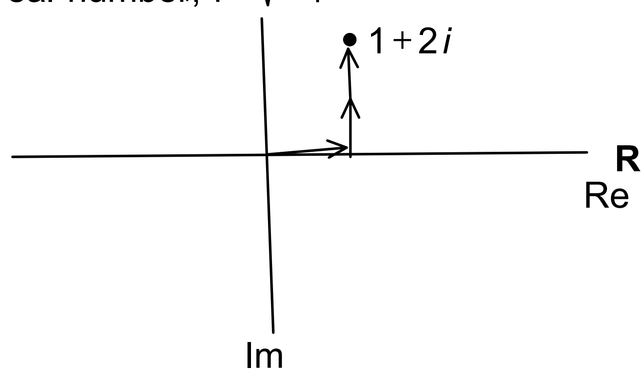
complex number: a+bi, a=real number,b=real number., i= $\sqrt{-1}$

examples: $1/2+2i,\ a=1/2,\ b=2$

$3+4i,\ a=3,\ b=4$

$2.3+5.6i,\ a=2.3,\ b=5.6$

$1+2i$ graph



Two complex numbers are equal when their real and imaginary parts are equal.
$a+bi \qquad c+di$
a=c, and b=d

ex: 2+4i $\qquad$ x+yi
say we want these to be equal: $x=2,\ y=4$

adding complex numbers: $(a+bi)+(c+di) \xrightarrow[\text{drop parenthesis}]{\text{combine like terms}} a+bi+c+di = a+c+bi+di \xrightarrow{\text{factor i out}} (a+c)+(b+d)i$

e xample: $(4+7i)+(1-6i) = 4+7i+1-6i = 4+1+7i-6i = 5+1i = 5+i$

subtraction: $(1+2i)-(4+2i) \xrightarrow{\text{it's really -1}} (1+2i)-1(4+2i) \xrightarrow[\text{drop parentehsis}]{\text{distribute -1}} 1+2i-4-2i$

$\xrightarrow{\text{combine like terms}} 1-4\cancel{+2i-2i} = -3 \Leftarrow$ notice the difference is a pure REAL number!

$4(-2+3i) \xrightarrow{\text{distribute 4}} 4(-2)+4(3i) \xrightarrow{\text{multiply}} -8+12i \Leftarrow$ goal is to make a+bi form
$a=-8,\ b=12$

$$(2-i)(4+3i)\xrightarrow[\text{binomial}\cdot\text{ binomial ..recall that } i^2=-1]{\text{relate to something familiar??}} 2(4)+2(3i)-i(4)-i(3i)$$

$$\xrightarrow{\text{multiply}} 8+6i-4i-3i^2$$

$$\xrightarrow{\text{replace } i^2 \text{ with -1}} 8+6i-4i-3(-1)$$

$$\xrightarrow{-(-1)=+1} 8+6i-4i+3$$

$$\xrightarrow{\text{combine like terms}} 8+3+6i-4i$$

$11+2i \Leftarrow$ complex number of the form a+bi
a=11, b=2

observation: $(a+bi)(a-bi)\xrightarrow{\text{FOIL}} a\cdot a+a(-bi)+bi(a)+bi(-bi)$

$$\xrightarrow{\text{multiply}} a^2-abi+abi-b^2i^2$$

$$\xrightarrow{\text{cancel middle terms}} a^2-b^2(i^2)$$

$$\xrightarrow{\text{replace } i^2 \text{ with -1}} a^2-b^2(-1)$$

$\xrightarrow{\text{multiply again}} a^2+b^2 \Leftarrow$ this is a real number for a and b

a+bi and a-bi are called complex conjugats. They multiply always to a^2+b^2 and add to 2a. $a+bi+a-bi=a+a\cancel{+bi-bi}=2a \Leftarrow$ sum is a purely real number!

division of complex numbers: Goal is to produce a+bi form!

$\frac{1}{i}$ not a+bi ofrm b/c i is in the bottom..bad!

$\frac{1}{i}\cdot\frac{i}{i}$ b/c i/i=1

$\frac{1\cdot i}{i\cdot i}$

$\frac{i}{i^2}$

$\frac{i}{-1}$

$-i \leftarrow a=0, b=-1 \ldots$ morefully it's $0-1i$ (a+bi form)

$\frac{1+i}{i}$

multiply by 1= i/i or -i/-i =1

$\frac{1+i}{i}\left(\frac{-i}{-i}\right)$ goal is to make a+bi form

$$\xrightarrow{\text{distribute in top}} \frac{1(-i)+i(-i)}{-i^2}$$

$$\xrightarrow[\text{replace } i^2 \text{ in bottom with -1}]{\text{multiply out}} \frac{-i-i^2}{-(-1)}$$

replace i^2 with -1 : $\frac{-i-(-1)}{+1}$

$\frac{-i+1}{1}=-i+1=1-i$ (a=1, b=-1)

$a+bi$

$(a+bi)(a-bi)=a^2+b^2 \Leftarrow$ real number (no i left)

$$\frac{2+3i}{4-2i} \xrightarrow{\text{not the a+bi form we want}} \xrightarrow[\text{make the bottom a real number}]{\text{gotta get rid of i in bottom}} \frac{2+3i}{4-2i}\left(\frac{4+2i}{4+2i}\right)$$

$$\xrightarrow{\text{4+2i b/c 4+2i and 4-2i are complex conjugates}} \frac{(2+3i)(4+2i)}{(4-2i)(4+2i)}$$

FOIL top
FOIL bottom $\dfrac{2(4)+2(2i)+3i(4)+3i(2i)}{4\cdot 4+4(2i)-2i(4)-2i(2i)}$

multiply out: $\dfrac{8+4i+12i+6i^2}{16+8i-8i-4i^2}$

replace i^2 with -1: $\dfrac{8+4i+12i+6(-1)}{16+8i-8i-4(-1)}$

multiply again: $\dfrac{8+4i+12i-6}{16+8i-8i+4}$

combine like terms: $\dfrac{2+16i}{20}$ goal is a+bi

separate into fractions: $\dfrac{2}{20}+\dfrac{16}{20}i$

simplify fractions: $\dfrac{1}{10}+\dfrac{4}{5}i,\ a=1/10,\ b=4/5$

$a+bi form$

$$\sqrt{-3}\ \sqrt{-12} \xrightarrow[\text{separate -1}]{\text{first handle i}} \sqrt{-1\cdot 3}\ \sqrt{-1\cdot 12} \xrightarrow{\text{apply } \sqrt{ab}=\sqrt{a}\sqrt{b}} \sqrt{-1}\ \sqrt{3}\ \sqrt{-1}\ \sqrt{12}$$

apply $\sqrt{-1}=i \rightarrow i\sqrt{3}\ i\sqrt{12} \xrightarrow{\text{multiply}} i^2\sqrt{3}\ \sqrt{12} \xrightarrow{\text{replace } i^2 \text{ with -1}} -1\sqrt{36}=-6$ (not +6)

~~bad: $\sqrt{-3}\ \sqrt{-12}=\sqrt{-3(-12)}=\sqrt{36}=6$~~

solving quadratic equations with i:

$x^2+4=0$

$x^2+4-4=0-4$

$x^2=-4$

root both sides: $\sqrt{x^2}=\pm\sqrt{-4}$

$x=\pm\sqrt{-1}\ \sqrt{4}$

$x=\pm i2$

$x=\pm 2i$

check:
$x=+2i$:

$(2i)^2+4=?0$

$2^2i^2+4=?0$

$4(-1)+4=?0$

$-4+4=?0$

$0=0$ is true!

repeat with x=-2i..
will also work!
$i^2=-1 \Leftarrow$ this makes it work
b/c it turns 4 into -4!!

$3x^2-2x+5=0$

$a=3, b=-2, c=5$

$ax^2+bx+c=0$

plug into quadratic formula:

$2x^2+0x+0=0$

$a=2, b=0, c=0$

$-3x^2+0x-4=0$

$a=-3, b=0, c=-4$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-2)\pm\sqrt{(-2)^2-4(3)(5)}}{2\cdot 3}$$

$$x=\frac{2\pm\sqrt{4-60}}{6}$$

$$x=\frac{2\pm\sqrt{-56}}{6}$$

$$x=\frac{2\pm\sqrt{-1}\ \sqrt{56}}{6}$$

$$x=\frac{2\pm i\sqrt{56}}{6}$$

$$x=\frac{2\pm i\sqrt{2\cdot 28}}{6}$$

2 is not a perfect, 28 is not a perfect square, so no good

$$x=\frac{2\pm i\sqrt{4\cdot 14}}{6}$$

b/c $\sqrt{4}=2$

$$x=\frac{2\pm i\sqrt{4}\ \sqrt{14}}{6}$$

$$x=\frac{2\pm 2i\sqrt{14}}{6}$$ ⇐ look at top and bottom..common factor?

$$x=\frac{2(1\pm i\sqrt{14})}{2\cdot 3}$$ ← factor 2 out, write 6 as $2\cdot 3$

$$x=\frac{1\pm i\sqrt{14}}{3}$$ (goal is a+bi)

$$x=\frac{1}{3}\pm i\frac{\sqrt{14}}{3}$$ $a=1/3, b=\sqrt{14}/3$

$$x=\frac{1}{3}+\frac{\sqrt{14}}{3}i,\ x=\frac{1}{3}-\frac{\sqrt{14}}{3}i$$ (complex conjugates)

69 in book on page 128(8th edition)

$1x^2-2x+2=0$

$a=1, b=-2, c=2$

put into formula: $x=\dfrac{-(-2)\pm\sqrt{(-2)^2-4(1)(2)}}{2(1)}$

$$x=\frac{2\pm\sqrt{4-8}}{2}$$

$$x=\frac{2\pm\sqrt{-4}}{2}$$

$$x=\frac{2\pm\sqrt{-1}\ \sqrt{4}}{2}$$

$$x=\frac{2\pm i\sqrt{4}}{2}\xrightarrow{\text{simplify}}x=\frac{2\pm i2}{2}\xrightarrow{\text{goal is a+bi}}x=\frac{2}{2}\pm\frac{2}{2}i\xrightarrow{2/2=1}x=1\pm 1i$$

$$x=1\pm i$$

$$x=1-i,\ x=1+i$$

one check with 1+i: $(1+i)^2 - 2(1+i) + 2 = ?0$

$(a+b)^2 = a^2 + 2ab + b^2$

$$1^2 + 2\cdot 1\cdot i + i^2 - 2 - 2i + 2 = ?0$$

$$1 + 2i - 1 - 2 - 2i + 2 = ?0$$

$$1 - 1 + 2i - 2i - 2 + 2 = ?0$$

$$0 = 0! \ \ true!$$