Make sure to take **DETAILED** notes and load them with your homework PDF.
Section 2.2/Functions: It's getting to be flu/cold season, so if you're coughing a lot, please cover your mouth or wear a mask.

Def: A function is a set of points such that the first member of each point is unique.
example 1: $f=\{(1,2),(3,4),(5,6)\}$, $1\neq 3\neq 5$ so f represents a function.

range is the set of outputs: R={2,4,6}(y-coords)
domain is the set of inputs: D={1,3,5}(x-coords)

example 1(book)

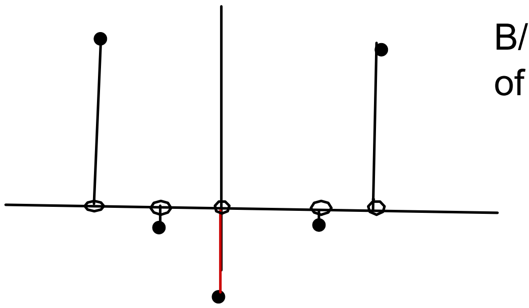
(a)

x	y
2	11
2	10
3	8
4	5
5	1

$\rightarrow(2,11)$
$\rightarrow(2,10)$

$\rightarrow$ 2 repeats! *This* set of points is NOT a function.

(b)



B/c we have different x coords, so plot of points is a function. Specific coords. don't matter.

example 2: book Does $x^2+y=1$ represent a function?
so lve for y: $y=1-x^2$
x=1: $y=1-1^2=1-1=0$ (1,0)
x=-1: $y=1-(-1)^2=1-1=0, (-1,0)$

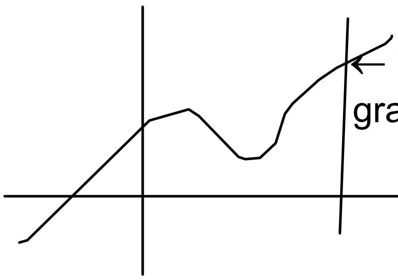
$(1,0)$ *vs* (-1,0)
$1\neq -1$, so $x^2+y=1$ is a function.

To each x there corresponds exactly one value of y.

(b) $-x+y^2=1$
$y^2=1+x$
$\sqrt{y^2}=\pm\sqrt{1+x}$
$y=\pm\sqrt{1+x}$

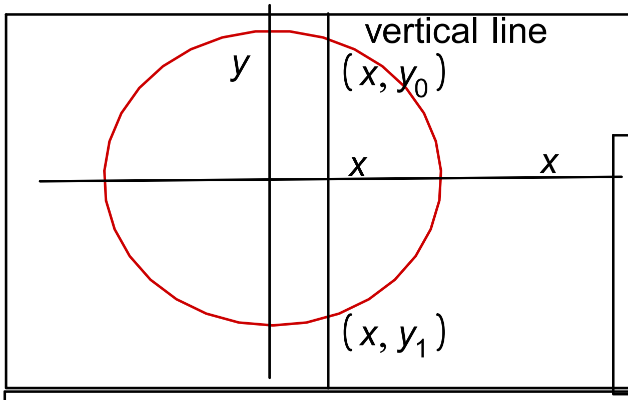
x=3:(random value)
$y=\pm\sqrt{1+3}=\pm\sqrt{4}=\pm 2$
we get (3,2) and (3,-2)
Single x (x=3) corresponds to two values of y.
So $-x+y^2=1$ is not a function.

our own example for graphs(c)



← When a graph intersects a vertical line ONLY ONCE, the graph represents a function. This is called the Vertical Line Test.

Please email Hannah .



Not a function b/c we have (x, y_0) and (x, y_1).
Both points have the same first member.

function notation: input x, output f(x) , "f of x"
point forms: (x, y), $(x, f(x))$, so y=f(x)=output!
$f(x) = 1 - x^2$
$x =$ independent variable, f(x) is the output
1-x^2 is how you find the output!

example 3 in book: $g(x) = -x^2 + 4x + 1$
$g(2) \leftarrow$ "g of 2"..replace every x with 2
$g(2) = -1(2)^2 + 4 \cdot 2 + 1$
$= -1 \cdot 4 + 8 + 1$
$= -4 + 8 + 1$
$= 5 \Leftarrow$ output

$-x^2$ means -1 $\cdot$ x^2 $= -1 \cdot (x)^2$
2nd 1st

point form: $(2, 5)$
more clearly: $(x = 2, y = 5)$

$3(b) : g(t)$ (g of t)~~(g times t)~~
every x gets replaced with t (on LHS and RHS)
$g(t) = -t^2 + 4t + 1$ (*not* this: ~~$g(x) = -t^2 + 4t + 1$~~)

3(c): $g(x+2)$ "g of x plus two" (~~not g times (x+2)~~)
every x gets replaced by x+2
$g(x+2) = -(x+2)^2 + 4(x+2) + 1$, (keep the minus out of the parenthesis)

$= -1[(x+2)^2] + 4x + 4 \cdot 2 + 1$

$(a+b)^2 = a^2 + 2ab + b^2$

$= -1[x^2 + 2 \cdot 2 \cdot x + 2^2] + 4x + 8 + 1$

~~$(a+b)^2 = a^2 + b^2$~~ (incorrect)

$= -1[x^2 + 4x + 4] + 4x + 9$

$= -x^2 \cancel{-4x} - 4 \cancel{+4x} + 9$ distribute -1 to each of the brackted terms

$= -x^2 + 5$ combine like terms

point form: (x+2 ,-x^2 +5)

example 5 in book: Finding values for which f(x)=0 $\leftarrow$ this means y=0
$f(x) = -2x + 10$
$-2x + 10 = 0$ (*set* f(x) equal to 0)

$-2x + 10 - 10 = 0 - 10$

$-2x = -10$

$x = 5 \Leftarrow$ This is not 0. This is a value of x that makes -2x+10 equal 0!

$f(5) = -2(5) + 10 = -10 + 10 = 0 \Leftarrow y$ coord! point: $(5, 0)$

5 is called the "zero" of the function not because it's 0, but because it makes y come out to be 0!!

example 7: domains of functions:

f(x)= $\sqrt{x}$, $x \geq 0 \Leftarrow$ inequality form

(a) $0 \longrightarrow \dots \infty$

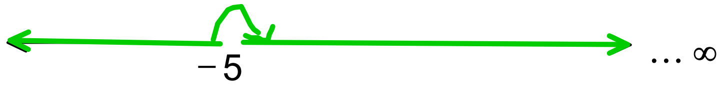
[0, ∞) ([b/c of the = part, infinty is a concept, not a number, so always))

$\sqrt{0.2}$, $\sqrt{0.001}$, $\sqrt{4}$, $\sqrt{10000343}$ (not allowed here: $\sqrt{-5}$)

$(b)\, g(x) = \frac{1}{x+5}$ $\frac{1}{0 \Leftarrow \text{not allowed here!}}$

$x + 5 \neq 0$ inequality

$x \neq -5$ number line: $-\infty \dots$



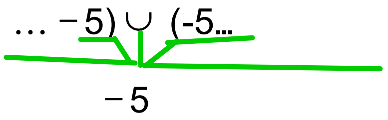
set-builder form:

$\{\boxed{\text{object}}\boxed{\text{such that}}\boxed{\text{condition}}\}$

$\{x \mid x \neq 5\}$

interval notation: (from left to right based on picture)

$(-\infty, -5) \cup (-5, \infty)$ (interval form)



"condition" in math means either true or false.

(c) $h(x) = \sqrt{4-3x}$ in general: $\sqrt{\text{expression}}$ (expression is whatever>=0)

$4 - 3x \geq 0$ apply defintion of domain for square root functions

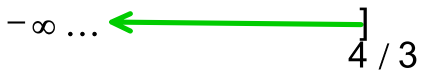
$-3x \geq -4$

$x \leq \frac{4}{3} \Leftarrow$ inequality form, set-builder form: {object such that condition}

$\{x : x \leq 4/3\}$

verbalize as " set of all x such that x is less than or equal to four-thirds"

PICTURE FORM FIRST:

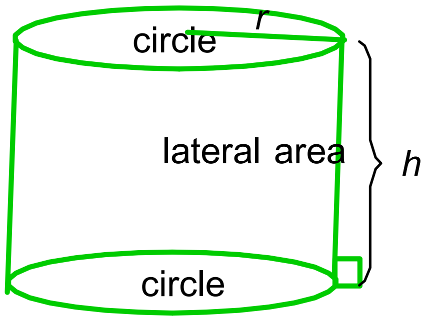


from left to right:

$(-\infty, 4/3]$ < says point left, = says use bracket!

INTERVAL FORM:

example 8: container dimensions:



right circular cylinder

$V(r) \Leftarrow$ "V of r"

V=volume of can=amount of 3D space an object takes up!

$V(r) = \pi r^2 h \Leftarrow$ formula!!,radius

$h =$ height

ratio of height to radius is 4: $\frac{h}{r} = 4 \rightarrow h = 4r \Leftarrow$ had to be given!!

$V(r) = \pi r^2 (4r) = 4\pi r^2 r = 4\pi r^{2+1} = 4\pi r^3 \Leftarrow$ only r remains!

can r=-.1? no..not physical

can r=0? We can measure 0 but there is no physical object still.

r>0 (then we have a real physical object we can hold) interval:(0, ∞)