Math 111 Notes 9/25. Make sure your class notes are loaded with your homework solutions

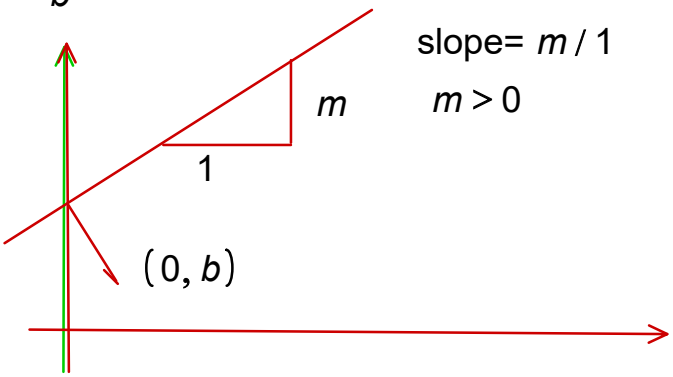
Section 2.1/Linear Equations in Two Variables:

$y = mx + b$, $m =$ slope, b= $y-intercept$ pt

$y = m \cdot 0 + b$, $x = 0$

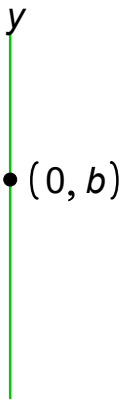
$= 0 + b$

$= b$



slope= $m / 1$

$m > 0$

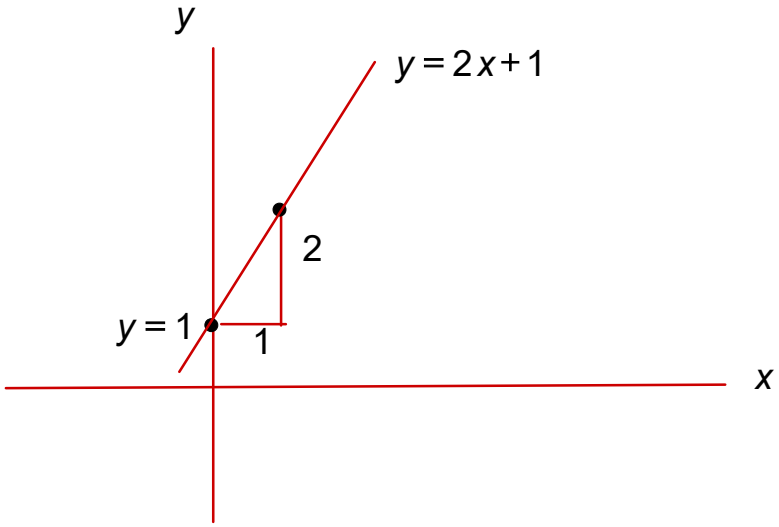


ex1: graph y=2x+1

$m = 2 = 2/1$ Every time x increases by 1, y increases by 2.

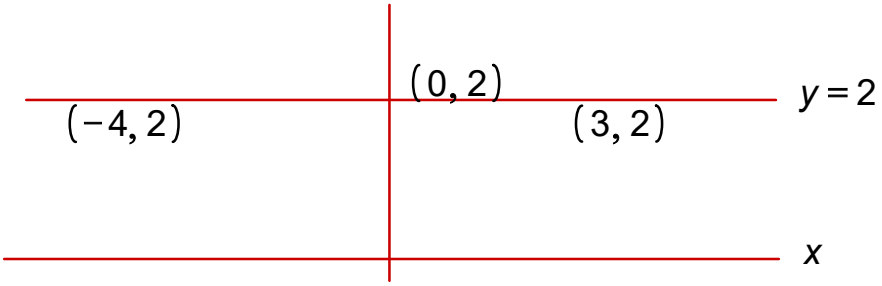
y intercept=1

1. mark y intercept
2. from y intercept go 1 right
3. turn 90 degrees
4. go 2 up
5. mark second point
6. connect points with line



ex 2: y=2 ..means y=0x+2, m=0, b=2, y=0x+2, y=0(1)+2 (1,2), y=0(3)+2, (3,2)

x can be any value but y is always 2!



ex 2: $x + y = 2$

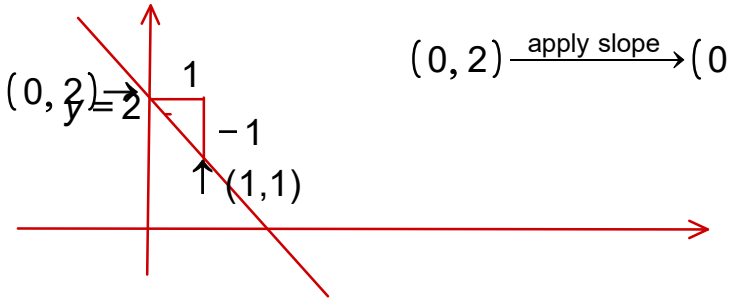
$x - x + y = 2 - x$

$0 + y = 2 - x$

$y = 2 - x$

$y = 2 - \frac{1}{1}x$

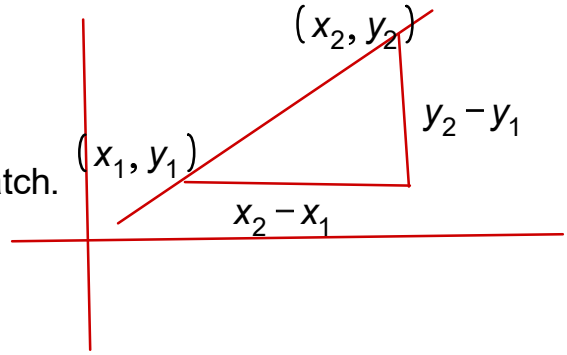
$y = 2 + \frac{-1}{1}x$, $b = 2$, $slope = -1/1$ (1 right , 1 down!)



$(0,2) \xrightarrow{\text{apply slope}} (0+1, 2-1) = (1,1)$

slope formula: $m =$ slope$= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{y_1 - y_2}{x_1 - x_2}$

but do not write $\dfrac{y_2 - y_1}{x_1 - x_2}$ ⇐ b/c subscripts don't match.



example 2 (book):
slope from (-1,2) to (2,2):

$(x_1 = -1, y_1 = 2), (x_2 = 2, y_2 = 2)$

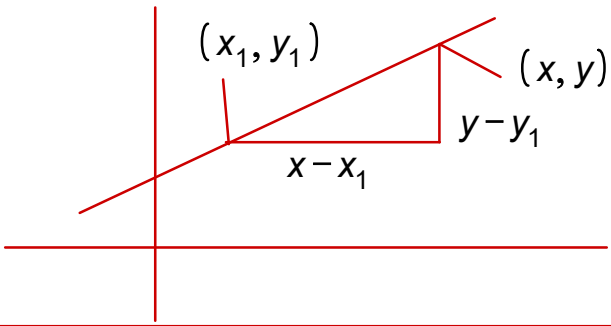
m= $\dfrac{2-2}{2-(-1)} = \dfrac{0}{2+1} = \dfrac{0}{3} = 0$ (should b/c our y-coords match)

slope from $(3,4)$ to (3,1) (x coords are the same for both)

$m = \dfrac{4-1}{3-3} = \dfrac{3}{0}$ ⇐ undefined! (~~slope= ∞~~) not a number

$\leftarrow (3,4)$

$\leftarrow (3,1)$



$\dfrac{m}{1} = \dfrac{y - y_1}{x - x_1}$ subscripted stuff is given!

cross multiply: $m(x - x_1) = 1(y - y_1)$

we get $m(x - x_1) = y - y_1$ ← point-slope b/c

we know (x_1, y_1) and m!

example 3(book): $m = 3, (1, -2)$

$y - (-2) = 3(x - 1)$ (this form reveals the slope and a point on the line)
we're going to make y=mx+b form(still has slope but has y intercept)
$y + 2 = 3x - 3$ (distribute 3 on RHS)

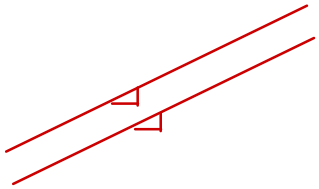
slope stays the same

$y + 2 - 2 = 3x - 3 - 2$

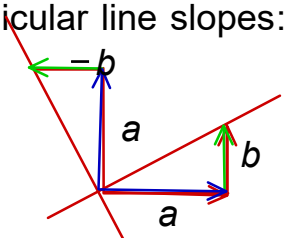
$y + 0 = 3x - 5$

$y = 3x - 5$ ⇐ slope=3, y intercept -5

example 4: parallel lines have the same slope



perpendicular line slopes:



slope=b/a

slope= $\frac{a}{-b}$

product: $\frac{a}{-b}\left(\frac{b}{a}\right)=-1$

line thru $(2,-1)$ and parallel to 2x-3y=5

parallel means same slope:

to get the slope, solve for y in 2x-3y=5:

$2x-2x-3y=5-2x$

$-3y=5-2x$

$\frac{-3y}{-3}=\frac{5}{-3}-\frac{2x}{-3}$

y= $-5/3+\frac{2}{3}x$ ⇐ slope is 2/3

$y=mx+b$ or y=b+mx

use this *with* (2,-1) and m=2/3 to find equation of parallel line:

$y-(-1)=\frac{2}{3}(x-2)$

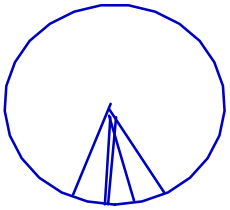
$y+1=\frac{2}{3}x-\frac{2}{3}\left(\frac{2}{1}\right)$

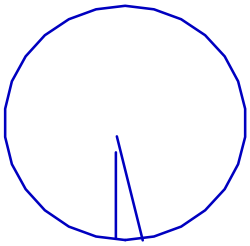
$y+1=\frac{2}{3}x-\frac{4}{3}$

$y=\frac{2}{3}x-\frac{4}{3}-1$

$y=\frac{2}{3}x-\frac{4}{3}-\frac{3}{3}$

$y=\frac{2}{3}x-\frac{7}{3}$





perp line: negative recp. $\frac{2}{3}\xrightarrow{\text{negate}}-\frac{2}{3}\xrightarrow{flip}-\frac{3}{2}$ ⇐ slope of perp. line

using still $(2,-1)$, slope=-3/2

$y-(-1)=-\frac{3}{2}(x-2)$

$y+1=-\frac{3}{2}x-\frac{3}{2}\left(\frac{-2}{1}\right)$

$y+1=-\frac{3}{2}x+\frac{6}{2}$

$y+1=-\frac{3}{2}x+3$

$y=-\frac{3}{2}x+2$

Using Slope as a Ratio:
The maximum recomennded slope of a wheelchair ramp is 1/12. A business is installing a wheelchair ramp that rises 22 inches over a length of 24 feet. Is the ramp steeper than recommended?

recommend slope is $\frac{1}{12}\approx 0.083$

our slope: convert 24 feet to inches so units match

$24\,\cancel{ft}\left(\frac{12\,in}{1\,\cancel{ft}}\right)=24\cdot 12\,in=288\,in$

slope= $\frac{22\,in}{288\,in}=\frac{22\,\cancel{in}}{288\,\cancel{in}}=\frac{22}{288}\approx 0.076<0.083$

Example 7/book:
A college buys exercise equipment worth 12,000. The equipment has a useful life of 8 years. The salvage value (at the end of the 8 years) is 2000. Werite a linear equation that describes the book value(value at any time t between the purchase date and salvage date) each year.

A=(0 year, 12000$) $B=$(year 8, 2000$)

$m=$ slope= rate of value loss= $\frac{\text{dollars}}{time}=\frac{2000-12000}{8-0}=\frac{-10000}{8}=\frac{-1250\$}{year}$

make an equation: (time,Value)

the formula $y-y_1=m(x-x_1)$ now becomes $V-V_1=m(t-t_1)$

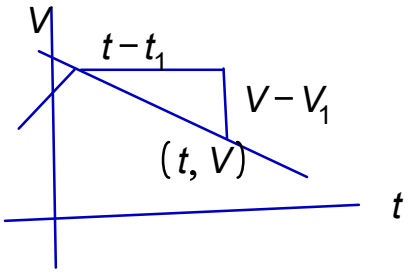
plug in: $V-12000=-1250(t-0)$ (using $(0, 12000)$) (t_1, V_1)

$V-12000=-1250t-1250(-0)$

$V-12000=-1250t+0$

$V-12000=-1250t$

$V=-1250t+12000$ ⇐ Give us the book value of equipment for any t.



$y=mx+b\rightarrow y-mx=b\rightarrow y-mx-b=0$

general form of line: $Ax+By+C=0$

$y=2x-4$

$y-2x+4=0$

$-2x+y+4=0$

$2x-y-4=0$ ← general form

As long as our moves are right, the forms are all equivalent.

$a=2, b=-1, c=-4$