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Page 60/Solving Systems with Three Variables:

Solve $\begin{cases} x+y-z=-1 \\ 4x-3y+2z=16 \\ 2x-2y-3z=5 \end{cases}$ $\Leftarrow$ three variables: x,y and z

Find the values the variables represent.

$\begin{cases} ax+by+cz=d \\ ey+fz=g \\ hz=i \end{cases}$ goal is to make this system

Goal is to produce an equivalent system at each stage. Equivalent means the solutions (x,y,z) remain the same even if the system looks different. The form above is good b/c once we know z , we can find y and then find x.

multiply eq1 by -4 and add to eq2:

$-4(x+y-z)=(-4)(-1)$

$-4x-4y+4z=4 \leftarrow$ add to second to make the new second equation

$$\begin{array}{r} 4x-3y+2z=16 \\ \hline -7y+6z=20 \end{array} \Leftarrow \text{ new second equation}$$

equivalent system: $\begin{cases} x+y-z=-1 \\ -7y+6z=20 \\ 2x-2y-3z=5 \end{cases}$

multiply eq1 by -2 and add to eq3:

$-2(x+y-z=-1) \rightarrow -2x-2y+2z=2$

$$\begin{array}{r} 2x-2y-3z=5 \\ \hline -4y-z=7 \end{array}$$

equivalent system: $\begin{cases} x+y-z=-1 \\ -7y+6z=20 \\ -4y-z=7 \end{cases}$

multiply -4y-z=7 by 6 and add to -7y+6z=20:

$6(-4y-z=7) \rightarrow -24y-6z=42$

$$\begin{array}{r} -7y+6z=20 \\ \hline -31y=62 \end{array}$$

equivalent system: $\begin{cases} x+y-z=-1 \\ -7y+6z=20 \\ -31y=62 \end{cases}$

solve for y :

$-31y=62$

$y=62/-31$

$y=-2$

use y=-2 in second:

$-7(-2)+6z=20$

$14+6z=20$

$6z=20-14$

$6z=6$

$z=1$

Use top equation x+y-z=-1 with y=-2 and z=1:

$x+(-2)-1=-1$

$x-3=-1$

$x=-1+3$

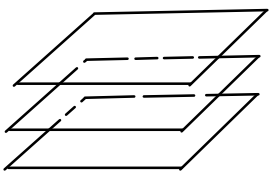
$x=2$

So the solution is

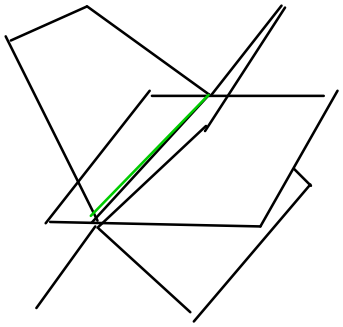
$(2,-2,1) \Leftarrow$ Triplet!

So if we plug these into the original system, the LHS in each equation will be = RHS!

A system with a solution of the form (x,y,z) is called consistent. A system with a solution like (x,y,z) (one point only) represents the intersection point of three planes b/c each equation in the system is a plane.



These planes do not intersect at a point, so no solution to whatever system represents them. Inconsistent.



In this case the three planes meet in the green line, so there are infinite solutions.

Consistent but infinite number of solutions.

Solve $\begin{cases} -x-3y-2z=22 \\ -3x-3y-2z=30 \\ -2x+y-z=7 \end{cases}$

goal is to produce
a system like
three variables=constant
two variables =constant
one variable=constant

multiply top by -3 to add to second equation so -3x vrom second equation disappears:

$-3(-x-3y-2z=22)\rightarrow 3x+9y+6z=-66$

eq2: $-3x-3y-2z=30\downarrow$ add

$6y+4z=-36\leftarrow$ new second equation

multiply top equation by -2 and add to third equation:

$-2(-x-3y-2z=22)\rightarrow 2x+6y+4z=-44$

eq3: $-2x+y-z=7\downarrow$ add

$7y+3z=-37\leftarrow$ new third equation

equivalent system: $\begin{cases} -x-3y-2z=22 \\ 6y+4z=-36 \\ 7y+3z=-37 \end{cases}$

Assuming all the algebra is correct, the solution to this system (x,y,z) is the same as the solution to the original system.

Could solve the y-z system using substitution or elimination and just use those values in eq1 to get x.

goal is to get rid of z in bottom two equations: 4 and 3..what's the LCM of 4 and 3? 12

$-3(6y+4z=-36)\rightarrow -18y-12z=108$

$4(7y+3z=-37)\rightarrow 28y+12z=-148\downarrow$ add

$10y=-40$

equivlaent system:

$\begin{cases} -x-3y-2z=22 \\ 6y+4z=-36 \\ 10y=-40 \end{cases}$

Back-substitution:

$10y=-40$

$y=-4$

Plug into second equation:

$6(-4)+4z=-36$

$-24-4z=-36$

$-4z=-36+24$

$-4z=-12$

$z=-12/4=-3$

Plug into top equation with z=-3 and y=-4:

$-x-3(-4)-2(-3)=22$

$-x+12+6=22$

$-x+18=22$

$-x=22-18$

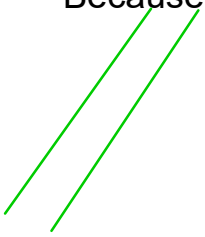
$-x=4\rightarrow x=-4!$

Solution point:

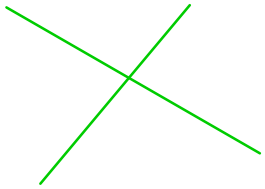
$(-4,-4,-3)$

Check in original to make sure it works!

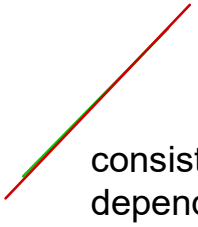
Because we have a single, unique solution, the system is consistent independent.



incosistent



consistent
independent



consistent
dependent