Make sure your class notes are saved together with your homework solutions in your PDF.
Section 2.2/Gaussian Elimination:

A matrix is a rectangular array of numbers.

$$\begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \\ a_{3,1} & a_{3,2} & a_{3,3} \end{bmatrix}$$

rows

column

$a_{1,3} \leftarrow$ entry in row 1, column 1

$a_{2,2}$ = entry in row 2, column 2

$a_{3,1}$ = entry in row 3, column 1

Goal is still to solve systems but do it in a more efficient way (more compact notation)

Augmented Matrix:

$$\begin{cases} 2x+3y=5 \\ -1x+4y=10 \end{cases} \Leftarrow$$ 2 by 2 system (two variables, two equations)

matrix form: $\begin{pmatrix} 2 & 3 & 5 \\ -1 & 4 & 10 \end{pmatrix} \Leftarrow$ augmented matrix for the system b/c 5 and 10 appear

All the variables get stripped away.

$$\begin{cases} 1x+1y+1z=3 \\ -1x+2y-1z=5 \\ 2x-1y+3z=10 \end{cases} \xrightarrow{\text{augmented matrix form}} \begin{bmatrix} 1 & 1 & 1 & 3 \\ -1 & 2 & -1 & 5 \\ 2 & -1 & 3 & 10 \end{bmatrix}$$

coeffcients RHS of system

Let's solve a system using this:
Allowed row operations on matrices: (matrix is singular, matrices is plural)
example 3/page 72:

$$\begin{cases} x+2y=3 \\ 4x-1y=2 \end{cases} \xrightarrow{\text{augmented matrix form}} \begin{pmatrix} 1 & 2 & 3 \\ 4 & -1 & 2 \end{pmatrix} \Leftarrow$$ same goal..make a matrix where

of the form $\begin{pmatrix} a & b & c \\ 0 & 1 & e \end{pmatrix}$

1 mean 1y=e

Where the 4 is we want 0:

$$\begin{pmatrix} 1 & 2 & 3 \\ 4-4(1) & -1-4(2) & 2-4(3) \end{pmatrix} \leftarrow R_2 = r_2 - 4r_1,\ R_2 = \text{new row 2}$$

r_2 = current version of row 2

$$= \begin{pmatrix} 1 & 2 & 3 \\ 4-4 & -1-8 & 2-12 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 & 3 \\ 0 & -9 & -10 \end{pmatrix} \xrightarrow{\text{divide row 2 by -9}} \begin{pmatrix} 1 & 2 & 3 \\ 0/-9 & -9/-9 & -10/-9 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 10/9 \end{pmatrix}$$

x y x y

$y = 10/9$ b/c $(0 \quad 1 \quad 10/9)$ means 1y=10/9

Since y=10/9, plug into top equation and get x: $x+2\left(\frac{10}{9}\right)=3$ solution point $\left(\frac{10}{9},\frac{7}{9}\right)$

$$x+\frac{20}{9}=3$$

$$x=3-\frac{20}{9}$$

$$x=\frac{27-20}{9}$$

$$x=\frac{7}{9}$$

allowed row operations on matrices:

1 . swapping rows b/c placemnt of any row in a system is made up, so you're free to swap rows.

2. dividing a row by a constant

3. multiplying rows by constants and adding them together

this is all just arithmetic: +,-,·, /

Solve using matrices:

$$\begin{cases} x-y+z=8 \\ 2x+3y-z=-2 \\ 3x-2y-9z=9 \end{cases}$$

basic goal:

$$\begin{bmatrix} a & b & c & d \\ 0 & e & f & g \\ 0 & 0 & 1 & h \end{bmatrix}$$

$x \quad y \quad z$

bottom row $(0\ 0\ 1\ h)$

$1z=h$

$z=h \Leftarrow$ we know z!!

augmented form $\rightarrow$

$$\begin{bmatrix} 1 & -1 & 1 & 8 \\ {\color{red}2} & 3 & -1 & -2 \\ {\color{red}3} & -2 & -9 & 9 \end{bmatrix}$$

$x \quad y \quad z \quad$ **R**HS

zero out the 2 and 3 in red.

$R_2=r_2-2r_1$

$R_3=r_3-3r_1$

$$\begin{bmatrix} {\color{red}1} & {\color{red}-1} & {\color{red}1} & {\color{red}8} \\ 2-2({\color{red}1}) & 3-2({\color{red}-1}) & -1-2({\color{red}1}) & -2-2({\color{red}8}) \\ 3-3({\color{red}1}) & -2-3({\color{red}-1}) & -9-3({\color{red}1}) & 9-3({\color{red}8}) \end{bmatrix} \Leftarrow R_2=r_2-2r_1$$

simplify $\rightarrow$

$$\begin{bmatrix} 1 & -1 & 1 & 8 \\ {\color{red}0} & 5 & -3 & -18 \\ {\color{red}0} & 1 & -12 & -15 \end{bmatrix} \Leftarrow \text{form} \begin{cases} 1x-1y+1z=8 \\ 5y-3z=-18 \\ 1y-12z=-15 \end{cases}$$

$R_2=r_3$

$R_3=r_2$

swap rows

$$\begin{bmatrix} 1 & -1 & 1 & 8 \\ 0 & 1 & -12 & -15 \\ 0 & {\color{red}5} & -3 & -18 \end{bmatrix}$$

Where the 5 is we want 0!

$$R_3=r_3-5\cdot r_2 \quad \begin{bmatrix} 1 & -1 & 1 & 8 \\ 0 & 1 & -12 & -15 \\ 0-5\cdot 0 & 5-5\cdot 1 & -3-5\cdot(-12) & -18-5(-15) \end{bmatrix}$$

simplify row 3 $\rightarrow$

$$\begin{bmatrix} 1 & -1 & 1 & 8 \\ 0 & 1 & -12 & -15 \\ 0 & 0 & 57 & 57 \end{bmatrix} \xrightarrow{R_3=r_3/57} \begin{bmatrix} 1 & -1 & 1 & 8 \\ 0 & 1 & -12 & -15 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

bottom row:

(0 0 1 1)

$1z=1$

$z=1!$

second row: $(0 \quad 1 \quad -12 \quad -15) \Leftarrow 1y-12z=-15$

$y-12(1)=-15$ (stick in z=1)

Solution point is
$(4, -3, 1)$

$y-12=-15$

$y=-15+12$

$y=-3$

top row: $(1 \quad -1 \quad 1 \quad 8)$

1x-1y+1z=8

$x-1(-3)+1=8$ (replace y and z)

x+3+1=8
x+4=8
x=8-4
x=4

$\mathbf{0\,x}$ (stuffed in) : $y+2z=0$ or similar

Homework Sample:

$$\begin{cases} x-2y+z=8 \\ \mathbf{0\,x}+1y+2z=0 \\ x+y+3z=4 \end{cases} \xrightarrow{\text{augmented matrix}} \begin{bmatrix} 1 & -2 & 1 & 8 \\ 0 & 1 & 2 & 0 \\ 1 & 1 & 3 & 4 \end{bmatrix}$$

coeffcients RHS

goal:

$$\begin{bmatrix} a & b & c & d \\ 0 & e & f & g \\ 0 & 0 & 1 & i \end{bmatrix}$$

$x \quad y \quad z \qquad z=i!\,!$

$$R_3 = r_3 - r_1 \xrightarrow{\text{subtract from row 3 the current version of row 1}}$$

$$\begin{bmatrix} 1 & -2 & 1 & 8 \\ 0 & 1 & 2 & 0 \\ 1-1 & 1-(-2) & 3-1 & 4-8 \end{bmatrix} = \begin{bmatrix} 1 & -2 & 1 & 8 \\ 0 & 1 & 2 & 0 \\ 0 & 3 & 2 & -4 \end{bmatrix}$$

$$R_3 = r_3 - 3r_2 \rightarrow \begin{bmatrix} 1 & -2 & 1 & 8 \\ 0 & 1 & 2 & 0 \\ 0-3\cdot 0 & 3-3\cdot 1 & 2-3\cdot 2 & -4-3\cdot 0 \end{bmatrix} = \begin{bmatrix} 1 & -2 & 1 & 8 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & -4 & -4 \end{bmatrix}$$

$$R_3 = r_3 / -4 : \quad \begin{bmatrix} 1 & -2 & 1 & 8 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

the bottom says (0 0 1 1):
$z=1!$

second row: $(0 \quad 1 \quad 2 \quad 0)$

$1y+2z=0$

$y+2(1)=0$

$y=-2$

top row: $(1 \quad -2 \quad 1 \quad 8)$

equation form: $1x-2y+1z=8$

We know y=-2 and z=1, so plug in:

$x-2(-2)+1(1)=8$

$x+4+1=8$

$x+5=8$

$x=8-5$

$x=3$

Solution point:
$(3, -2, 1)$

For homework, you are to show a sequence of steps very similar (not the same) to these.

1. Matrices are shown.
2. And things like $R_3 = r_3 - 3r_1$ are shown!