Make sure your class notes get loaded with your homework solutions.

1 Solve $\begin{cases} x-2y+z=-1 \\ y+2z=5 \\ x+y+3z=6 \end{cases}$ $\xrightarrow{\text{augmented system}}$ $\left[\begin{array}{cccc} 1 & -2 & 1 & -1 \\ 0 & 1 & 2 & 5 \\ 1 & 1 & 3 & 6 \end{array}\right]$ 2

4 coefficients RHS

$\xrightarrow[3]{R_3 = r_3 - r_1}$ $\left[\begin{array}{cccc} 1 & -2 & 1 & -1 \\ 0 & 1 & 2 & 5 \\ 1-1(1) & 1-1(-2) & 3-1(1) & 6-1(-1) \end{array}\right] = \left[\begin{array}{cccc} 1 & -2 & 1 & -1 \\ 0 & 1 & 2 & 5 \\ 0 & 3 & 2 & 7 \end{array}\right]$ 5

6 $\xrightarrow{R_3 = r_3 - 3r_2}$ $\left[\begin{array}{cccc} 1 & -2 & 1 & -1 \\ 0 & 1 & 2 & 5 \\ 0-3(0) & 3-3(1) & 2-3(2) & 7-3(5) \end{array}\right] = \left[\begin{array}{cccc} 1 & -2 & 1 & -1 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & -4 & -8 \end{array}\right]$ 8

7

9 $\xrightarrow{R_3 = r_3 / -4}$ $\left[\begin{array}{cccc} 1 & -2 & 1 & -1 \\ 0 & 1 & 2 & 5 \\ 0/-4 & 0/-4 & -4/-4 & -8/-4 \end{array}\right] = \left[\begin{array}{cccc} 1 & -2 & 1 & -1 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 1 & 2 \end{array}\right]$ (11)

x y z RHS

10 x y z RHS

(12)

back-substitution:

from (11) $z=2$

plug into row 2 of (11)

$1y+2(2)=5$

$y+4=5$

$y=5-4$

$y=1$

In row 1 of (11) (13)

replace y with 1 and z with 2:

$x-2(1)+2=-1$

$x-2+2=-1$

$x=-1$

Solution point: (14)

$(-1, 1, 2)$

Echelon Form

(Echelon means ladder in French)

Application:

A movie theater has a seating capacity of 179. The theater charges 5 for children, 7 for students and 12 for adults . There are half as many adults as there are children. If the total ticket sales was 1280, how many children, students and adults attendend that night?

(1) a=number of adults, c=number of children, s=number of students (introduce and define variables)

(2) a+c+s=179 (every seat is filled ..total people fill all the seats)

(3) $a = 1/2\,c$ (half as many adults as kids)

(4) money earned: $12a+5c+7s=1280$ (12a means 12 per adult · number of adults a and so on)

(5) transform (3) : $2a=c \rightarrow 2a-c=0 \xrightarrow{\text{want equation with three variables}} 2a-c+0s=0$

(6) system: $\begin{cases} a+c+s=179 \\ 12a+5c+7c=1280 \\ 2a-c+0s=0 \end{cases}$ $\xrightarrow{\text{augmented matrix}}$ $\left[\begin{array}{cccc} 1 & 1 & 1 & 179 \\ 12 & 5 & 7 & 1280 \\ 2 & -1 & 0 & 0 \end{array}\right]$ $\Leftarrow$ *must* be in HWORK!

(7)

(8) $R_2 = r_2 - 12r_1$

$R_3 = r_3 - 2r_1$

(9) $\left[\begin{array}{cccc} 1 & 1 & 1 & 179 \\ 12-12(1) & 5-12(1) & 7-12(1) & 1280-12(179) \\ 2-2(1) & -1-2(1) & 0-2(1) & 0-2(179) \end{array}\right]$ $\Leftarrow$ two operations!

(10) $\left[\begin{array}{cccc} 1 & 1 & 1 & 179 \\ 0 & -7 & -5 & -868 \\ 0 & -3 & -2 & -358 \end{array}\right]$ $\xrightarrow{(11)\ R_3 = -3r_2 + 7r_3}$ $\left[\begin{array}{cccc} 1 & 1 & 1 & 179 \\ 0 & -7 & -5 & -868 \\ 0 & -3(-7)+7(-3) & -3(-5)+7(-2) & -3(-868)+7(-358) \end{array}\right]$ (12)

(13) $\begin{bmatrix} 1 & 1 & 1 & 179 \\ 0 & -7 & -5 & -868 \\ 0 & 0 & 1 & 98 \end{bmatrix}$

a c s

(14) $s = 98$

(15) using row 2 of (13):
$-7c - 5(98) = -868$
$-7c - 490 = -868$
-7c= $-868 + 490$
$-7c = -378$
$c = -378 / -7 = 54$

(16) using c=54 from (15) and s=98 from (14) plug into row 1 of (13):
$a + 54 + 98 = 179$
$a + 152 = 179$
$a = 179 - 152$
$a = 27$

(17) So we have 27 adults, 54 children and 98 students!

Your homework should should between 15 and 20 steps. Your work should not show substitution as the method of solution until the very end where you have a matrix of the form $\begin{bmatrix} a & b & c & d \\ 0 & e & f & g \\ 0 & 0 & h & i \end{bmatrix}$ ⇐ This form should be in your notes!