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Section 3.1/Matrix Algebra:
Def: a matrix is a rectangular array of numbers.

$\begin{pmatrix} 1 & 2 & 3 \\ -5 & 6 & 7 \end{pmatrix}$ $\begin{bmatrix} -4 & 5 \\ 4 & 6 \end{bmatrix} \leftarrow$ 2 row by 2 column matrix

example 1: In a survey of 900 people, we got this information:
200 males said federal defense spending was too high
150 males said federal spending was too low
45 males had no opinion
315 feamls said federal defenese spending was too high
125 females said defense spending was too low
65 females had no opinion

tabular display:

	Too High	Too Low	No Opinion
Male	200	150	45
Female	315	125	65

Just one possible display format.

Matrix form: $\begin{pmatrix} 200 & 150 & 45 \\ 315 & 125 & 65 \end{pmatrix} \Leftarrow$ 2 row by 3 column matrix 2×3

example 2(our own)
$[2 \quad 1] \leftarrow$ 1 *row* by 2 column matrix

$\begin{bmatrix} 1 \\ 2 \end{bmatrix} \leftarrow$ 2 row by 1 column matrix

2×1

$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \leftarrow$ 3 row by 3 column matrix 3×3

$[1] \leftarrow$ 1 row by 1 column, 1×1

In general, for a matrix, the dimension is $m \times n$ (m=number of rows, n=number of cols!)

$$\begin{bmatrix} a_{11} & a_{1,2} & a_{1,3} & a_{1,4} \\ a_{2,1} & a_{2,2} & \boxed{a_{2,3}} & a_{2,4} \\ \boxed{a_{3,1}} & a_{3,2} & a_{3,3} & a_{3,4} \end{bmatrix}$$

$a_{2,3}$ = entry in row 2, column 3

$a_{3,1}$ = entry in row 3, column 1

Matrix equality: Two matrices (not ~~matrixes~~) are equal if their corresponding(entries in the same position) are equal.

example 4: $\begin{bmatrix} p & q \\ 1 & 0 \end{bmatrix}$ $\begin{bmatrix} 2 & 4 \\ n & 0 \end{bmatrix}$ $p = 2$ b/c they're both at row 1, index 1

$q = 4$ b/c they're both in row 1, column 2

$1 = n$ b/c they're both in row 2, column 1

$0 = 0$ for reasons above!

example 5:

$$A=\begin{bmatrix} x+y & 6 \\ 2x-3 & \boxed{2-y} \end{bmatrix} \qquad B=\begin{bmatrix} 5 & 5x+2 \\ y & \boxed{x-y} \end{bmatrix}$$

$$\left.\begin{array}{ll} x+y=5 & 5x+2=6 \\ 2x-3=y & \boxed{2-y=x-y} \end{array}\right\}$$

We have 4 equations but only TWO unknowns!

solve 5x+2=6 for x: $5x=4 \to x=4/5$

solve 2-y=x-y for x: add y: $2\cancel{-y+y}=x\cancel{-y+y} \to 2=x$

Our system has no solution.

Contradiction

x=4/5

x=2

cannot be!

example 6: Motors, Inc. makes three models of cars: a sedan, a convertible, and an SUV.

Table 1 (given):

Month 1:	Sedan	Convertibles	SUV
Units of Material	23	16	10
Units of Labor	7	9	11

column header (SUV)

row header ↑ (Units of Labor)

$$A=\begin{bmatrix} 23 & 16 & 10 \\ 7 & 9 & 11 \end{bmatrix}$$ dimension: 2×3 (total of 6 entries)

Month 2: $$B=\begin{pmatrix} 18 & 12 & 9 \\ 14 & 6 & 8 \end{pmatrix}$$

18 means 18 units of material for sedans!

6 means 6 units of labor for convertibles

$$C=A+B=\underset{2\times 3}{\begin{pmatrix} 23 & 16 & 10 \\ 7 & 9 & 11 \end{pmatrix}}+\underset{2\times 3}{\begin{pmatrix} 18 & 12 & 9 \\ 14 & 6 & 8 \end{pmatrix}}=\underset{2\times 3}{\begin{pmatrix} 23+18 & 16+12 & 10+9 \\ 7+14 & 9+6 & 11+8 \end{pmatrix}}=\begin{pmatrix} 41 & 28 & 19 \\ 21 & 15 & 19 \end{pmatrix}$$

The value 28 in the sum means 28 units of material for convertibles!

The value 21 means the total units of labor for the two months for sedans.

example 7(book): $\underset{1\times 2}{[1 \quad 2]}+\underset{2\times 1}{\begin{bmatrix} 1 \\ 3 \end{bmatrix}}$ ⇐ can't add these 1+1, but 2+?

2 is in position row 1, column 2

the column matrix doesn't have a row 1, column 2!

$$\underset{2\times 2}{\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}}+\underset{1\times 2}{[-6 \quad 5]}$$

4 is in position row 2, column 2 but

in [-6   5] there is now row 2, column 2!

properties of matrix addition: $A+B=B+A$ (matrix addition is commutative) $\begin{array}{l} 2+3=3+2 \\ a+b=b+a \end{array}$

$A+(B+C)=(A+B)+C$ (floating parenthesis to regroup is called the "associative property of addition") based on $2+(3+4)=(2+3)+4$ $\quad a+(b+c)=(a+b)+c$

example 9: Zero Matrix: recall $a+0=a$ (0 is called the additive identity element)

$$\begin{bmatrix}1 & 2\\ 4 & 3\end{bmatrix}+\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}=\begin{bmatrix}1+0 & 2+0\\ 4+0 & 3+0\end{bmatrix}=\begin{bmatrix}1 & 2\\ 4 & 3\end{bmatrix}$$

$A+\mathbf{0}=A$ (A=matrix)

2×2 $\quad 2\times 2$

$\mathbf{0}$ (bold zero) ..it's the zero matrix

additive inverse: $2+(-2)=0$ (2 and -2 are additive inverses)

$-3+(3)=0$ (3 and -3 are additive inverses)

$a+(-a)=0$ (so a and -a are additive inverses b/c their sum is 0)

$[1 \quad 2]+[-1 \quad -2]=[1-1 \quad 2-2]=[0 \quad 0]$ (row matrix)

1×2 $\quad 1\times 2$

column matrix

$$\begin{bmatrix}-3\\ 4\end{bmatrix}+\begin{bmatrix}3\\ -4\end{bmatrix}=\begin{bmatrix}-3+3\\ 4-4\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

we add this b/c -3+3=0, 4-4=0 the matrix $\begin{bmatrix}3\\ -4\end{bmatrix}$ is the additive inverse of $\begin{bmatrix}-3\\ 4\end{bmatrix}$

$$\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}+\begin{bmatrix}-1 & -2\\ -3 & -4\end{bmatrix}=-A$$

A

putting this in
b/c 1-1=0, 2-2=0
3-3=0, 4-4=0

$A+(-A)=\mathbf{0}$

$A\,and-A$ are additive inverses b/c their sum is the zero matrix.

Example 11: Differences of Matrices:

$$[1 \quad 2]-\begin{bmatrix}3\\ 4\end{bmatrix}$$

Could do 1-3=-2, but 2-? ??

1by2 $\quad 2\times 1$

Subtraction needs to have matrices of the same size.

$[1 \quad 2]-[3 \quad 4]=[1-3 \quad 2-4]=[-2 \quad -2]$

1×2 $\quad 1\times 2$ $\quad 1\times 2$

$$\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}-\begin{bmatrix}4 & -3\\ -3 & 2\end{bmatrix}=\begin{bmatrix}1-4 & 2-(-3)\\ 3-(-3) & 4-2\end{bmatrix}=\begin{bmatrix}-3 & 5\\ 6 & 2\end{bmatrix}$$

2×2 $\quad 2\times 2$

$$\underset{2\times 4}{\begin{pmatrix} 2 & 4 & 8 & -3 \\ 0 & 1 & 2 & 3 \end{pmatrix}} - \underset{2\times 4}{\begin{pmatrix} -3 & 4 & 0 & 1 \\ 6 & 8 & 2 & 0 \end{pmatrix}} = \underset{2\times 4}{\begin{pmatrix} 2-(-3) & 4-4 & 8-0 & -3-1 \\ 0-6 & 1-8 & 2-2 & 3-0 \end{pmatrix}} = \underset{2\times 4}{\begin{pmatrix} 5 & 0 & 8 & -4 \\ -6 & -7 & 0 & 3 \end{pmatrix}}$$