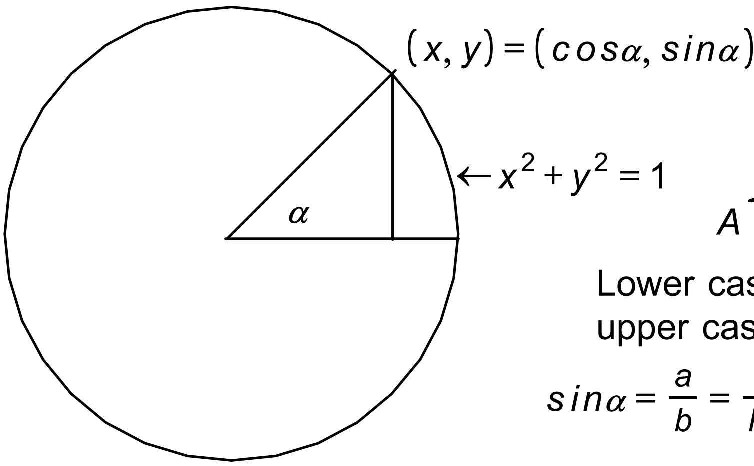
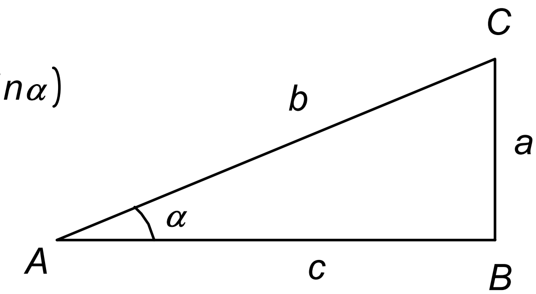
Section 4.3/Right Triangle Trig.





Lower case letters are placed opposite the upper case letters that indicate the angles.

$$\sin\alpha = \frac{a}{b} = \frac{\text{opposite}}{hypotenuse} \qquad \csc\alpha = \frac{b}{a} = 1/\sin\alpha = \frac{1}{\frac{a}{b}}$$

$$\cos\alpha = \frac{c}{b} = \frac{\text{adjacent}}{hypotenuse} \qquad \sec\alpha = \frac{b}{c} = \frac{1}{\cos\alpha} = \frac{1}{\frac{c}{b}}$$

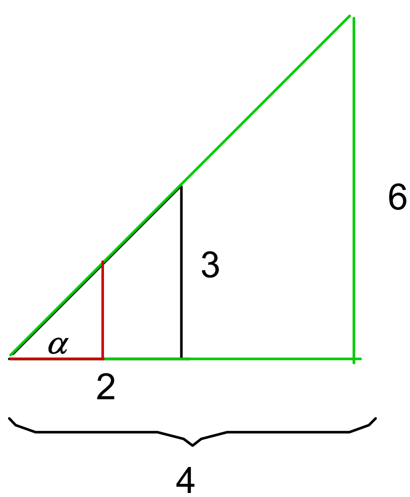
$$\tan\alpha = \frac{a}{c} = \frac{\text{opposite}}{\text{adjacent}}$$

$$\cot(\alpha) = \frac{\text{adjacent}}{\text{opposite}} = \frac{c}{a} = \frac{1}{\tan\alpha} = \frac{1}{\frac{a}{c}}$$

Mnemonic Device:
S OH C AH T OA $\Leftarrow$ *it's* got to be spelled this way!!

"Some old HOG came around here and took our apples".

These are all ratios, so infinitely many possible side combinations exist for any ONE value of a trig function.



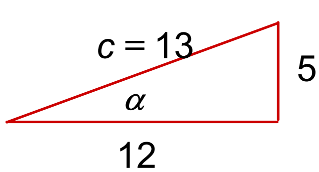
$$\tan\alpha = \frac{3}{2} = 1.5$$

$$\tan(\alpha) = \frac{6}{4} = \frac{3\cdot 2}{2\cdot 2} = \frac{3}{2} = 1.5$$

$$\tan(\alpha) = \frac{3/2}{2/2} = \frac{1.5}{1} = 1.5$$

Any fraction can expressed in infinitely many ways.

example 1: Evaluate trig functions:



$\alpha = ??$ Not given!!!

$c^2 = 12^2 + 5^2$ by our friend the Pytagorean Theorem!

$$c^2 = 144 + 25$$

$$c^2 = 169$$

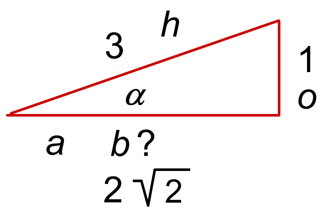
$$\sqrt{c} = \sqrt{169} = 13$$

$$\cos\alpha = \frac{12}{13} \qquad \tan\alpha = \frac{5}{12} \qquad \sec\alpha = \frac{13}{12}$$

$$\sin\alpha = \frac{5}{13} \qquad \cot\alpha = \frac{12}{5} \qquad \csc\alpha = \frac{13}{5}$$

Example 2:

Write down all the trig function values:)



What's opposite and what's adjacent depends on the position of the angle!!!

$a^2 + b^2 = c^2$

$b = ?,\ c = 3,\ a = 1$

$1^2 + b^2 = 3^2$

$b^2 = 9 - 1$

$b^2 = 8$

$\sqrt{b^2} = \sqrt{8}$

$b = \sqrt{4 \cdot 2} = \sqrt{4}\,\sqrt{2} = 2\sqrt{2}$

$cos\alpha = \dfrac{2\sqrt{2}}{3}$

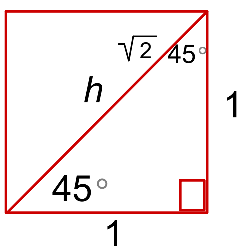
$sin\alpha = \dfrac{1}{3}$

$tan\alpha = \dfrac{1}{2\sqrt{2}} \xrightarrow{\text{rationalize}}$

$\dfrac{1}{2\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{1\sqrt{2}}{2\sqrt{2 \cdot 2}} = \dfrac{\sqrt{2}}{2\sqrt{4}} = \dfrac{\sqrt{2}}{2 \cdot 2} = \dfrac{\sqrt{2}}{4}$

sec, csc and cot ...:) on your own !!

Function values for some angles that have turned out to be useful, so we call them "special".



$1^2 + 1^2 = h^2$

$1 + 1 = h^2$

$2 = h^2$

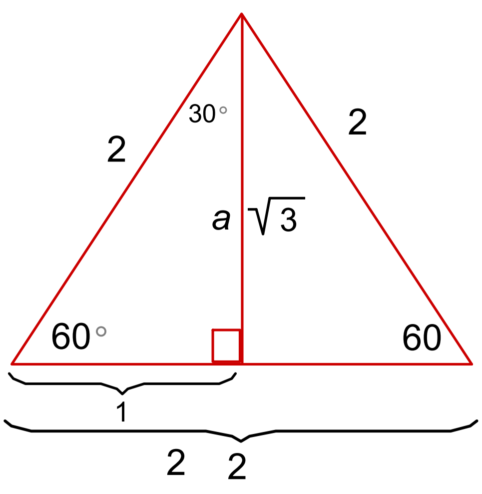
$h = \sqrt{2}$

recall π rad = $180^\circ \xrightarrow{\text{divide by 4}} \dfrac{\pi}{4}$ rad= 45°

$sin45^\circ = \dfrac{1}{\sqrt{2}} \xrightarrow{\text{rationalize}} \dfrac{1}{\sqrt{2}}\dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{\sqrt{2}}{2} \Leftarrow$ unit circ. ver.

$cos45^\circ = \dfrac{1}{\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{\sqrt{2}}{\sqrt{4}} = \dfrac{\sqrt{2}}{2}$

$tan(45^\circ) = \dfrac{1}{1} = 1$



$2\pi = 360^\circ$, π rad= 180°

divide by 3: $\dfrac{\pi}{3}$ rad= 60°

Fact sides are 2 is not important! Could be any number. Pretend it says a.By choosing 2, we get 2/2=1 , an easy value for the leg there.

$a = ?$

$1^2 + a^2 = 2^2$

$1 + a^2 = 4$

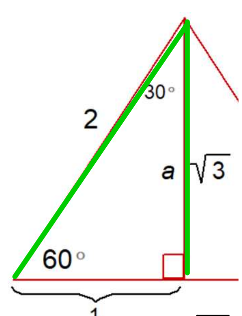
$a^2 = 4 - 1$

$a^2 = 3$

$\sqrt{a^2} = \sqrt{3}$

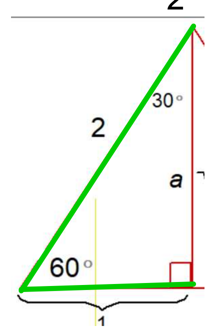
$a = \sqrt{3}$

$sin60^\circ = \dfrac{\sqrt{3}}{2}$

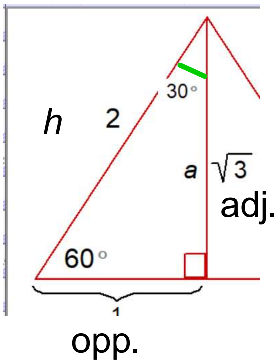


$sin\left(\dfrac{\pi}{3}\right) = \dfrac{\sqrt{3}}{2}$

$cos60^\circ = \dfrac{1}{2}$



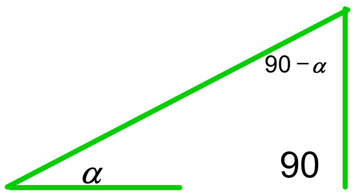
$cos\left(\dfrac{\pi}{3}\right) = \dfrac{1}{2}$



$$\sin(30^\circ) = \frac{opp}{hyp.} = \frac{1}{2} \qquad \sin(30^\circ) = \frac{1}{2} = \cos(60^\circ)$$

$$\cos(30^\circ) = \frac{adj}{hyp} = \frac{\sqrt{3}}{2} \qquad \cos(30^\circ) = \frac{\sqrt{3}}{2} = \sin(60^\circ)$$

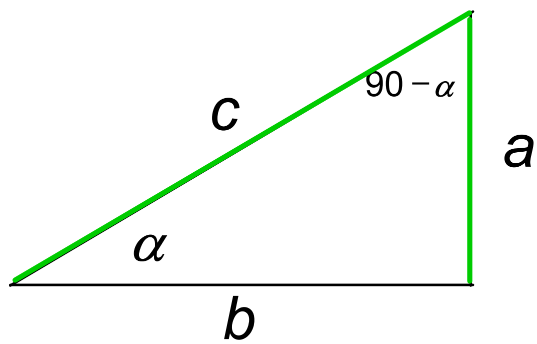
$30 + 60 = 90$ ... b/c it's a right triangle, one angle is already 90 degrees, so the other two, regardless of how we assign values, must be 90degrees



$\alpha + 90 - \alpha = 90^\circ$!

$\cos(\alpha) = \sin(90 - \alpha)$

α	$30^\circ = \frac{\pi}{6}$	$45^\circ = \frac{\pi}{4}$	$60^\circ = \frac{\pi}{3}$
$\sin\alpha$	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$
$\cos\alpha$	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$
$\tan\alpha$	$\sqrt{3}/3$	1	$\sqrt{3} = \frac{\sqrt{3}/2}{1/2}$



$\cos(\alpha) = \frac{b}{c}$ same $\frac{b}{c} = \sin(90 - \alpha)$

$\cos(\alpha) = \sin(90 - \alpha)$

$\tan(\alpha) = \frac{a}{b} \qquad \frac{a}{b} = \cot(90 - \alpha)$

Since both are a/b, we get that

$\tan\alpha = \cot(90 - \alpha)$

$\sin(\alpha) = \frac{a}{c}$ How else to get a/c?

$\cos(90 - \alpha) = \frac{a}{c}$

$\sin\alpha = \cos(90 - \alpha)$

These are called cofunction identities.
b/c cosine is like CO-SINE and SINE!
CO-TANGENT AND TANGENT!

Example 5: $\sin 72^\circ$

$\sin 72^\circ = \cos(90^\circ - 72^\circ) = \cos(18^\circ)$

$$\csc\left(\frac{\pi}{3}\right) = \sec\left(\frac{\pi}{2} - \frac{\pi}{3}\right)$$

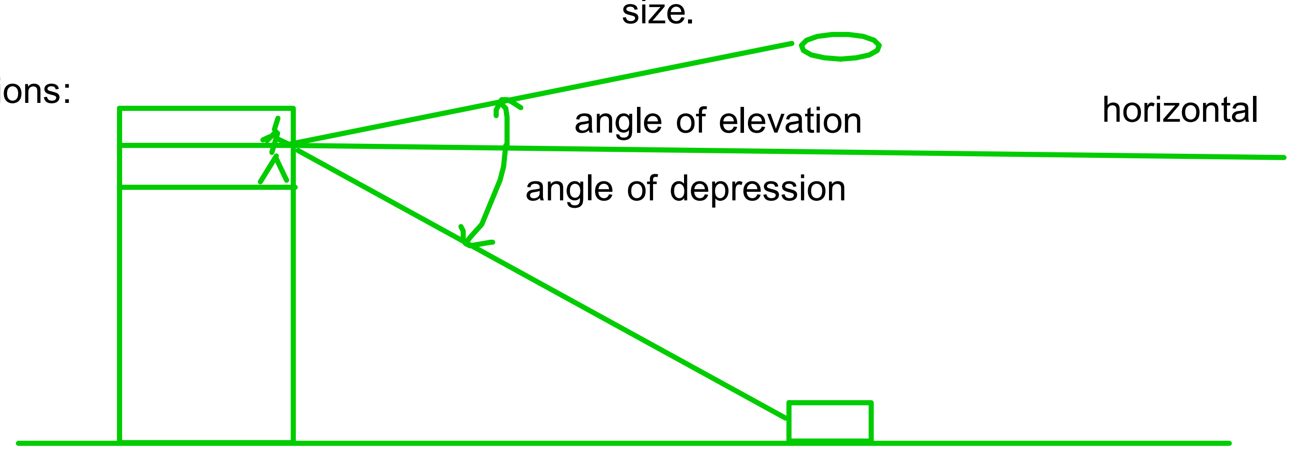
$$= \sec\left(\frac{3\pi}{6} - \frac{2\pi}{6}\right) = \sec\left(\frac{\pi}{6}\right)$$

π rad= 180°

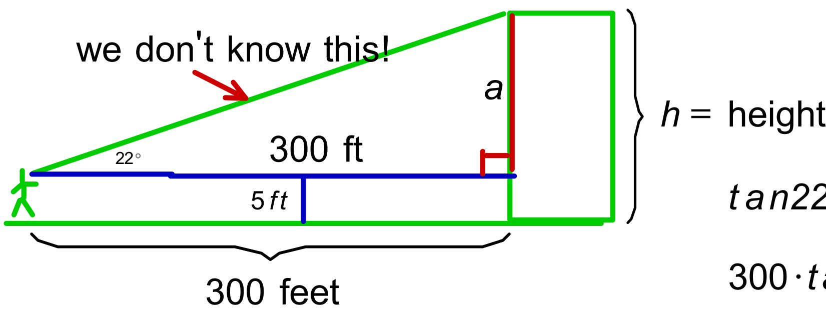
$\div 2 \rightarrow \frac{\pi}{2}$ rad= 90°

Imagine a pie and you cut pieces out.
We want to stack the pieces so they fit.
This happens only if they're the same size.

Applications:



Example 6: Problem Solving Using an Angle of Elevation:
Sighting the top of a building , a surveyor measured the angle of elevation to be 22 °. *The* transit (telescope on a tripod) is 5 feet about the ground and 300 feet from the building. Find the building's height.



knowns:
relative to 22 degrees:
adjancent side=300ft
opposite is called a

$$\tan 22^\circ = \frac{a}{300}$$

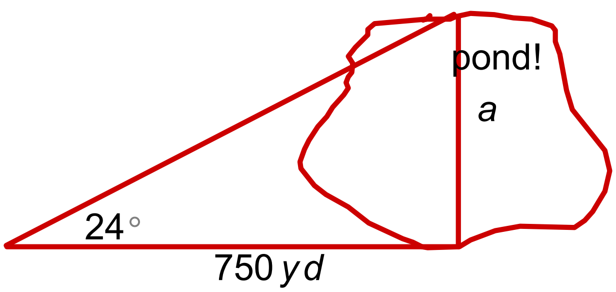
$$300 \cdot \tan 22^\circ = a$$

$$h = 5 + a = 5 + 300\tan(22^\circ)$$

$$h = 126 \text{ ft.}$$

Here, don't use cos or sine b/c the hyp. is not known.

example 2 of using trig:

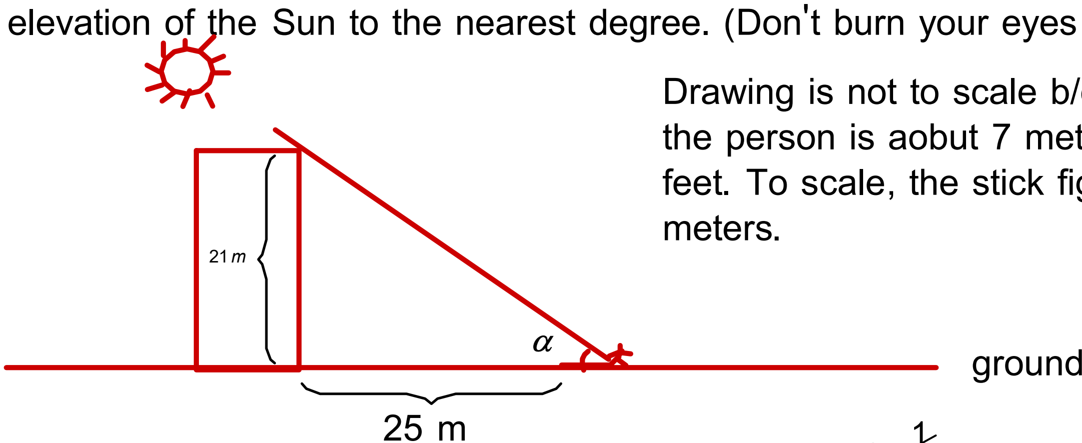


$$\tan 24^\circ = \frac{a}{750}$$

$$a = 750 \cdot \tan 24^\circ$$

Make sure it's in degree mode, so usually across top of calc. screen says **DEG** vs. RAD

Applied example 3: Determining the angle of elevation .
A building that is 21 meters tall casts a shadow 25 meters long. Find the angle of elevation of the Sun to the nearest degree. (Don't burn your eyes out!!!)



Drawing is not to scale b/c according to this the person is aobut 7 meters tall , which is 21 feet. To scale, the stick figure should be about 2 meters.

$$\tan(\alpha) = \frac{21\,m}{25\,m} = \frac{\text{opp}}{\text{adj}} = \frac{21}{25} \Rightarrow \tan(\alpha) = \frac{21}{25}$$

$$2 \cdot \frac{1}{2}x = x$$

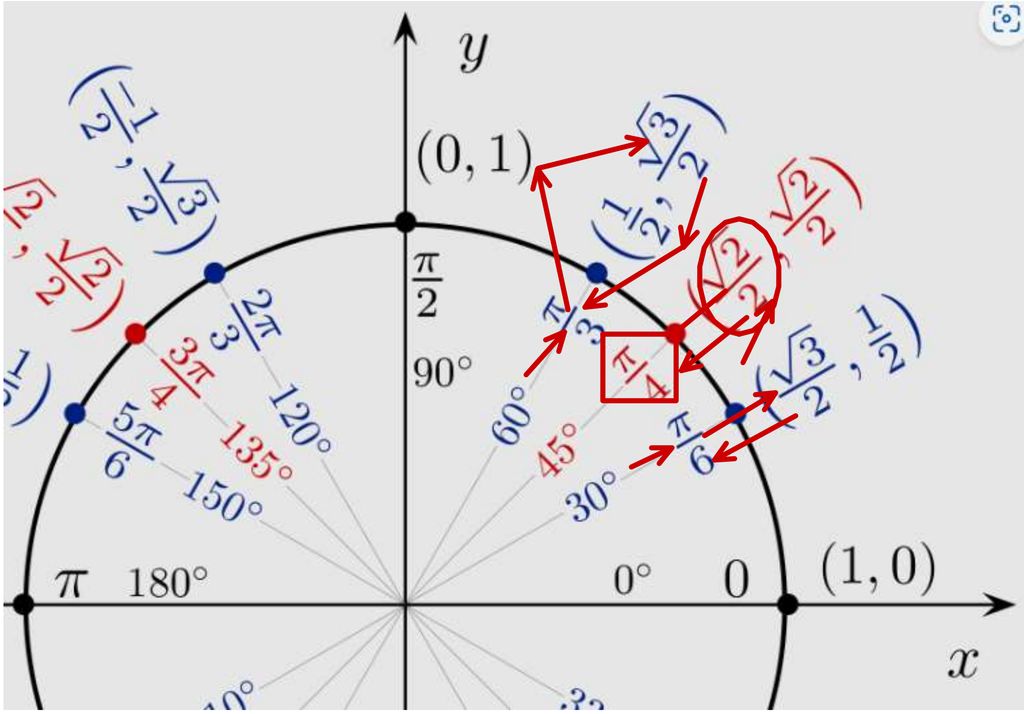
$$e^{\ln x} = x$$

We want to get α?

apply the INVERSE tangent function: $\tan^{-1}(\tan\alpha) = \tan^{-1}(21/25)$

wrap both sides in $\tan^{-1}$: $\tan^{-1}(\tan(\alpha)) = \tan^{-1}(21/25)$

$$\alpha = \tan^{-1}(21/25) \Leftarrow \text{calc. work} = 40^\circ.$$



$$\cos(\pi/4) = \frac{\sqrt{2}}{2}$$

$$\cos^{-1}\left(\frac{\sqrt{2}}{2}\right) \xrightarrow{\text{angle}} \frac{\pi}{4}$$

$$\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

$$\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$$

$$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

$$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$$