Math 111 Notes 9/18/2023. Make sure your class notes are loaded with your homework PDF.

Good notes are notes that, when reviewed two YEARS later, still make perfect sense.

1.6 in book:

$3x^4 = 48x^2$

do not divide by x b/c x=0 could be a solution.

We can divide by 3 b/c it's not 0.

$\frac{3x^4}{3} = \frac{48x^2}{3}$

$x^4 = 16x^2$

$x^4 - 16x^2 = 0$

factor x^2 out: $x^2(x^2-16)=0$

x^2-16 is a difference of two squares: $x^2(x^2-4^2)=0$

apply $x^2-y^2=(x-y)(x+y)$ $\quad x^2(x-4)(x+4)=0 \Leftarrow$ a· b· c=0, a=x^2,b=x-4, c=x+4

$x^2 \Leftarrow$ factor, x-4 is a factor, x+4 is a factor

Since only multiplication is left, set each factor to 0: $x^2=0 \to \sqrt{x^2} = \pm\sqrt{0} \to x=0$

$x-4=0 \xrightarrow{\text{add } 4} x=4$

$x+4=0 \xrightarrow{\text{subtract } 4} x=-4$

check with x=0:

safest is the original equation:

$3(0)^4 = ?48\cdot 0^2$

$3\cdot 0 = ?48\cdot 0$

$0=0$ true!

example 2:

$x^3-3x^2+3x-9=0$

we have all different exponents

what's the GCF in x^3 and $-3x^2$: x^2

what's the GCF between 3x and -9? : 3

reminder: $x^2\left(\frac{x^3}{x^2}-\frac{3x^2}{x^2}\right)=x^2(x-3)$

$(x\cdot x\cdot x-3\cdot x\cdot x)=x\cdot x(x-3)=x^2(x-3)$

group with these observations in mind: $(x^3-3x^2)+(3x-9)=0$

parenthesis around terms with common factors, and a + (not a minus) in the middle

factor from each set of parenthesis: $x^2(x-3)+3(x-3)=0$ $\qquad 3\left(\frac{3x}{3}-\frac{9}{3}\right)$

what's the GCF now? $(x-3)$: $\quad (x^2+3)(x-3)=0$

entire (x-3) is treated like a single unit. $\qquad =3(x-3)$

set each factor equal to 0: $\quad a\cdot b=0,\ a=0\ or\ b=0$

$x^2+3=0 \qquad x-3=0$

$x^2=-3 \qquad x=3$

$\sqrt{x^2} = \pm\sqrt{-3}$

$x=\pm i\sqrt{3}$

$x=\pm\sqrt{3}\,i$

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example 4 in book of Mathmagic:

$\sqrt{2x+7}-x=2$

square both sides:...LHS will be very hard to handle

add x: $\sqrt{2x+7}=2+x \Leftarrow$ b/c squaring $(\sqrt{\ldots})^2 = \ldots$

$(\sqrt{2x+7})^2=(2+x)^2 \Leftarrow$ both sides...maintain balance

$2x+7=2^2+2\cdot 2\cdot x+x^2$ FOIL RHS!

$2x+7=4+4x+x^2$ multiply out

make ax^2 +bx+c=0 form b/c we have x^2 present

$2x-2x+7-7=4-7+4x-2x+x^2$

$0=-3+2x+x^2$

$0=x^2+2x-3$ (move terms..not shift between sides..so don't change sign)

$0=(\ )(\ )$ two values that multiply to -3 and add to 2:

$0=(x-1)(x+3)$ (works b/c 3(-1)=-3 and -1+3=2)

$0=x-1 \qquad 0=x+3$

$1=x \qquad -3=x$

check x=-3: $\sqrt{2(-3)+7}-(-3)=?2$

$\sqrt{-6+7}+3=?2$

$\sqrt{1}+3=?2$

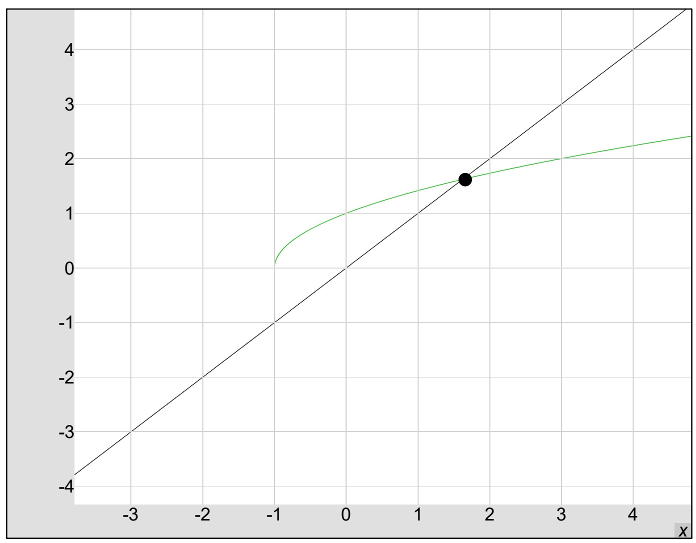
$1+3=?2$

$4=?2$ NO!

So x=-3 is not a solution. Why does it exsist then? B/c squaring make the equation into a shape like a parabola and this introduces one extra solution.

simple example:

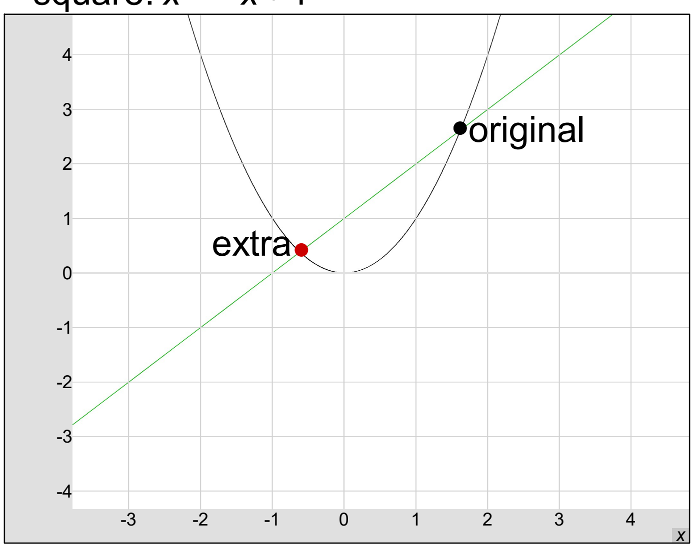
x



$\sqrt{x+1}$

$x=\sqrt{x+1}$

square: $x^2=x+1$



Squaring makes an extra solution appear that doesn't relate to the original equation.

example 6:
$|x-2|=3$

absolute value...to make 3, you can do |3|= 3 or |-3|= 3

either x-2=3 or x-2=-3

$x=3+2 \qquad x=-3+2$

$x=5 \qquad x=-1$

example 6 (in book)

$\frac{2}{x} = \frac{3}{x-2} - 1$ $\Leftarrow$ b/c we have fractions, it's called a rational equation (ratio) (fractional eqs)

clear the fractions away. How? multiply by the LCM!

$x(x-2)$

$\cancel{x}(x-2)\cdot\frac{2}{\cancel{x}} = x\cancel{(x-2)}\cdot\frac{3}{\cancel{x-2}} - 1(x)(x-2)$ (multiply every term by x(x-2))

$2(x-2) = 3x - x(x-2)$ cancel x, x-2

$2x - 4 = 3x - x^2 + 2x$ (distribute 2 and -x)

$2x - 4 = 5x - x^2$ $(3x + 2x = 5x)$

now make the form $ax^2 + bx + c = 0$

b/c we have $-x^2$, and -1 on x^2 makes things complicated when factoring, move $-x^2$ to RHS

$2x - 5x - 4 + x^2 = 5x - 5x - x^2 + x^2$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$ b/c (-4)(1)=-4, -4+1=-3

$x = 4$ $x = -1$

Since neither x is 0 or 2, both work.

In this case:

$\frac{2}{x} = \frac{3}{x-2} - 1 \Leftarrow$ rational equation

$x \neq 0$ or $x \neq 2$ or we'd have division by 0, which is not allowed.

Example 9: Compound Interest

Formula: $A = P(1 + r/n)^{nt}$, $A =$ future money, P=money being invested right now r=rate of interest (convert to decimal form), n=number of times we compound compounding means calculating interest and adding to the original amount, t=time over which we hold our investment

Imagine we invest P=1000 at a rate of 4% (.04) for 2 years(t=2) and we compound 4 times per year. (n=4).

$A =$ how much money we will have= $1000\left(1 + \frac{0.04}{4}\right)^{4\cdot 2} = 1000(1 + 0.01)^8$

Effective rate at the end of each period is .01 or 1% and we compound a total of 8 times.

$= 1000(1.01)^8$

$= 1082.86$ units of currency.

Interest=free money
$= 1082.86 - 1000 = 82.86$ units of currency.

$n = 1$ (annual=once per year)

$n = 2$ (semi-annual=twice per year)

n=4 (quarterly=4 times per year)

$n = 12$ (monthly=12 times per year)

$n = 365$ (daily=365 times per year)

Example 9:
Imagine when you were born , your grandparents deposted \$5000 in a long-term investment in which the interest was compounded quarterly.

Today, on your 25th birthday, the value of your investment is 25,062.59.
What is the annual interest rate for this investment?
P=5000(back 25 years ago we invested 5000)
Our A= 25062.59 (money present right now)
$n =$ quarterly=n, t=25 (25 years since year 0 of our birth)

r=? replace what we can:

$$25062.59 = 5000(1+r/4)^{4\cdot 25} \Leftarrow$$ replace as much as possible

$$\frac{25062.59}{5000} = \frac{5000}{5000}\left(1+\frac{r}{4}\right)^{100}$$ divide by 5000, $4 \cdot 25$=100 in top

$$5.0125 = \left(1+\frac{r}{4}\right)^{100}$$ Now we have 100 in the exponent.

What's the opposite of raising to the 100th?
so take the 100th root of both sides

$$\sqrt[100]{5.0125} = \sqrt[100]{\left(1+\frac{r}{4}\right)^{100}}$$

$$\sqrt[100]{5.0125} = 1+\frac{r}{4}$$ on RHS, 100 index of radical cancel with 100 in exponent

$$\sqrt[100]{5.0125} - 1 = \frac{r}{4}$$ subtract 1

$$4\left(\sqrt[100]{5.0125^{1}} - 1\right) = r$$ (multiply by 4 on both sides)

something multiplied by itself 100 times is 5.0125

$$4\left(5.0125^{1/100} - 1\right) = r$$ $\sqrt[2]{x^{1}} = x^{1/2}$, $\sqrt[3]{x^{1}} = x^{1/3}$

$r = 0.065$ or in percent form r= 6.5%