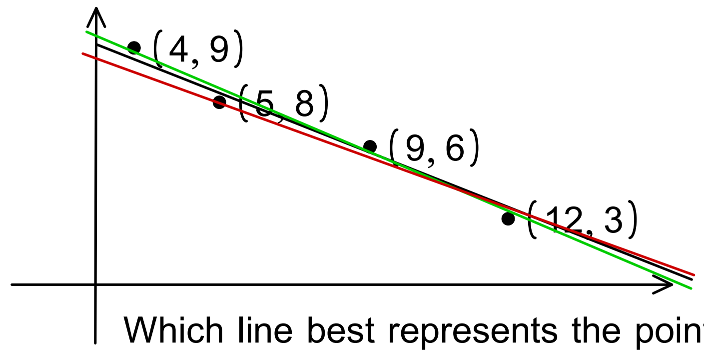
The Method of Least Squares:
Page 163:

Example 1: Finding the transpose of a matrix:

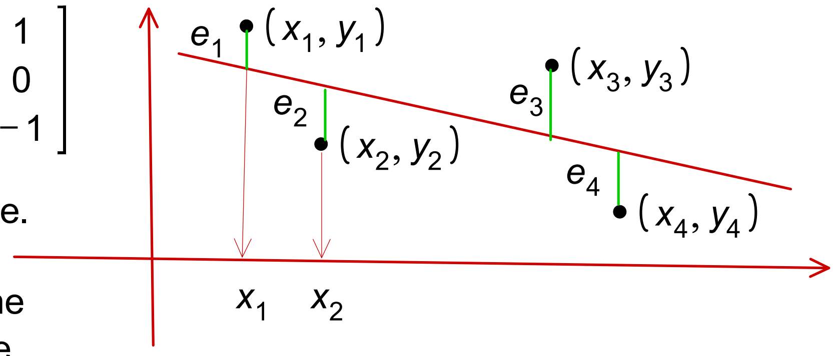
$$A=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}\xrightarrow[\text{flip rows and columns}]{A^T\text{ (read as A transpose)}}\begin{bmatrix}1 & 3\\ 2 & 4\end{bmatrix}$$

$$B=\begin{bmatrix}1 & 1\\ 0 & 1\\ 2 & 3\end{bmatrix}\xrightarrow{A^T\text{ (flip rows and columns)}}\begin{pmatrix}1 & 0 & 2\\ 1 & 1 & 3\end{pmatrix}$$

$$C=\begin{bmatrix}1 & 0 & -1\end{bmatrix}\xrightarrow{A^T\text{ (flip this row into a column)}}\begin{bmatrix}1\\ 0\\ -1\end{bmatrix}$$



Which line best represents the points?
y=mx+b ...we need m and b!
how to find m and b so all the points play some role?



e_1 =error between y_1 and y coord on line.

e_2 = error between y_2 and y coord on line
e_3 =error between y_3 and y coord on line
e_4 =error between y_4 and y coord on line

y_i for i=1 to 4 is the observed value
mx_i +b is the estimated value from line (i=1 to 4)

$$e_1=y_1-(mx_1+b) \qquad e_4=y_4-(mx_4+b)$$

$$e_2=y_2-(mx_2+b)$$

$$e_3=y_3-(mx_3+b)$$

Since $e_3=y_3-(mx_3+b)\rightarrow y_3=e_3+(mx_3+b)=$ error+ estimate=observed value

We want the sum $e_1^2+e_2^2+e_3^2+e_4^2$ to be as small as possible! This is just based on experience and some theory we're not going to cover here.

Method of Least Squares: to make $e_1^2+e_2^2+e_3^2+e_4^2$ as small as possible , we use the following process: y=mx+b and we have to find m and b and make sure all the points have an influence on the values of m and b.
Assume for simplicity: $(x_1, y_1), (x_2, y_2), (x_3, y_3), (x_4, y_4)$

$A^TAX=A^TY$ (why...not explaining here the reason for this formula)

$$A=\begin{bmatrix}x_1 & 1\\ x_2 & 1\\ x_3 & 1\\ x_4 & 1\end{bmatrix} \qquad X=\begin{bmatrix}m\\ b\end{bmatrix} \qquad Y=\begin{bmatrix}y_1\\ y_2\\ y_3\\ y_4\end{bmatrix}\Leftarrow \text{ each entry here is known!}$$

unknowns!

x 's are known

Example 1: Use the method of least squares to find the line of best fit for $(4,9),(5,8),(9,6)$ and (12,3)

$$A=\begin{bmatrix}4&1\\5&1\\9&1\\12&1\end{bmatrix}\quad X=\begin{bmatrix}m\\b\end{bmatrix}\quad Y=\begin{bmatrix}9\\8\\6\\3\end{bmatrix}$$

Formula:
$A^T AX=A^T Y$

$$\begin{pmatrix}4&5&9&12\\1&1&1&1\end{pmatrix}\begin{bmatrix}4&1\\5&1\\9&1\\12&1\end{bmatrix}\begin{bmatrix}m\\b\end{bmatrix}=\begin{pmatrix}4&5&9&12\\1&1&1&1\end{pmatrix}\begin{bmatrix}9\\8\\6\\3\end{bmatrix}$$ (no cancelling matrices)

2×4 $\quad$ 2×2 $\quad$ 2×4 $\quad$ 4×1

2×1

2×2

$$\begin{bmatrix}16+25+81+144 & 4+5+9+12\\ 4+5+9+12 & 1+1+1+1\end{bmatrix}\begin{bmatrix}m\\b\end{bmatrix}=\begin{bmatrix}4\cdot9+5\cdot8+9\cdot6+12\cdot3\\ 9+8+6+3\end{bmatrix}$$

$$\begin{bmatrix}266&30\\30&4\end{bmatrix}\begin{bmatrix}m\\b\end{bmatrix}=\begin{bmatrix}166\\26\end{bmatrix}$$

2×2 $\quad$ 2×1

$$\begin{bmatrix}266m+30b\\30m+4b\end{bmatrix}=\begin{bmatrix}166\\26\end{bmatrix}$$

set entries and solve the system for m and b:

$266m+30b=166$

$30m+4b=26$

doing some algebra........compute....
$m=-29/41$ *and* b= $484/41$

line of best fit is $y=\frac{-29}{41}x+\frac{484}{41}$

"I can't do it "cancer of the mind!
"It's too hard". cancel of the mind!!

Mindset

"You reap what you sow. "

Let's say we want to use this model: $x=5$ (notice x=5 is one of the values in the list)

$y=\frac{-29}{41}\cdot5+\frac{484}{41}=8.27$ (real value=observed value was 8..error= $8.27-8=0.27$)

x=6 is not part of the x coordinates given...

estimate: y= $\frac{-29}{41}\cdot6+\frac{484}{41}=7.56$ (estimated value..we don't know the real one)

Say we have the points $(0,1),(1,3),(2,2)$ Find the line of best fit.

$A^T AX=A^T Y$

$$A=\begin{bmatrix}0&1\\1&1\\2&1\end{bmatrix}\quad X=\begin{bmatrix}m\\b\end{bmatrix}\quad Y=\begin{bmatrix}1\\3\\2\end{bmatrix}$$

$$\begin{pmatrix} 0 & 1 & 2 \\ 1 & 1 & 1 \end{pmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 1 & 1 \end{pmatrix} \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$$

2×3 $\quad$ 3×2 $\quad$ 2×3 $\quad$ 3×1

$$\begin{bmatrix} 0\cdot 0+1\cdot 1+2\cdot 2 & 0\cdot 1+1\cdot 1+2\cdot 1 \\ 1\cdot 0+1\cdot 1+1\cdot 2 & 1\cdot 1+1\cdot 1+1\cdot 1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} = \begin{bmatrix} 0\cdot 1+1\cdot 3+2\cdot 2 \\ 1\cdot 1+1\cdot 3+1\cdot 2 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 3 \\ 3 & 3 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} 5m+3b \\ 3m+3b \end{bmatrix} = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$$

$$5m+3b=7$$

$$3m+3b=6 \xrightarrow{\text{divide by 3}} m+b=2 \xrightarrow{\text{subtract m}} b=2-m$$

$$\xrightarrow{\text{plug into top equation}} 5m+3(2-m)=7$$

$$\xrightarrow{\text{solve for m}} 5m+6-3m=7$$

$$5m-3m=7-6$$

$$2m=1$$

$$m=1/2 \xrightarrow{\text{solve for b}}$$

$$b=2-m$$

$$b=2-1/2$$

$$b=4/2-1/2$$

$$b=3/2$$

Line of Best Fit:

$$y=\frac{1}{2}x+\frac{3}{2}$$

our original points: $(0,1),(1,3),(2,2)$

estimate y when x=1/2:

$$y=\frac{1}{2}\left(\frac{1}{2}\right)+\frac{3}{2}=\frac{1}{4}+\frac{3}{2}=\frac{1}{4}+\frac{6}{4}=\frac{7}{4} \Leftarrow \text{estimated value of y!}$$