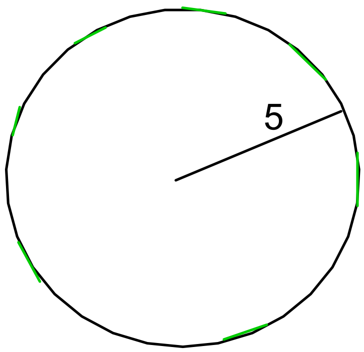
We do have class on Thursday. Ignore the syllabus. It's a mistake!!
Please be sure to put all phones and computers away while taking notes.

Section 3.5 :

If we write $x^2 + y^2 = 25$, here y is not isolated. $x^2 + y^2 = 5^2$ circle of radius 5 with center (0,0).



Mostly we write y in this context but mean y(x)!

How to find the slope on a shape like this? y is not solved for!
y=2x+4 (y is isolated)
$x^2 + y^2 = 25$ (y is NOT isolated)
We can use Implicit Differentiation. $y = y(x) \Leftarrow$ y is some unknown function of x! $y = 2x+4$

$x^2 + [y(x)]^2 = 25$

differentiate both sides with respect to x:

$\frac{d}{dx}(x^2) + \frac{d}{dx}[y(x)]^2 = \frac{d}{dx}(25)$ (each term gets $\frac{d}{dx}$)

$2x + 2[y(x)]^{2-1} y'(x) = 0$ x^2 power rule, $[y(x)]^2$ chain rule

$2x + 2[y(x)]y'(x) = 0$
$2x + 2y(x)y'(x) = 0$
$2y(x)y'(x) = -2x$

$y'(x) = \frac{-2x}{2y(x)} = -\frac{x}{y(x)} \xrightarrow{\text{now we write } y=y(x)} -\frac{x}{y}$

using y=y(x) notation:

$x^2 + y^2 = 25$
$(x^2)' + (y^2)' = (25)'$

$2x + 2yy' = 0$

$2y\, y' = -2x$

$y' = \frac{-2x}{2y} = -\frac{x}{y} \Leftarrow$ Measures slope on the graph of $x^2 + y^2 = 25$ at any point (x,y) that is on the graph!

$\frac{d}{dx} y^3$ means $\frac{d}{dx}[y(x)]^3 = 3[y(x)]^{3-1} y'(x)$ chain rule $(\ )^3$ =outside, y(x)=inside

$\frac{d}{dx} y^4$ means $\frac{d}{dx}[y(x)]^4 = 4[y(x)]^{4-1} y'(x)$..outside= $[\ldots]^4$, $inside =$ y(x)

Given that the point $(3,4)$ belongs to the graph of $x^2 + y^2 = 25$, find $y'\Big|_{(3,4)}$

$\frac{dy}{dx}\Big|_{(3,4)} = -\frac{3}{4} \Leftarrow$ slope at the point (3,4) on the graph of $x^2 + y^2 = 25$

example 2/page 211 in my book:

Find y' if $x^3 + y^3 = 6xy$ y^3 means $[y(x)]^3$, $6xy$ means $6xy(x) \Leftarrow$ product rule!

hit every term with ' : $(x^3)' + (y^3)' = (6xy)'$

$3x^2 + 3y^2 y' = 6[xy]'$

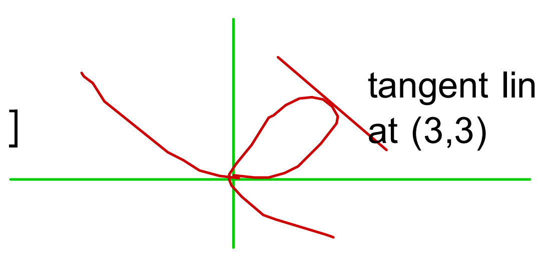
$3x^2 + 3y^2\ y' = 6[(x)'\, y + xy']$

$3x^2 + 3y^2\ y' = 6[y + xy']$

divide 3 away: $x^2 + y^2 y' = 2[y + xy']$

$x^2 + y^2 y' = 2y + 2xy'$

$x^3 + y^3 = 6xy$ is called the folium of Descartes



tangent line at (3,3)

$$y^2 y' - 2xy' = 2y - x^2$$
$$y'[y^2 - 2x] = 2y - x^2$$
$$y' = \frac{2y - x^2}{y^2 - 2x}$$

When x=3, y=3:
(given information)

$$y'\Big|_{(3,3)} = \frac{2(3) - 3^2}{3^2 - 2\cdot 3} = \frac{6-9}{9-6} = \frac{-3}{3} = -1$$

The slope on the graph of $x^3 + y^3 = 6xy$ at the point (3,3) is -1.

equation of tangent line: $y - b = y'\Big|_{(a,b)} (x-a)$

$y - 3 = -1(x-3)$ (a=3, b=3, y' at (3,3) is -1)
$y - 3 = -x + 3$
$y = -x + 3 + 3$
$y = -x + 6 \Leftarrow$ equation of tangent line!

example 3/page 212:

Find y' if $sin(x+y) = y^2 cos(x)$, $\quad y = y(x)$, y^2 means $[y(x)]^2$, $(..)^2 = ouside, y(x) = insi$

$sin(x+y)$ really means $sin(x+y(x))$

outside=sin(...)
inside is $x + y(x)$

$$[sin(x+y)]' = [y^2 cos(x)]'$$
$$\underbrace{cos(x+y)[x+y]'}_{\text{chain rule}} = \underbrace{[y^2]' cos(x) + y^2 (cos x)'}_{\text{product rule}}$$
$$cos(x+y)[1+y'] = \underbrace{2y\, y'}_{\text{chain rule}} cos(x) + y^2(-sin(x))$$

must isolate y': (......algebra......)

$$\xrightarrow{\text{distribute cos(x+y) on LHS}} cos(x+y) + cos(x+y)y' = 2yy' cos(x) - y^2 sin(x) \leftarrow \text{pull -1 to middle}$$

terms with y' on LHS, terms without y' on RHS:
$$cos(x+y)y' - 2yy' cos(x) = -y^2 sin(x) - cos(x+y)$$

factor y' out: $y'[cos(x+y) - 2ycos(x)] = -y^2 sin(x) - cos(x+y)$

divide by cos(x+y)-2ycos(x): $y'\left[\frac{cos(x+y) - 2ycos(x)}{cos(x+y) - 2ycos(x)}\right] = \frac{-y^2 sin(x) - cos(x+y)}{cos(x+y) - 2ycos(x)}$

factor -1 out of top: $y' = \frac{-1[y^2 sin(x) + cos(x+y)]}{-2ycos(x) + cos(x+y)}$

factor -1 out of bottom too: $y' = \frac{-1[y^2 sin(x) + cos(x+y)]}{-1[2ycos(x) - cos(x+y)]}$

cancel off -1 : $y' = \frac{y^2 sin(x) + cos(x+y)}{2ycos(x) - cos(x+y)}$

example 4(our own): Find y' if $e^y + x^2 = y$ $\quad y = y(x)$, e^y means $e^{y(x)}$

$$(e^y)' + (x^2)' = (y)'$$
$$e^y\, y' + 2x = y' \xrightarrow{\text{gather terms with y ' on one side}} 2x = y' - e^y y' \xrightarrow{\text{factor y' out}} 2x = y'(1 - e^y)$$

$\xrightarrow{\text{divide by } 1-e^y} \frac{2x}{1-e^y} = y'\left(\cancel{\frac{1-e^y}{1-e^y}}\right) \xrightarrow{\text{we get...}} y' = \frac{2x}{1-e^y}$

example 4 in book page 213:

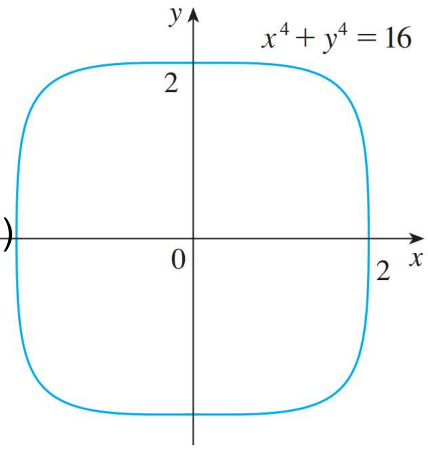
Find y'' for $x^4 + y^4 = 16$

$(x^4)' + (y^4)' = (16)'$

$4x^3 + 4y^3 y' = 0$

subtract $4x^3$: $4y^3 \ y' = -4x^3$

divide by $4y^3$: $y' = \frac{-4x^3}{4y^3} = -\frac{x^3}{y^3}$



$y'' = \frac{d}{dx}(y') = \frac{d}{dx}\left(\frac{-x^3}{y^3}\right)$

$= -1\frac{d}{dx}\left(\frac{x^3}{y^3}\right)$ (quotient rule)

$= -\frac{[y^3 \cdot 3x^2 - x^3 \cdot 3y^2 y']}{(y^3)^2}$

$= -\frac{[3x^2 y^3 - 3x^3 y^2 y']}{y^6}$

$\xrightarrow{\text{recall that } y' = -x^3/y^3} = -\frac{[3x^2 y^3 - 3x^3 y^2(-x^3/y^3)]}{y^6}$

$= -\frac{[3x^2 y^3 + 3x^6/y]}{y^6}$

$x^4 + y^4 = 16$

$= \frac{-3\left[x^2 y^3 + \frac{x^6}{y}\right]}{y^6} = \frac{3y[x^2 y^3 + x^6/y]}{y \cdot y^6}$ $\quad y/y = 1$

$= \frac{-3[x^2 y^4 + x^6 \cancel{y/y}]}{y^7} = \frac{-3[x^2 y^4 + x^6]}{y^7} = \frac{-3x^2[y^4 + x^4]}{y^7}$

$y = \frac{-3x^2[16]}{y^7} = \frac{-48x^2}{y^7} \Leftarrow$ this contains only x and y..no y'!

What is the derivative of $|x|$? $\frac{d}{dx}|x| = ?$

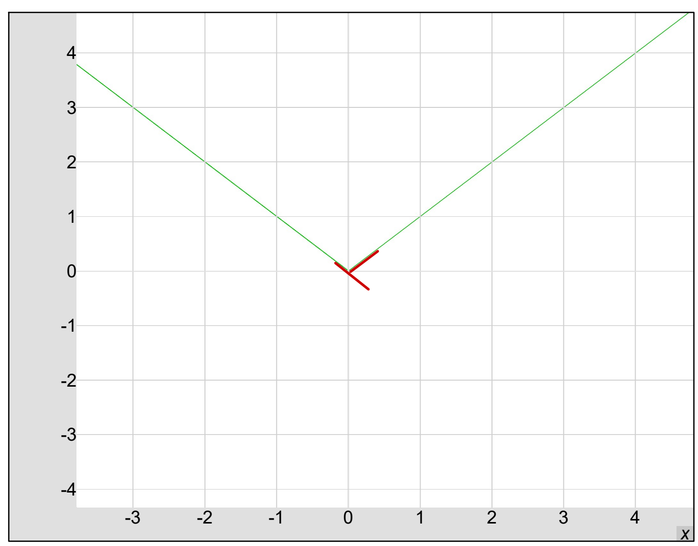
The graphs of $|x|$ and $\sqrt{x^2}$ match!

$\frac{d}{dx}|x| = \frac{d}{dx}\sqrt{x^2} \xrightarrow{\text{chain rule}} \frac{d}{dx}(x^2)^{1/2}$

outside$=(\dots)^{1/2}$, $inside = x^2$

$= \frac{1}{2}(x^2)^{1/2-1}\frac{d}{dx}(x^2) = \frac{1}{\cancel{2}}(x^2)^{-1/2}(\cancel{2}x)$

$= \frac{1}{(x^2)^{1/2}}x = \frac{x}{\sqrt{x^2}} = \frac{x}{|x|} \Leftarrow \ x \neq 0!$



"bad..." $\cancel{(x^2)^{1/2} = x^{2/2}} = x$

homework like questions:

Q1: $x^2 - y^9 = 10$..find y':

prime each term: $(x^2)' - (y^9)' = (10)'$

$2x - 9y^8 y' = 0$

$2x = 9y^8 y' \rightarrow \frac{2x}{9y^8} = y'$

Q2: $5x^2+4x+xy=-8$ $find$ y'(1) :

we're given only x=1..y' is likely to involve y also..

$5\cdot 1^2+4\cdot 1+1y=-8 \Leftarrow$ repalce x with 1 and get y:

5· 1+4+y=-8

$5+4+y=-8$

$9+y=-8$

$y=-8-9$

$y=-17$

point$(1,-17)$:

derivative: $(5x^2)'+(4x)'+(xy)'=(-8)'$

$5\cdot 2\cdot x^1+4+(x)'y+xy'=0$

$10x+4+1y+xy'=0$

$10x+4+y+xy'=0$

$xy'=-10x-4-y$

$y'=\frac{-10x-4-y}{x}$, y' at $x=1, y=-17$: $\frac{-10(1)-4-(-17)}{1}$

$=-10-4+17$

$=-14+17=3$

Slope at (1,-17) on the graph of $5x^2+4x+xy=-8$

is 3!..so graph is going up b/c slope>0

$-2y=xe^{3y}$

y=y(x), can't solve for y...y=stuff with x only!!

$(-2y)'=(xe^{3y})'$ (prime both sides)

$(e^{p(x)})'=e^{p(x)}p'(x)$

-2y'=$(x)'$ $e^{3y}+x$ $(e^{3y})'$ LHS power rule, RHS=product

$-2y'=1e^{3y}+x(e^{3y}\cdot 3y')$ Chain rule on (e^{3y})'

$-2y'=e^{3y}+3xe^{3y}y'$ rewrite a little

$-2y'-3xe^{3y}y'=e^{3y}$ gather terms with y' on LHS

$y'=\frac{e^{3y}}{-2-3xe^{3y}}$ divide by -2-3xe^{3y}

$-2y=xe^{3y}$

$y=\frac{xe^{3y}}{-2} \Leftarrow$ bad b/c y in top!

~~ln(-2y) = $ln(xe^{3y})$~~