Make sure you load your class notes and homeworks solutions as a PDF. Remember you can use an app like FastScanner or CamScanner to make images, and then export the images as a single PDF. Images don't show up sometimes in the box.

On with section 3.2:

Scalar Multiplication of Matrices

$$A=\begin{bmatrix}1&2\\3&4\end{bmatrix} \text{ 2 by 2 matrix, } 4A=4\begin{bmatrix}1&2\\3&4\end{bmatrix}\xrightarrow{\text{distribute the 4 to each entry}}\begin{bmatrix}4\cdot1&4\cdot2\\4\cdot3&4\cdot4\end{bmatrix}=\begin{bmatrix}4&8\\12&16\end{bmatrix}$$

The red 4 is the scalar. A scalar is a real number.

$$\text{ex2}:\frac{1}{3}\begin{pmatrix}6&9&12\\8&4&-9\end{pmatrix}\xrightarrow{\text{distribute the 1/3}}\begin{pmatrix}\frac{1}{3}\cdot6&\frac{1}{3}\cdot9&\frac{1}{3}\cdot12\\\frac{1}{3}\cdot8&\frac{1}{3}\cdot4&\frac{1}{3}(-9)\end{pmatrix}=\begin{pmatrix}2&3&4\\8/3&4/3&-3\end{pmatrix}$$

$\frac{1}{3}$ = scalar $\quad 2\times3 \qquad\qquad 2\times3 \qquad\qquad 2\times3$

$$\text{ex3: }2\begin{bmatrix}1&2\\3&4\end{bmatrix}-3\begin{bmatrix}-1&2\\3&5\end{bmatrix}$$

think about it: 2(x-3)-3(x+5)

$$\xrightarrow{\text{distribute the 2 and -3 first}}\begin{bmatrix}2\cdot1&2\cdot2\\2\cdot3&2\cdot4\end{bmatrix}+\begin{bmatrix}(-3)(-1)&(-3)(2)\\-3(3)&-3(5)\end{bmatrix}\leftarrow$$ carry the - with the 3, leave +

$$\xrightarrow{\text{multiply out}}\begin{bmatrix}2&4\\6&8\end{bmatrix}+\begin{bmatrix}3&-6\\-9&-15\end{bmatrix}$$

$$\xrightarrow{\text{add}}\begin{bmatrix}2+3&4-6\\6-9&8-15\end{bmatrix}\xrightarrow{\text{finalize the additions}}\begin{bmatrix}5&-2\\-3&-7\end{bmatrix}$$ final result

each is 2 by 2

The action of multiplying by a scalar and adding /subtracting is called forming a linear combination of matrices.

Ex4: To get good at math, just do a lot of exercises.

$$5\left(2\begin{bmatrix}4&-2\\3&4\end{bmatrix}\right)\xrightarrow{\text{distribute the 2}}5\begin{bmatrix}2\cdot4&2(-2)\\2\cdot3&2\cdot4\end{bmatrix}=5\begin{bmatrix}8&-4\\6&8\end{bmatrix}=\begin{bmatrix}5\cdot8&5(-4)\\5\cdot6&5\cdot8\end{bmatrix}$$

$$=\begin{bmatrix}40&-20\\30&40\end{bmatrix}\Leftarrow$$ First distribute 2 and second distribute 5.

What if we first multiply 5 by 2 to make 10 and distribute 10?

$$5\left(2\begin{bmatrix}4&-2\\3&4\end{bmatrix}\right)\xrightarrow{\text{multiply 5 by 2}}10\begin{bmatrix}4&-2\\3&4\end{bmatrix}\xrightarrow{\text{distribute 10}}\begin{bmatrix}10\cdot4&10(-2)\\10\cdot3&10\cdot4\end{bmatrix}=\begin{bmatrix}40&-20\\30&40\end{bmatrix}$$

in general: $a\left(b\begin{bmatrix}c&d\\e&f\end{bmatrix}\right)\xrightarrow{\text{do ab first}}ab\begin{bmatrix}c&d\\e&f\end{bmatrix}\xrightarrow{\text{distribute ab}}\begin{bmatrix}abc&abd\\abe&abf\end{bmatrix}$

Matrix multiplication /Section 3.2:

ex 1: $\begin{bmatrix} 1\$ & 5\$ \end{bmatrix}$ prices $\begin{bmatrix} 5 \\ 6 \end{bmatrix}$ number of items

$5 \cancel{\text{apples}} \cdot \dfrac{1\$}{\cancel{\text{apple}}} + 6 \cancel{\text{pineapples}} \cdot \dfrac{5\$}{\cancel{\text{pineapple}}}$

$5 \cdot 1\$ + 6 \cdot 5\$ = 5\$ + 30\$ = 35\$$ total money spent

$$\begin{bmatrix} 1 & 5 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ 6 \end{bmatrix} = 1 \cdot 5 + 5 \cdot 6 = 5 + 30 = 35 \text{ (same value as above)}$$

in general: $\begin{bmatrix} a_1 & a_2 & \dots & a_n \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ \dots \\ b_n \end{bmatrix} = a_1 b_1 + a_2 b_2 + \dots + a_n b_n$ (dot product)

e x 2: $\underset{1 \times 3}{\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}} \underset{3 \times 1}{\begin{bmatrix} -1 \\ 2 \\ 4 \end{bmatrix}} = 1(-1) + 2(2) + 3(4) = -1 + 4 + 12 = 3 + 12 = 15 \Leftarrow$ scalar result

dot product

$[15] = 15$ (1 by 1)

Number of columns in first matches the number of rows in second.

ex3: $\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = 1 \cdot 1 + 2 \cdot 2 + ?3$ (can't multiply a 1×2 with a 3×1, 2 is not 3!!)

ex 4: $\begin{bmatrix} \boxed{1 \quad 2} \\ \boxed{3 \quad 4} \end{bmatrix} \cdot \begin{bmatrix} \boxed{\begin{matrix} 4 \\ -2 \end{matrix}} & \boxed{\begin{matrix} 5 \\ 3 \end{matrix}} \end{bmatrix} \xrightarrow{\text{multiply rows by columns again}} \begin{array}{cc} 1 & 2 \\ \multicolumn{2}{c}{\begin{bmatrix} 1(4)+2(-2) & 1(5)+2(3) \\ 3(4)+4(-2) & 3(5)+4(3) \end{bmatrix}} \\ 3 & 4 \end{array}$

$\underbrace{2 \times 2 \qquad 2 \times 2}$

these match so meaningful operation

entry 1: dot product of row 1, column 1
entry 2: dot product of row 1, column 2
entry 3: dot product of row 2, column 2
entry 4: dot product of row 2, column 2

2×2

ex 3 /page 122/123 (from book)

$$\underset{2 \times 3}{\begin{pmatrix} 2 & 4 & -1 \\ 5 & 8 & 0 \end{pmatrix}} \underset{3 \times 4}{\begin{bmatrix} 2 & 5 & 1 & 4 \\ 4 & 8 & 0 & 6 \\ -3 & 1 & -2 & -1 \end{bmatrix}}$$

3 and 3 tell us multiplication will work
resulting matrix will be **2 by 4**

$$\begin{pmatrix} \overset{1}{2(2)+4(4)-1(-3)} & \overset{2}{2(5)+4(8)-1(1)} & \overset{3}{2(1)+4(0)-1(-2)} & \overset{4}{2(4)+4(6)-1(-1)} \\ \underset{5}{5(2)+8(4)+0(-3)} & \underset{6}{5(5)+8(8)+0(1)} & \underset{7}{5(1)+8(0)+0(-2)} & \underset{8}{5(4)+8(6)+0(-1)} \end{pmatrix}$$

$$= \begin{pmatrix} 4+16+3 & 10+32-1 & 2+2 & 8+24+1 \\ 10+32 & 25+64 & 5 & 20+48 \end{pmatrix} \xrightarrow{\text{finalize the additions}} \begin{pmatrix} 23 & 41 & 4 & 33 \\ 42 & 89 & 5 & 68 \end{pmatrix} \Leftarrow \text{result}$$

example 4(our own)

$$\underset{1\times 2}{\begin{bmatrix}1 & 2\end{bmatrix}}\underset{2\times 3}{\begin{pmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{pmatrix}} = \begin{array}{ccc} 1 & 2 & 3 \\ \multicolumn{3}{c}{\begin{bmatrix}1\cdot 1+2\cdot 4 & 1\cdot 2+2\cdot 5 & 1\cdot 3+2\cdot 6\end{bmatrix}} \end{array} = \begin{bmatrix}1+8 & 2+10 & 3+12\end{bmatrix}$$

1×3

$$=\begin{bmatrix}9 & 12 & 15\end{bmatrix}$$

2=2,so compat. for multiplication

1×3 matrix!