Math 111 Notes 10/11/2023. Please close all computers and put away all phones while taking notes in class. It's important to write detailed notes where every tiny symbol is shown the respect it deserves. Make sure you do the Midterm Acknowledgement assignment.

Section 3.1/Quadratic Functions:

Function of the form $f(x)=ax^2+bx+c$ (highest degree is 2)

$f(x)=x^2+6x+2$ contains $1x^2$

$g(x)=2(x+1)^2-3 \xrightarrow{\text{we can multiply out}} 2[x^2+2\cdot x\cdot 1+1^2]-3=2[x^2+2x+1]-3$

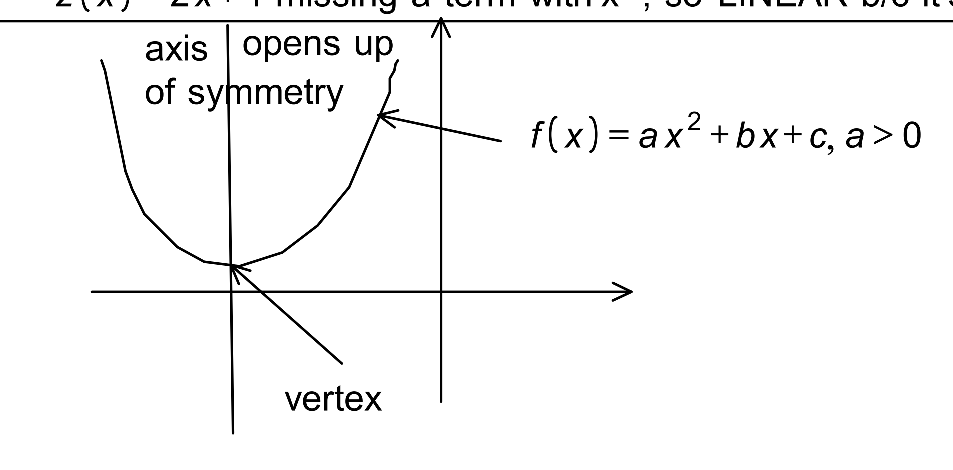
$=2x^2+4x+2-3=2x^2+4x-1$, so looks like ax^2+bx+c

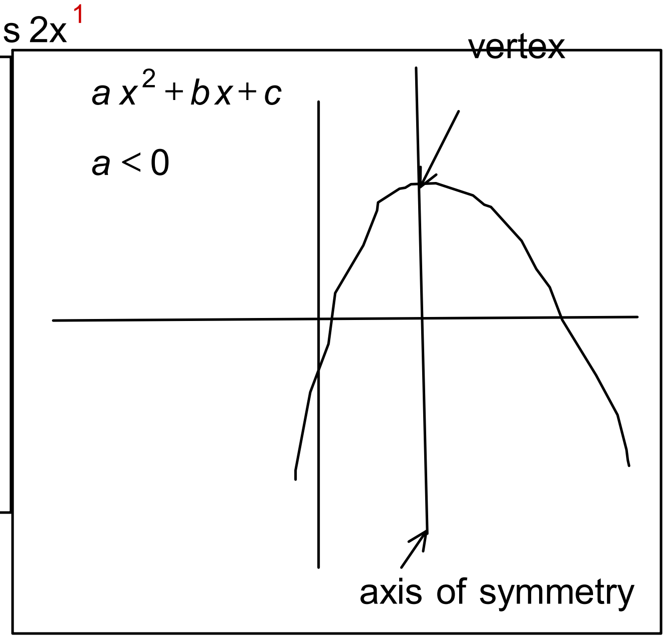
$h(x)=9+\frac{1}{4}x^2=9+0x+\frac{1}{4}x^2$, so has the form $c+bx+ax^2$

$k(x)=-3x^2+4=-3x^2+0x+4$, ax^2+bx+c

$m(x)=(x-2)(x+1)\xrightarrow{\text{FOIL}} x^2+1x-2x-2=x^2-x-2$, ax^2+bx+c

$z(x)=2x+4$ missing a term with x^2, so LINEAR b/c it's $2x^1$





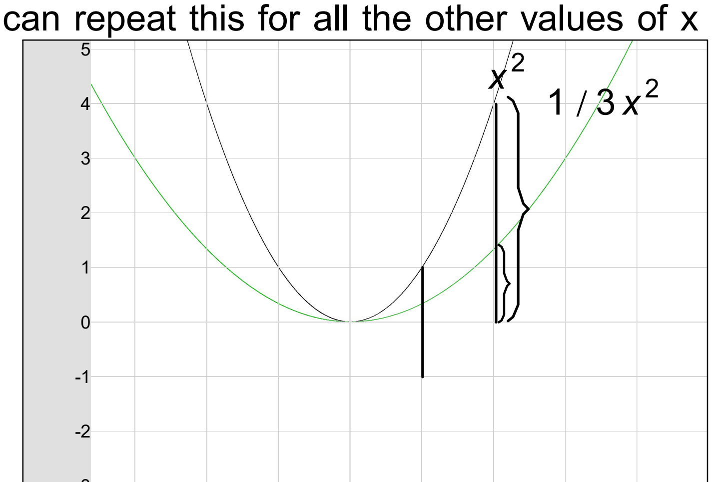
example 1 :book:

make a picture of $y=x^2$ and $f(x)=\frac{1}{3}x^2=\frac{x^2}{3}$

This will divide each original y coordinate by 3, so $1/3x^2$ will appear to be closer to x axis.

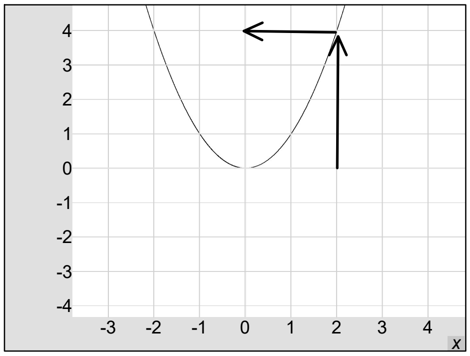
$\underline{x=1}$: $y=1^2=1$, $(1, 1)$, $f(1)=\frac{1}{3}(1)^2=\frac{1}{3}$, $(1,1/3)$..same x coords but y coords change!

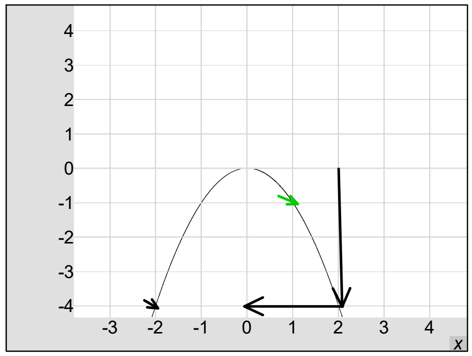
We can repeat this for all the other values of x and y!

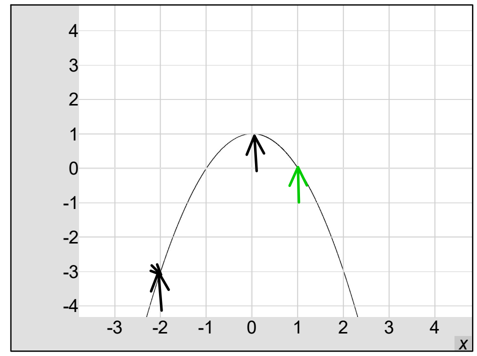


Green graph, for a given x , appears to be closer to x axis than black graph.

$f(x) = x^2$ $\qquad$ $g(x) = -1x^2 = -f(x)$ $\qquad$ $h(x) = -x^2 + 1$



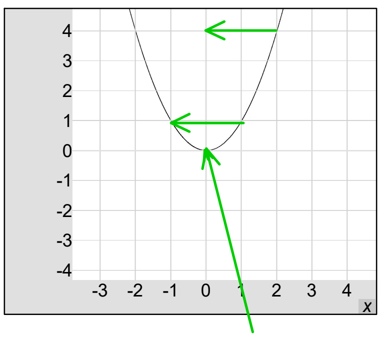




y coords flip signs →

add 1 to each y coordinate →

$f(x) = x^2$

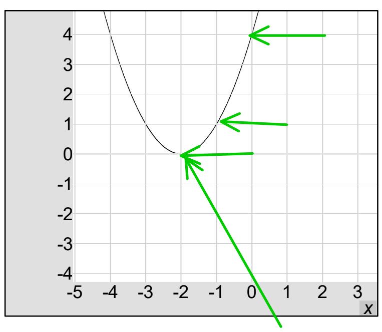


vertex is (0,0)

$g(x) = (x+2)^2$

$= (x - \ -2)^2$

-2 means subtract 2 from each x coord:

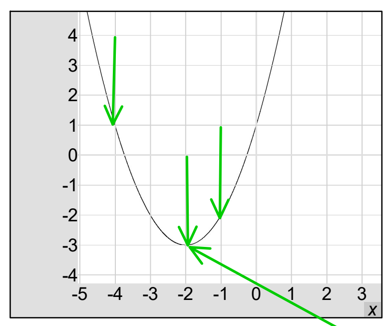


vertex is (-2,0)

h(x)= $(x+2)^2 - 3$

shift whole graph down by 3

subtract 3 from each y coord.



vertex is (-2,-3)

So in general for the form $f(x) = (x+2)^2 - 3$, the vertex is at (-2,-3) (NOT x=+2)

So truly in general: $f(x) = (x-a)^2 + b$, vertex is (a,b)

$= [x - (+\mathbf{a})]^2 + b$ (not x=-a for vertex)

Example (not in book):

Complete the square to reveal the vertex form:

$f(x) = ax^2 + bx + c \Leftarrow$ vertex is not visible!! (we need $y = (x-a)^2 + b$ form)

So let's learn how to reveal the vertex:

$f(x) = x^2 + 4x + 6 \Leftarrow$ vertex not visible!!

$= \left(x + \frac{4}{2}\right)^2 + 6 - \left(\frac{4}{2}\right)^2$

$= (x+2)^2 + 6 - 2^2$

$= (x+2)^2 + 6 - 4 = (x+2)^2 + 2 \Leftarrow$ this shows the vertex is $(-2, 2)$

$check:$

(x+2)^2 +2..does it equal the original?

$(x^2 + 2 \cdot 2 \cdot x + 2^2) + 2$

$x^2 + 4x + 4 + 2$

$x^2 + 4x + 6 \Leftarrow$ this hides the vertex

$g(x) = x^2 - 4x + 5$

$= \left(x - \frac{4}{2}\right)^2 + 5 - \left(\frac{-4}{2}\right)^2$

$= (x-2)^2 + 5 - (-2)^2$

$= (x-2)^2 + 5 - 4$

$= (x-2)^2 + 1 \Leftarrow$ vertex is (2,1) not (-2,1)

check:

$(x-2)^2 + 1$

$(x^2 + 2(x)(-2) + (-2)^2) + 1$

$x^2 - 4x + 4 + 1$

$x^2 - 4x + 5$

write left parenthes, right parenthesis

write variable as x^1, follow by middle coefficient divided by 2

square the whole thing

$x^2 - 4x + 5 \longrightarrow \left(x - \frac{4}{2}\right)^2 + 5 - \underbrace{\left(\frac{-4}{2}\right)^2}$

always minus
half the middle
coefficient squared!

Homework Q1:

$y = x^2 + 2x - 8$

y intercept: $(0, y)$: $y = 0^2 + 2(0) - 8 = 0 + 0 - 8 = -8$ put in the -8 (always the same steps)

x intercepts: $f(x) = 0$

$x^2 + 2x - 8 = 0$

two numbers that multiply to -8 and add to 2:

$(x+4)(x-2) = 0$ (factor LHS)

x+4=0 x-2=0

x=-4 x=2 in MOM: 2,-4

v ertex part: $y = x^2 + 2x - 8$

$= \left(x + \frac{2}{2}\right)^2 - 8 - \left(\frac{2}{2}\right)^2$

$= (x+1)^2 - 8 - 1^2 \qquad (-1)^2 = 1, -1^2 = -1$

$= (x+1)^2 - 8 - 1$

$= (x+1)^2 - 9$, vertex is $(-1, -9)$ NOT (+1,-9)

$= (x - -1)^2 - 9$

I n homework mark the vertex and one more point , meaning the y intercept.

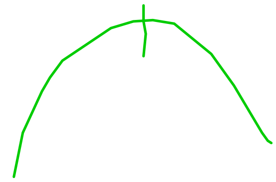
Q2/Homework :NASA shots a rocket into space at t=0 seconds.

$h(t) = -4.9t^2 + 148t + 374$

$h(t) = 0$ (what time is the height =0)

$-4.9t^2 + 148t + 374 = 0$ (you can do this b/c we have done this before)

The rocket peaks at .... meters above sea-level?



vertex formula: $\left(-\dfrac{b}{2a}, f\left(\dfrac{-b}{2a}\right)\right)$

$a=-4.9,\ b=148$

t coord of vertex$=-\dfrac{b}{2a}=\dfrac{-148}{2(-4.9)}=15.10204$ and then plug this into the function:

$h(15.10204)$ (put in a lot of decimal places to be sure to avoid errors)

$-4.9(15.10204)^2+148(15.10204)+374\approx 1491.55102$ meters

---

Question 3:

$y=x^2+6x+8$

factored form: $y=(x+4)(x+2)$ b/c $2\cdot 4=8$ and $2+4=6$

complete the square: $y=\ldots\ldots\ldots\ldots\ldots$..

$y=x^2+6x+8$

$y=\left(x+\dfrac{6}{2}\right)^2+8-\left(\dfrac{6}{2}\right)^2$

$y=(x+3)^2+8-3^2$

$y=(x+3)^2+8-9$

$y=(x+3)^2-1$ (answer)

When we say "find the zeros of the function", we mean the values of x that make y come out to be 0!(Roots or x-intercepts)

(c) identify the vertex

$(-3,-1)$ (NOT (+3,-1))

(d) y intercept:

$y=0^2+6\cdot 0+8=8$ but MOM here wants (0,8).

(e) identify x intercpets as points:

factor form and set to 0:

$(x+4)(x+2)=0$

$x+4=0 \qquad x+2=0$

$x=-4 \qquad x=-2$

input into MOM: $(-4,0),(-2,0)$

When x=-4, y=0.

When x=-2, y=0.