Math 111 Notes 11/15/2023. Please take detailed notes and put away all other distractions.
Remember next week class will be on zoom. I will send the link out via MyOpenMath.
Applications of Exponential Functions:

Simplest Interest= $I = Prt$, $P = principal$, $r = rate$, $t = time$

If we set t=1 for the sake of simplicty, we get $I = Pr$

amount at the end of 1 year: $A =$ P+Pr (principal + interest)

$$A = P(1+r) \Leftarrow \text{factor } P \text{ out} \qquad P = 1 \cdot P$$

We invest this amount , P(1+r) into the next year:

amount at end of year 2: $A = P(1+r)\cdot 1 + P(1+r)r$

notice P(1+r) is present in both, so factor P(1+r) out

$= P(1+r)[1+r]$

$= P(1+r)^2$ (b/c 1+r repeats twice)

We invest $P(1+r)^2$ into year 3:

amount at the end of year 3: $A = P(1+r)^2 \cdot 1 + P(1+r)^2 r$

$= P(1+r)^2(1+r)$

$= P(1+r)^3$ (b/c now 1+r repeats 3 times)

We invest $P(1+r)^3$ into year 4:

amount at end of year 4: $A = P(1+r)^3 \cdot 1 + P(1+r)^3 r$

$= P(1+r)^3(1+r)$

$= P(1+r)^4$ (b/c now 1+4 is present 4 times)

Amount at the end of t years:

$A = P(1+r)^t$

discrete compounding formula (simplified version)

Defintion of the number "e ":

Invest 1 dollar, for 1 year, at 100% interest and change the number of times we compound the interest

$A = 1\left(1+\frac{1}{n}\right)^n$, 1 = 1 dollar, 1 =rate of 100%

$n =$ variable

$n = 1$: $1\left(1+\frac{1}{1}\right)^1 = 1(1+1) = 1\cdot 2 = 2$ (begin with 1 dollar and end up with 2)

$n = 100$: $1\left(1+\frac{1}{100}\right)^{100}$ (invest 1 dollar, compound 100 times over 1 year but the 1/100 interest rate per period)

$= 2.704813829$

2.7 repeats...

$n = 10,000$: $1\left(1+\frac{1}{10000}\right)^{10000} = 2.718145927$

$n = 100,000$: $1\left(1+\frac{1}{100,000}\right)^{100,000} = 2.718268237\ldots$

$n = 10,000,000$: $1\left(1+\frac{1}{10,000,000}\right)^{10,000,000} = 2.718281693$

As $n \to \infty$, $1\left(1+\frac{1}{n}\right)^n \to 2.718281693\ldots$

Invest 1 dollar, for 1 year, at 100%, and get only about 2.718 dollars. This number is called "e".

Euler but we say it as "Oiler".

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Most general form of discrete compounding: $A = P\left(1+\frac{r}{n}\right)^{nt} \Leftarrow$ take math 200.

$A =$ future value, P=principal ...money being invested right now, r=rate of interest

n=number of times we compound, t=time

$\frac{r}{n} \to$ interest rate at the end of each period

nt= *total* number of times we compound

Say we invest 10000 at 3% for 4 years and we compound 4 times per year.

P=10000, 3%=.03, t=4 years, compound 4 times per year means n=4

A= $10000\left(1+\frac{0.03}{4}\right)^{4\cdot 4} = 10000(1+0.0075)^{16} = \$11,269.92$

Interest= A-P= $11269.92 - 10000 = \$1,269.92$

going from discrete compounding to continuous compouding: $A = P\left(1+\frac{r}{n}\right)^{nt}$

not obvious: A= $P\left(1+\frac{1}{\frac{n}{r}}\right)^{nt}$ $b/c\ \frac{1}{n/r} \xrightarrow{\text{KCF}} 1\cdot\frac{r}{n}=\frac{r}{n}$

set n/r=m: A= $P\left(1+\frac{1}{m}\right)^{mrt}$ (replace n with mr in expo.)

$n=mr$ $A=P\left(1+\frac{1}{m}\right)^{m\cdot rt}$

recall that $\left(1+\frac{1}{m}\right)^{m} \to e$ when m becomes huge!

$A=P(e)^{rt}$ (replace $\left(1+\frac{1}{m}\right)^{m}$ with e)

Example 8/Compound Interest , Page 386:

A total of $12,000 is invested at an annual interest rate of 9%. Find the balance after 5 years:

a. annual (n=1) $A=12000\left(1+\frac{0.09}{1}\right)^{1\cdot 5}=12000(1+0.09)^{5}=$ 18463.49 Interest $=A-P=18463.49-12000=6463.49$

b. quarterly: (n=4) (every three months) $A=12000\left(1+\frac{0.09}{4}\right)^{4\cdot 5}=$ 18726.11 interest= $18726-12000=6726.11$

c. monthly (end of each month) n=12: $A=12000\left(1+\frac{0.09}{12}\right)^{12\cdot 5}=$ 18788.17, interest= $18788.17-12000=6788.17$

d. continuous compounding: $A=Pe^{rt}$

$A=12000\,e^{0.09\cdot 5}=$ 18, 819.75

This is the most we can make using P=12000, r=9% and t=5.

IRA= Individual Retirement Account! Successful people think 5,10,20,25 years ahead...

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