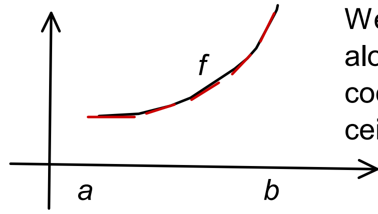
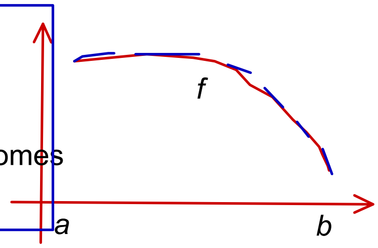
Please be sure to take detailed notes as usual. Please be sure your **camera is on** so I can be sure you're here.
I will check the attendance at some point. We'll start a little after 4 once more people join.
Please be sure your microphone is off so I don't get background sound.

Section 4.3/What does f'' say about f?
$\cup$ $f' < 0$, $f'=0$, $f'>0$
$\cap$ $f'>0$, $f'=0$, $f'<0$
or $f' =$ constant



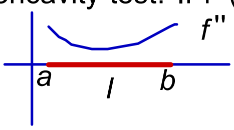
We see f is always above the tangent lines along f. When this is true, f is said to be cocave up. $\cup$ (cupped hand with finger tips towards ceiling.)

1st derivative test(review)
When x makes $f'=0$, and $f'<0$ and $\cup$ becomes $f'>0$, we have a local min.
When x makes $f'=0$, and $f'>0$ and becomes $f'<0$, we have a local max. $\cap$

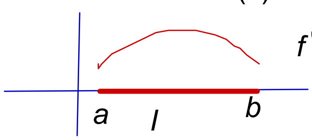


We see that f is always below its tangent lines. When this is true, f is said to be concave down. *f* looks like $\cap$.

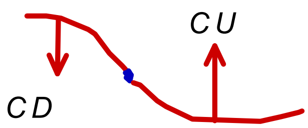
Concavity test: If $f''(x)>0$ for all x in I, then the graph of f is concave up over I.



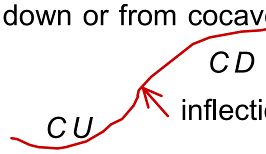
If $f''(x)<0$ for all x in I, then the graph of f is concave down over I.



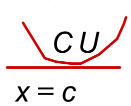
What happens at $f''=0$? A point P (x,y) is called an inflection point if f is continous there and the curve changes from concave up to concave down or from cocave down to concave up.

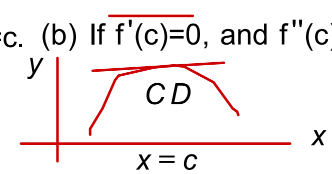


or

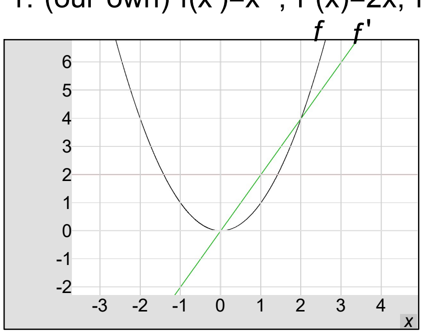


Second Derivative Test: Suppose f'' is continuous at x=c.
(a) If $f'(c)=0$ and $f''(c)>0$, then f has a local minimum at x=c. (b) If $f'(c)=0$, and $f''(c)<0$, the f has a local max at x=c.





example 1: (our own) $f(x)=x^2$, $f'(x)=2x$, $f''(x)=2$



$f' = 0$ at x=0.
f'' at x=0 is 2>0
second derivative test tells us that at x=0, f has a local min.

$f'' = 2 > 0$ for all x
$\Rightarrow$ f is always concave up!
(f looks like a cupped hand with fingers towards ceiling)
(smiley parabola)

example 2: $f(x) = x^4 - 4x^3$

$\lim_{x\to\infty} f(x) = ?$ *notice* x^4 leads, so we have $\lim_{x\to\infty} x^4 = \infty^4 = +\infty$ $\uparrow$

$\lim_{x\to-\infty} f(x) = ?$ notice that x^4 leads (4>3 in exponents), so we have $\lim_{x\to-\infty} x^4 = (-\infty)^4 = +\infty$ f is $\uparrow$

$f'(x) = (x^4 - 4x^3)' = (x^4)' + (-4x^3)' = 4x^3 - 12x^2$ (power rule)

$f''(x) = (f'(x))' = (4x^3 - 12x^2)' = (4x^3)' + (-12x^2)' = 12x^2 - 24x$

critical numbers are found by setting f'=0 and solving for x:

$4x^3 - 12x^2 = 0$

factor 4x out: $4x^2(x-3) = 0$

$4x^2 = 0$ or $x-3=0$

$x = 0$ $\quad$ $x = 3$

in short:

$4 \cdot (+) > 0$ unless x=0

$\cup$ *or* $\cap$ rough concavity pictures for remembering what things mean!

Does x=0 give a local max or local min?

checking f'' is faster here:

$f''(0) = 12 \cdot 0^2 - 24 \cdot 0 = 0 - 0 = 0$ (when f''=0, second derivative test fails)

Does x=3 give us a local max or min? $f''(3) = 12 \cdot 3^2 - 24 \cdot 3$

$= 12 \cdot 9 - 72$

$= 36 > 0$

Since f''(3)>0, and f'(3)=0 , we have a local min at x=3. Graph of f looks like $\cup$.

Where is the graph concave up? To answer, solve f''(x)=0 and check the signs around these values.

$12x^2 - 24x = 0$ (set f''=0)

factor 12x: $12x(1x-2) = 0$

set 12x=0 $\quad$ x-2=0

$x = 0$ $\quad$ $x = 2$

number line: $-\infty \ldots \leftarrow$ -------------------0 ------------------------------- 2------------------$\rightarrow\infty$

$x < 0$, use x=-1:

$12(-1)^2 - 24 \cdot (-1)$

$= 12 + 24$

$= 36 > 0$

$CU =$ concave up

$\boxed{f \text{ is CU } f \text{ looks like } \cup}$

$x = 1:$

$12 \cdot 1^2 - 24 \cdot 1$

$= 12 - 24$

$= -12 < 0$

f is CD or $\cap$

$x = 3$

$12 \cdot 3^2 - 24 \cdot 3$

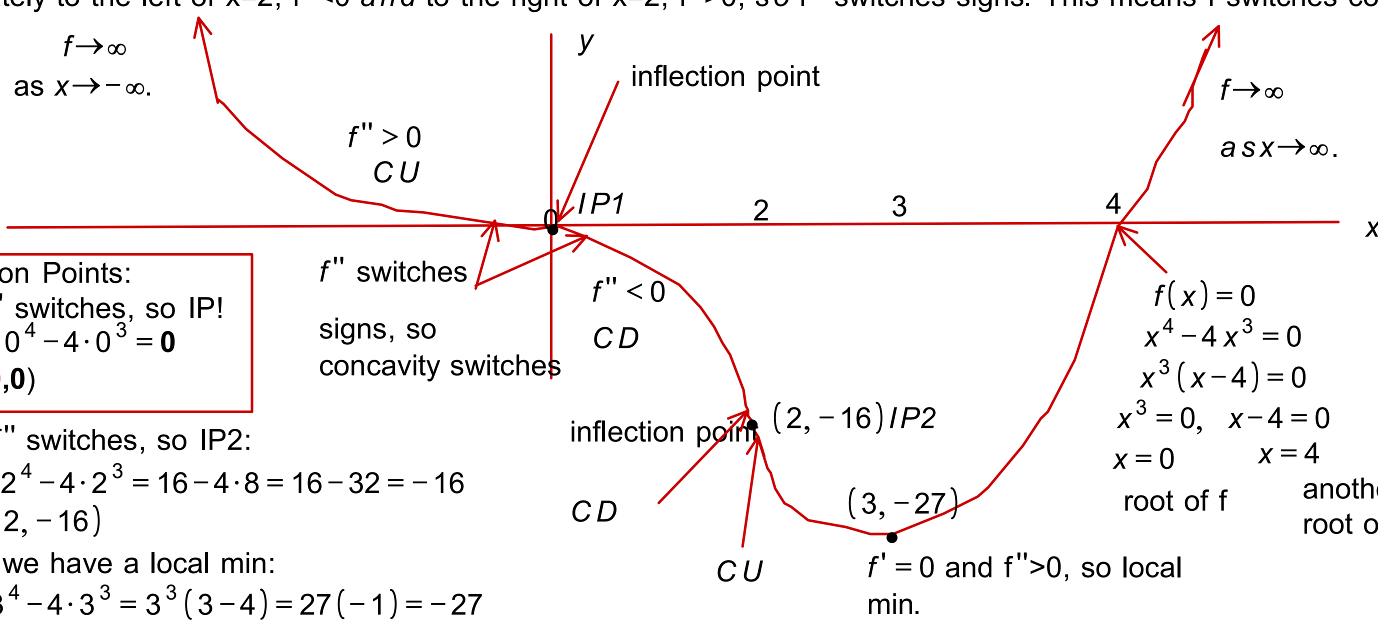
$= 12 \cdot 9 - 72$

> 0 f is CU

At x=0, notice f'' has this behavior:

left of x<0, f''>0 *and* for x>0, f''<0, so f'' switches its sign. This means f switches concavity.

immediately to the left of x=2, f''<0 *and* to the right of x=2, f''>0, *so* f'' switches signs. This means f switches concavity.



Inflection Points:

x=0, f'' switches, so IP!

$\boldsymbol{f}(0) = 0^4 - 4 \cdot 0^3 = \mathbf{0}$

IP1: (0,**0**)

$x = 2, f''$ switches, so IP2:

$\boldsymbol{f}(2) = 2^4 - 4 \cdot 2^3 = 16 - 4 \cdot 8 = 16 - 32 = -16$

$IP2: (2, -16)$

At x=3, we have a local min:

$f(3) = \underbrace{3^4 - 4 \cdot 3^3 = 3^3(3-4)}_{\text{factored } 3^3 \text{ out b/c it's common}} = 27(-1) = -27$

summary: end behavior on right, end behavior on left, roots of function, local max, local min, concavity, inflection points!

right end behavior: $\lim_{x \to \infty} f(x) = ?$

left end behavior: $\lim_{x \to -\infty} f(x) = ?$

roots: f(x)=0 and solve for x.

concavity: f''(x)>0 means CU

$f''(x) < 0$ means CD

local max: $f' = 0$ and $f'' < 0$

get this x from f'(x)=0 (solve this equation)

local min: f'= and f''>0

get this x from f'(x)=0

special case: when f''(c)=0 at some x=c (f'(c)=0) , then f'' gives no inform. In this case, check behavior as in example above by looking at signs of f'' around x=c.

Inflection points: Solve f''(x)=0. Plot the roots of f'' on a line. Check behavior of f'' around the roots.

If f''<0 and goes to f''>0, it's an inflection point on f.

If f''>0 and goes to f''<0, it's an inflection point on f at the value of x that makes f''=0.

Q2 homework:

Let $g(x) = 9x^3 - x^2 + 7x + 7$:

Find $g'(x) = (9x^3)' + (-x^2)' + (7x)' + (7)' = 27x^2 - 2x + 7$

Find g''(x)= $[f'(x)]' = (27x^2)' + (-2x)' + 7' = 54x - 2$

For which value(s) of x does f''(x)=0?

f''(x)=0

$54x - 2 = 0$

$54x = 2$

$x = 2/54 = 1/27$

Does x=1/27 give us an inflection point? check signs of f'' around x=1/27:

$-\infty \ldots \leftarrow$ ------------------------ 1/27 -------------------------- $\rightarrow$

$x = 0$ b/c <1/27	x	x=1 b/c >1/27
$f''(0) = 54 \cdot 0 - 2$	$f'' = 0$	$f''(1) = 54 \cdot 1 - 2$
$= 0 - 2$		$= 54 - 2$
$= -2 < 0$		$= 52 > 0$
SO it's CD		so CU

At x=1/27, f'' switches signs. This means **f** switches concavity.
So x=1/27 does give an inflection point.

Q3: Given g(x)= $6x^3 - 81x^2 + 360x$

find g'(x)= $18x^2 - 162x + 360$

set g'(x)=0 and solve for x:

$18x^2 - 162x + 360 = 0$

divide 18 away: $x^2 - 9x + 20 = 0$

factor: $(x-4)(x-5) = 0$

x=4, x=5 where f'(4)=0, or f'(5)=0

find g''(x): $g''(x) = [g'(x)]'$

$= (18x^2 - 162x + 360)'$

$= 36x - 162$

Evaluate g''(5)..why? b/c x=5 says f'=0 and we have seen that if g''(5)>0, we have a $\cup$ or , if g''(5)<0, we have $\cap$.

$g''(5) = 36 \cdot 5 - 162 = 18 > 0$, f is CU.

Since g'(5)=0 and g''(5)>0, so we have a local min.

Q1: C(x)= $x^3 - 18x^2 + 105x + 24$

a) Find $C'(x) \xrightarrow{\text{power rule each term}} 3x^2 - 36x + 105$

b) Find C''(x) $\xrightarrow{\text{power rule each term of C'(x)}} 6x - 36$

c) Find any critical points of C(x), found by writing C'(x)=0 or C'(x) is DNE!)

$C'(x) = 0$

$3x^2 - 36x + 105 = 0$

divide 3: $x^2 - 12x + 35 = 0$

factor : $(x-7)(x-5) = 0$

$x = 7, x = 5$

d) critical point of C'': C''(x)=0

$6x - 36 = 0$

$6x = 36$

$x = 6$

e)Find the local max:

either first derivative or second derivative test with f'=0.

$x = 5$: (from first derivative)

$C'(5) = 0$ and C''(5) $= 6 \cdot 5 - 36 = 30 - 36 = -6 < 0$!

CD local max at x=5.

f) find the local min:

$x = 7$ from first derivative :

$C'(7) = 0$, so horizontal tangent line on f and

$C''(7) = 6 \cdot 7 - 36 = 42 - 36 = 6 > 0$, so CU..

CU local min at x=7.

g) Find the inflection point:

check behavior of C'' around its own critical x value. x=6:

$-\infty \leftarrow$ -------------------- 6----------------------- $\rightarrow \ldots \infty$

$x = 0$ b/c <6

$C''(0) = 6 \cdot 0 - 36$

$= 0 - 36$

$= -36 < 0$

$x = 7$ b/c 7>6:

$C''(7) = 6 > 0$ (above we have C''(7) already)

C'' switches signs at x=6, so C (original function) has an inflection point at x=6.

my habit is to write f(x) , or f' or f'' ...but sometimes functions have different names!!!