Calculus 1 notes..11/2/2023...more from section 3.7:

1. A particle moves according to $f(t)=t^3-12t^2+36t$ (position function for any $0\le t\le 8$) seconds/feet

(a) velocity= $s'(t)=(t^3)'+(-12t^2)'+(36t)'=3t^2-24t+36$

(b) velocity at t=3: $v(3)=s'(3)=3\cdot 3^2-24\cdot 3+36=3\cdot 9-72+36=27-72+36=-9\,m/s\,The$ minus tells us it's going left. Velocity is speed but signed so we know left or right motion.

(c) When is the particle at rest? At rest means velocity =0.

$3t^2-24t+36=0$ (set $s'(t)$ equal to 0)

divide by 3: $t^2-8t+12=0$

factor the LHS: $(t-2)(t-6)=0$

solve for t: $t=2, t=6$..both fall within 0 $to\cdot 8$ from above.

(d) when is it going right? $v(t)>0$.. 0 ---$\rightarrow$---------2------------$\leftarrow$-------------6---$\rightarrow$--------8

$t=1$ $\quad t=3$ $\quad t=7$

draw a number line and mark the roots of v..and test values like t=1, t=3 and t=7.

$v(1)=3\cdot 1^2-24\cdot 1+36=3-24+36=15>0$ (right pointing arrow)

$v(3)=3\cdot 3^2-24\cdot 3+36=-9$ (from above)

$v(7)=3\cdot 7^2-24\cdot 7+36\xrightarrow{\text{calculator work....}}15>0$ (right pointing arrow)

(e) total distance traveled during the first 8 seconds: The particle turns twice at t=2 and t=6.

total distance= distance from 0 to 2 seconds + distance from 2 to 6 seconds + distance from 6 to 8 seconds.

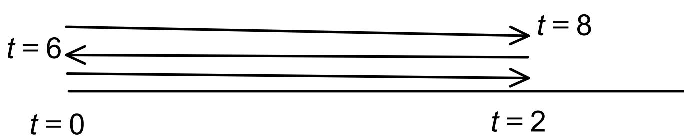
distance as absolute value of displacement= $|f(\text{later time})-f(earlier\text{ time})|$ distance= $|\ldots|\ge 0$

compute f(0), f(2), f(6), f(8)

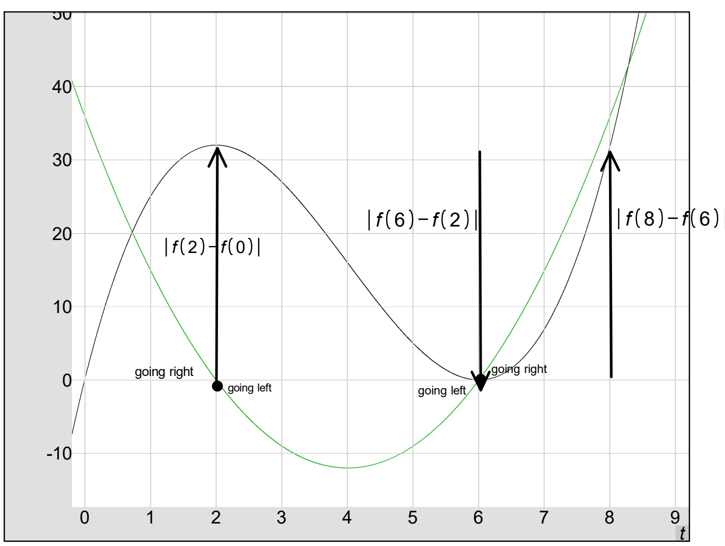
$f(0)=0^3-12\cdot 0^2+36\cdot 0=0$, $\quad f(2)=2^3-12\cdot 2^2+36\cdot 2=8-12\cdot 4+72=8-48+72=-40+72=32$

$f(6)=6^3-12\cdot 6^2+36\cdot 6=0$ $\quad f(8)=8^3-12\cdot 8^2+36\cdot 8=32$

distance $=|\boldsymbol{f}(\mathbf{2})-f(0)|+|f(6)-\boldsymbol{f}(\mathbf{2})|+|f(8)-f(6)|=|32-0|+|0-32|+|32-0|=32+32+32=96$ feet



This is A visual interpreation of the motion. DOn't take this diagram literally. A more realistic picture , since the particle is going left or right only, would show the arrows overlapping and then it would be messy.



Example 2: A particle moves along a straight line and its position at time t is given by s(t)= $2t^3-18t^2+30t$

(a) Find the velocity (in ft/sec) of the particle at any time t=0:

$s'(t)=(2t^3)'+(-18t^2)'+(30t^1)' \xrightarrow{\text{power rule one each term}} 2\cdot3\cdot t^{3-1}-18\cdot2\cdot t^{2-1}+30\cdot1\cdot t^{1-1}=6t^2-36t+30$

$s'(0)=v(0)=6\cdot0^2-36\cdot0+30=6\cdot0-0+30=30$ ft/sec (object is moving at t=0 at 30feet/second)

(b). The particle stops moving (is at rest) twice, first when t=..... and then again when t=......

s'(t)=0. $\xrightarrow{\text{solve}} 6t^2-36t+30=0 \xrightarrow{\text{divide by 6}} t^2-6t+5=0 \xrightarrow{\text{factor LHS}} (t-1)(t-5)=0 \xrightarrow{\text{solve for t}} t=1, t=5$

(c) What is the position of the particle at t=12? $s(12)=2\cdot12^3-18\cdot12^2+30\cdot12 \xrightarrow{\text{calculator work...}} 1224$ feet

(d) find the total distance the particle traveles from t=0 to t=12:

number line: 0 ---→-----------1 -----------←---------------------5-----------→---------------------12 (t axis)

test values: $t=0.5$ $t=2$ $t=6$ ← these numbers are not unique or special... just between the given values 0,1,5 and 12.

$s'(0.5)=6\cdot0.5^2-36\cdot0.5+30=13.5$

$s'(2)=6\cdot2^2-36\cdot2+30=-18$ $s'(6)=6\cdot6^2-36\cdot6+30=30$

total distnce= distance from t=0 to t=1+ distance from t=1 to t=5 + distance from t=5 to t=12

$s(0)=2\cdot0^3-18\cdot0^2+30\cdot0=0$, $s(1)=2\cdot1^3-18\cdot1^2+30\cdot1=14$ ft, s(5) $=2\cdot5^3-18\cdot5^2+30\cdot5=-50$

$s(12) \xrightarrow{\text{from above}} 1224$ ... distance= $|s(1)-s(0)|+|s(5)-s(1)|+|s(12)-s(5)|$ ..$|1224-(-50)|$

$=|14-0|+|-50-14|+|1224-(-50)|$

$=|14|+|-64|+|1274|$

$=14+64+1274=1352$ feet