Sketch a graph of $y = \frac{2x^2}{x^2-1}$

a. $y(0) = \frac{2\cdot 0^2}{0^2-1} = \frac{0}{-1} = 0$, so $(0,0)$ is on the graph.

b. domain: $x^2-1 \neq 0 \Rightarrow x^2 \neq 1 \Rightarrow$ $-\infty \ldots \leftarrow$ ------------ -1 -------------------- 1 ------------ $\rightarrow \ldots \infty$

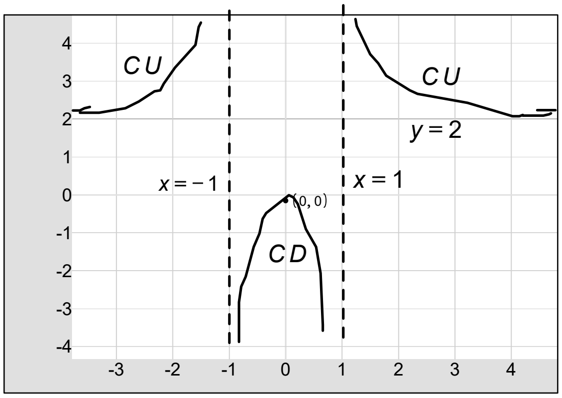
interval form: $(-\infty,-1)\cup(-1,1)\cup(1,\infty)$

c. symmetry: $f(-x) = \frac{2(-x)^2}{(-x)^2-1} = \frac{2x^2}{x^2-1} = f(x)$. So we have y-axis symmetry.

d. $\lim_{x\to\infty} \frac{2x^2}{x^2-1} = \lim_{x\to\infty} \frac{2x^2}{x^2} = 2$, so $y=2$ is the horizontal asymptote going towards positive infinity along x.

e. $\lim_{x\to-\infty} \frac{2x^2}{x^2-1} = \lim_{x\to-\infty} \frac{2x^2}{x^2} = 2$, so y=2 is the horizontal asymptote as x goes to negative infinity.

f. Since x=-1 and x=1 are excluded from the domain, x=-1 and x=1 are vertical asymptotes.



g. roots: $y = 0 \Rightarrow \frac{2x^2}{x^2-1} = 0 \Rightarrow 2x^2 = 0(x^2-1) \Rightarrow 2x^2 = 0 \Rightarrow x = 0$

Notice we already have (0,0) from the y intercept.

$y' = \left(\frac{2x^2}{x^2-1}\right)' = [2x^2(x^2-1)^{-1}]' = (2x^2)'(x^2-1)^{-1} + (2x^2)[(x^2-1)^{-1}]'$

h.
$$= 4x(x^2-1)^{-1} + 2x^2(-1)(x^2-1)^{-2}(x^2-1)'$$
$$= 4x(x^2-1)^{-1} - 2x^2(x^2-1)^{-2}(2x)$$
$$= 4x(x^2-1)^{-1} - 4x^3(x^2-1)^{-2}$$
$$= \frac{4x}{x^2-1} - \frac{4x^3}{(x^2-1)^2} = \frac{4x(x^2-1)}{(x^2-1)^2} - \frac{4x^3}{(x^2-1)^2}$$
$$= \frac{4x^3-4x-4x^3}{(x^2-1)^2} = \frac{-4x}{(x^2-1)^2}$$

Since bottom is $(x^2-1)^2$, it's always>0, so -4x decides sign. When $x<-1$, $-4x>0$, so y'>0. Function is increasing over $(-\infty,-1)$. -1<x<0, we have $-4x>0$, so f is again increasing. Over 0<x<1, we ahve -4x<0, so f decreases over (0,1). Over (1, ∞), we have -4x<0, so f decreases over (1, ∞).

i. second derivative: $y'' = \left(\frac{-4x}{(x^2-1)^2}\right)' = -4\left[\frac{x}{(x^2-1)^2}\right]' = -4[x(x^2-1)^{-2}]' = -4[(x)'(x^2-1)^{-2} + x((x^2-1)^{-2})']$

$$= -4[1(x^2-1)^{-2} + x(-2)(x^2-1)^{-3}(2x)]$$
$$= -4\left[\frac{1}{(x^2-1)^2} - \frac{4x^2}{(x^2-1)^3}\right]$$
$$= -4\left[\frac{(x^2-1)}{(x^2-1)^3} - \frac{4x^2}{(x^2-1)^3}\right]$$
$$= -4\left[\frac{x^2-1-4x^2}{(x^2-1)^3}\right]$$
$$= -4\left[\frac{-3x^2-1}{(x^2-1)^3}\right]$$
$$= \frac{12x^2+4}{(x^2-1)^3}$$

For $\frac{12x^2+4}{(x^2-1)^3}$, $12x^2 + 4>0$ for all x,so bottom, which changes signs, decides the sign of the overall expression.

For x<-1, use x=-2:

$\frac{12(-2)^2+4}{((-2)^2-1)^3} = 1.9 > 0$, *so* f is concave up on $(-\infty,-1)$

over (-1,1), we get using x=0 as a test value:

$\frac{12(0)^2+4}{(0^2-1)^3} = \frac{4}{(-1)^3} = -4 < 0$, so f is concave down.

over $(1,\infty)$, using x=2 as a test value, we get

$x=2$: $\frac{12\cdot 2^2+4}{(2^2-1)^3} = 1.9 > 0$, so f is concave up

j. $\lim_{x\to-1^-} \frac{2x^2}{x^2-1} \Rightarrow$ set x=- 1.00001 (left of -1 but very close) $\frac{2(-1.0001)^2}{(-1.0001)^2-1} = 10001.5000$. So $\lim_{x\to-1^-} \frac{2x^2}{x^2-1} = \infty$.

$x = -1$

k. $\lim_{x\to-1^+} \frac{2x^2}{x^2-1} \Rightarrow$ set $x = -0.9999$ (right of -1 but very close): $\frac{2(-0.9999)^2}{(-0.9999)^2-1} < 0$, so $\lim_{x\to-1^+} \frac{2x^2}{x^2-1} = -\infty$.

$x = -1$

l. $\lim_{x\to1^-} \frac{2x^2}{x^2-1} \Rightarrow$ set $x = 0.9999$ (left of x=1 but very close) $\frac{2(0.9999)^2}{(0.9999)^2-1} < 0$, so $\lim_{x\to1^-} \frac{2x^2}{x^2-1} = -\infty$

$x = 1$

m. $\lim_{x\to1^+} \frac{2x^2}{x^2-1} \Rightarrow$ set x= 1.0001 (*right* of x=1 but very close) $\frac{2(1.0001)^2}{(1.0001)^2-1} > 0$ and very big, so $\lim_{x\to1^+} \frac{2x^2}{x^2-1} = \infty$.