Section 5.4/Exponential and Logarithmic Equations:

recall: $a^x = a^y$ if and only if x=y (If bases are equal, then exponents must be equal)

$2^3 = 8 \Rightarrow$ rewrite as $\mathbf{2^3 = 2^3}$

$log_a x = log_a y$ if and only if $x = y$.

$log_2(8) = log_2(x) \Rightarrow$ then x=8!!

Things that go into functions are called arguments.

$a^{log_a x} = x$ recall: $2^{log_2(8)} = 2^3 = \mathbf{8}$

$log_a(a^x) = x$ recall: $log_2(8) = log_2(2^3) = \mathbf{3}$

example 1: $2^x = 32$

$2^x = 2^5$

$x = 5$

$ln x - ln 3 = 0$

$ln x - ln 3 + ln 3 = 0 + ln 3$

$ln x = ln 3$

$x = 3$

$\left(\frac{1}{3}\right)^x = 9$

$(3^{-1})^x = 3^2$

$\mathbf{3^{-x} = 3^2}$

$-x = 2$

$x = -2$

$e^x = 7$

inverse of e^x is ln function: take ln of both sides: $ln e^x = ln 7$

apply power rule on LHS/bring x down: $x ln e = ln 7$

recall that $ln e$ means $\log_e$ (e)=1

$x \cdot 1 = ln 7$

$x = ln 7$

$ln x = -3$

exponentiate both sides: $e^{ln x} = e^{-3}$

recall that e and ln are inverse, so they cancel: $\not{e}^{\not{ln(x)}} = e^{-3}$

$x = e^{-3}$

$log x = -1$

rewrite as $10^{log x} = 10^{-1}$

log and 10 cancel: $x = 10^{-1} = \frac{1}{10}$

$\log_3$ (x)=4

exponentiate: $3^{log_3(x)} = 3^4$

3 *and* $\log_3$ cancel: x=3^4

$x = 81$

example 2 in book: $e^{-x^2} = e^{-3x-4}$

set exponents equal: $-x^2 = -3x - 4$

$-x^2 + 3x + 4 = 0$

$x^2 - 3x - 4 = 0$ (divide by -1)

factor: $(x-4)(x+1) = 0$

$x - 4 = 0 \qquad x + 1 = 0$

$x = 4, \qquad x = -1$ (they do work i original equation)

$(b)\ 3 \cdot 2^x = 42$

$\frac{\not{3}}{\not{3}} \cdot 2^x = \frac{42}{3}$

$2^x = 14 \Rightarrow 2^x$, so take base 2 logs: $log_2(2^x) = log_2(14) \Rightarrow x = log_2(14)$

$x = \frac{ln14}{ln2}$ (change bases)

(3 in book) $e^x + 5 = 60$

$e^x + 5 - 5 = 60 - 5$

$e^x = 55$

take ln: $\ln e^x = \ln 55$

bring x down: x lne=ln55

$x \cdot 1$ =ln55

x=ln55 ≈ 4.007

example 6 in book:

$\ln x = 2$

$e^{\ln x} = e^2$ exponentiate both sides

$x = e^2$

b. $\log_3(5x-1) = \log_3(x+7)$

logs have same bases and each side has one log so set arguments equal:

$5x - 1 = x + 7$

$5x - x - 1 = x - x + 7$

$4x - 1 = 7$

$4x - 1 + 1 = 7 + 1$

$4x = 8$

$x = 2$

c. $\log_6(3x+14) - \log_6(5) = \log_6(2x)$

first combine LHS: $\log_6\left(\frac{3x+14}{5}\right) = \log_6(2x)$

same bases, so equate arguments! $\frac{3x+14}{5} = 2x$

$5 \cdot \frac{3x+14}{5} = 2x \cdot 5$

$3x + 14 = 10x$

$3x - 3x + 14 = 10x - 3x$

$14 = 7x$

$2 = x$

example 7: $5 + 2\ln x = 4$

$5 - 5 + 2\ln x = 4 - 5$

$2\ln x = -1$

$\frac{2\ln x}{2} = \frac{-1}{2}$

$\ln x = \frac{-1}{2}$

$e^{\ln x} = e^{-1/2}$ $\Rightarrow a^{\log_a(x)} = \boldsymbol{x}$

$x = e^{-1/2}$ $e^{\ln x}$ really means $e^{\log_e(x)} = x$

$\log 5x + \log(x-1) = 2$ recall $\log_{10}(x)$ is logx

$\log(5x(x-1)) = 2$

exponentiate: $10^{\log(5x(x-1))} = 10^2$ $\Leftarrow$ by $a^{\log_a(x)} = x$

$5x(x-1) = 100$

$5x \cdot x - 5x \cdot 1 = 100$

$5x^2 - 5x = 100$

divide by 5: $\frac{5x^2}{5} - \frac{5x}{5} = \frac{100}{5}$

$1x^2 - x = 20$

$\Rightarrow$ quad. equation.. $x^2 - 1x - 20 = 0$

(x-5)(x+4)=0 check: -5(4)=-20, and -5+4=-1

$x = 5, x = -4$

check with x=-4: $log(5 \cdot -4) + log(-4-1) = ?2$

$log(-20) + log(-5)$ cannot be equal to 2 b/c

log(-20) doesn't exist. log(-5) doesn't exist!

for any base log function, $\log_a$ (argument), arguemnt>0

So x=-4 is not a solution. x=5 does work.

example 10 in book: Doubling an investment:

You have deposited 500 in an account that pays 6.75% interest, compounded continuously. Formula: $A = Pe^{rt}$

$P = principal,\ e = 2.718$ r=rate(decimal form), t=time

math 200 we derive A=Pe^{rt}

How long (**t=?**) will it take for your money to double?

A= $500e^{0.0675\boldsymbol{t}}$ to double means 500 is now $2 \cdot 500$

$2 \cdot 500 = 500e^{0.0675\boldsymbol{t}}$ (LHS gets $2 \cdot 500$)

cancel off 500: $2 \cdot \frac{500}{500} = \frac{500}{500}e^{0.0675\boldsymbol{t}}$

$2 = e^{0.0675\boldsymbol{t}}$

since we have **e** , take ln: $ln2 = lne^{0.0675\boldsymbol{t}}$

$ln2 = 0.0675\boldsymbol{t} \cdot lne$ $\Leftarrow$ power rule on RHS

$ln2 = 0.0675\boldsymbol{t} \cdot 1$

$ln2 = 0.0675 \times \boldsymbol{t}$

$\frac{ln2}{0.0675} = \boldsymbol{t} \Rightarrow$ calculator $t = 10.27$ years!!

application: retail sales in billions of e-commerce companies in the US from 2002 to 2007 can be modeled by $y = -549 + 236.7lnt$, where $12 \le t \le 17$

(model is derived from some survey data)

During which year did the sales reach 108 billion?

what is t? $-549 + 236.7ln(t) = 108$? (units of billions on LHS and RHS would cancel)

$-549 + 549 + 236.7ln(t) = 108 + 549$

$236.7ln(t) = 657$

divide: $\frac{236.7}{236.7}ln(t) = \frac{657}{236.7}$

$ln(t) = 657/236.7$

$\Rightarrow pick$ up here : $e^{lnt} = e^{657/236.7}$

$\boldsymbol{t} = e^{657/236.7}$

$t \approx 16.$

our t is $12 \le t \le 17$

12 means 2002, 17 means 2007

t=16, 2006! !