Section 4.1/Rational Functions and Asymptotes:

A rational function is a quotient of two polynomials: $f(x) = \dfrac{N(x)}{D(x)}$, N(x) is poly. and D(x) is a poly.

Example 1: Graph $f(x) = \dfrac{1}{x}$, $N(x) = 1$, $D(x) = x$

$x = -2: f(-2) = \dfrac{1}{-2} = .-5 \xrightarrow{point} (-2, -0.5)$

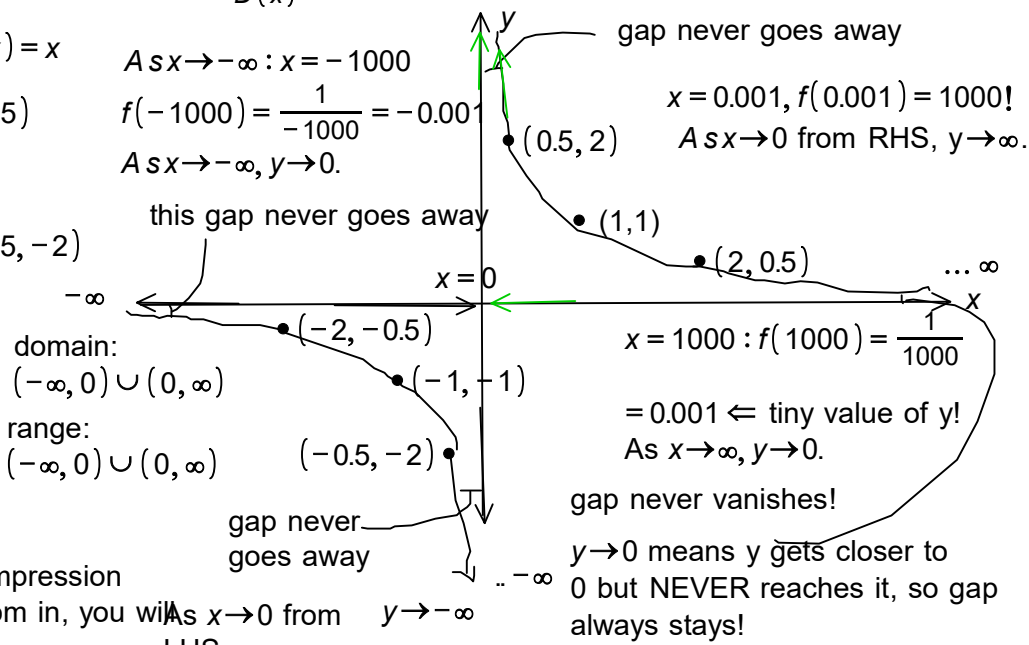$x = -1: f(-1) = \dfrac{1}{-1} = -1 \xrightarrow{point} (-1, -1)$

$x = -0.5: f(-0.5) = \dfrac{1}{-0.5} = -2 \xrightarrow{point} (-0.5, -2)$
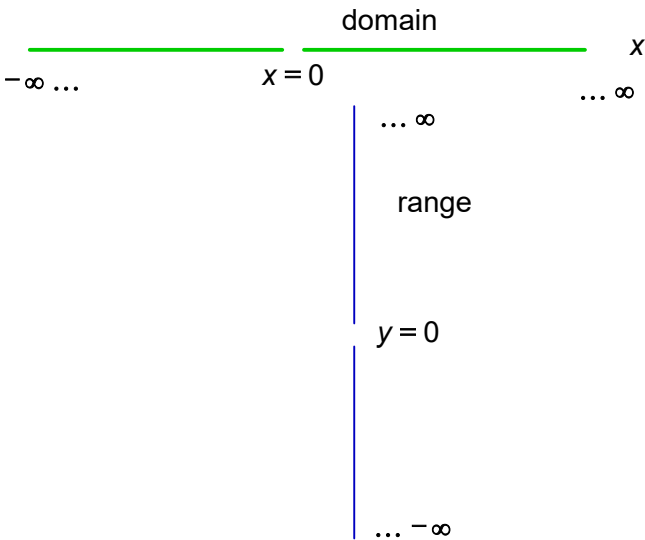
$x = 0: f(0) = \dfrac{1}{0} \leftarrow$ undefined!

$x = 0.5: f(0.5) = \dfrac{1}{0.5} = 2 \xrightarrow{point} (0.5, 2)$

$x = 1: f(1) = \dfrac{1}{1} = 1 \xrightarrow{point} (1, 1)$

$x = 2: f(2) = \dfrac{1}{2} = 0.5 \xrightarrow{point} (2, 0.5) x$

When graphing 1/x, you might get the impression graph touches x or y axes, but if you zoom in, you will see this is false!
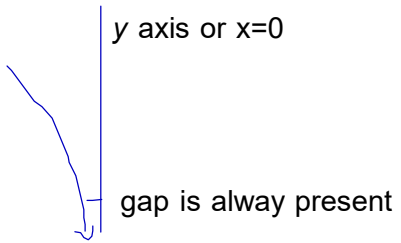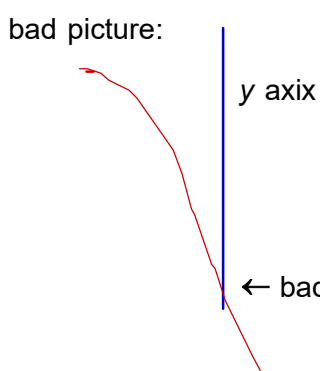
As $x \to -\infty$: $x = -1000$

$f(-1000) = \dfrac{1}{-1000} = -0.001$

As $x \to -\infty$, $y \to 0$.

this gap never goes away

domain:
$(-\infty, 0) \cup (0, \infty)$

range:
$(-\infty, 0) \cup (0, \infty)$

gap never goes away

gap never goes away

$x = 0.001, f(0.001) = 1000!$
As $x \to 0$ from RHS, $y \to \infty$.

$(0.5, 2)$

$(1, 1)$

$(2, 0.5)$ ... ∞

$x = 0$

$(-2, -0.5)$

$(-1, -1)$

$(-0.5, -2)$

$x = 1000: f(1000) = \dfrac{1}{1000}$

$= 0.001 \Leftarrow$ tiny value of y!
As $x \to \infty$, $y \to 0$.

gap never vanishes!

$y \to 0$ means y gets closer to 0 but NEVER reaches it, so gap always stays!

gap never goes away

As $x \to 0$ from LHS
$x = -0.001$

$f(-0.001) = \dfrac{1}{-0.001} = -1000$

$y \to -\infty$

domain

$-\infty$ ... $x = 0$ ... ∞ x

... ∞

range

$y = 0$

... $-\infty$

If you cross x=0 (y axis), you're saying, for example, that

$f(0) = \dfrac{1}{0} = -1000$ now..

bad picture:

y axix

Think of $f(x) = \dfrac{1}{x}$ as the fundamental rational function.

It's called the parent rational function.
When we move along the x axis, every y is 0, so it makes sense to refer to the x axis as the line y=0. So y=0 is called the HORIZONTAL ASYMPTOTE. So an asymptote is a line we approach but we don't cross.

When we move on the y axis, every x is 0, so we can call this x=0. The line x=0 is called the VERTICAL ASYMPTOTE.

y axis or x=0

gap is alway present

$\leftarrow$ bad!! can't be!..so 1/x doesn't cross y axis.

Homework Q1:

Graph the function $f(x) = \dfrac{x+2}{x-2}$, $N(x) = x+2$, $D(x) = x-2$ (ratio of two binomials)

1. input the vertical asymptote: value x that leads to division by 0:
$x - 2 \neq 0$ (set D(x)=0)
$x - 2 + 2 \neq 0 + 2$
$x \neq 2 \Leftarrow$ exclude from domain! in box on myopenamth put x=2(vertical asymptote)

2. input the horizontal asymptote: $y =$ some number

evaluate at x=2:
$f(2) = \dfrac{2+2}{2-2} = \dfrac{4}{0}$ (undefined)

For horizontal asymptotes, we're asking what happens to y when $x \to \infty$ or $x \to -\infty$:

choose x= 1000 : $f(1000) = \frac{1000+2}{1000-2} = \frac{1002}{998} = 1.004$

choose x=10000: f(10000)= $\frac{10000+2}{10000-2} = 1.0004$

As $x \to \infty$, $y \to 1$, so y=1 is the horizontal asymptote!

In box input y=1

3. Input the point where the asymptotes meet: x=2 is VA, y=1 is the HA, so they meet at (2,1) gap

. 4. Input the root as a number: $f(x) = 0$

$\frac{x+2}{x-2} = 0 \leftarrow 0$ represents y=0!

$\frac{x+2}{x-2} = \frac{0}{1}$ proportion

$1(x+2) = 0(x-2)$ cross multiplication
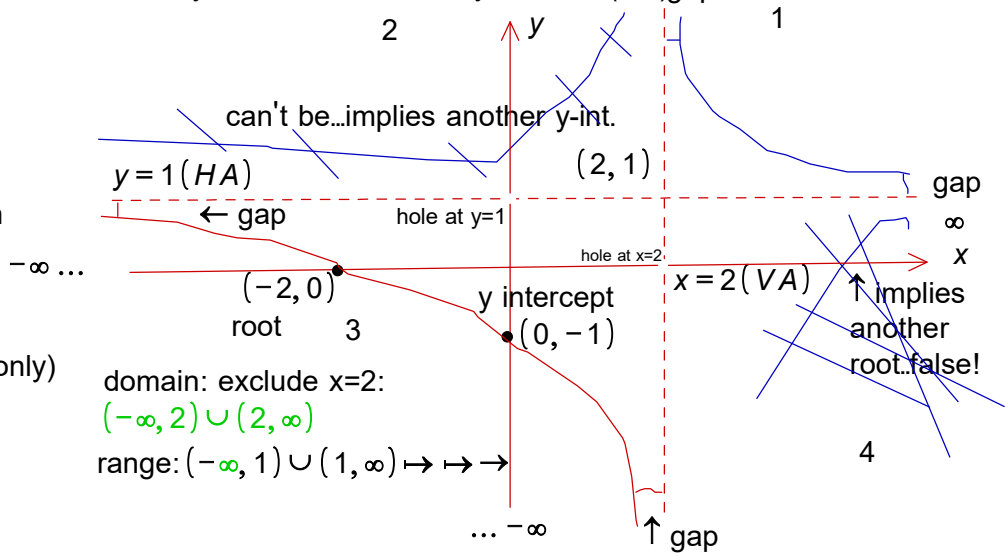
$x+2 = 0$

$x = -2 \xrightarrow{point} (-2, 0)$

graph crosses x-axis at (-2,0).(one point only)

5. y-intercept: $f(0) = \frac{0+2}{0-2} = \frac{2}{-2} = -1 = y$

$point = (0, -1)$ (must be on graph)

can't be...implies another y-int.

$y = 1(HA)$ (2,1)   gap

$\leftarrow$ gap   hole at y=1 ∞

$-\infty$ ...   hole at x=2 x

$(-2, 0)$ y intercept $x = 2(VA)$ ↑ implies

root   3 $(0, -1)$ another root..false!

domain: exclude x=2:

$(-\infty, 2) \cup (2, \infty)$

range: $(-\infty, 1) \cup (1, \infty) \mapsto \mapsto \to$

... $-\infty$ ↑ gap   4

summary: f(x)= $\frac{N(x)}{D(x)}$, $D(x)$ cannot be 0..solve D(x)=0 and this tells you location of VA.

root: $\frac{N(x)}{D(x)} = 0 \to N(x) = 0 \leftarrow y = 0$ and solve for x!

HA: plug in x=1000, x=10000 until the values of y begin to show a stable pattern!

$\frac{N(x)}{D(x)}$ is a function, so it must pass the vertical line test.

For yours, there is only a single point where the graph crosses the x axis.

For any $\frac{N(x)}{D(x)}$, there is only a single y intercept.