Math 111 Notes-11/13/2023. Please put away all phones and computers and write down detailed notes. This helps the information to get written to the brain. The real price of watching TikTok vidoes is not 0. It's the wasted hours that could be invested into something succesful. Value your time. It's limited.

Section 5.1/Exponential Functions:

Def: $f(x)=a^x$. a>0, a $\neq 1$ and x is any real number.

examples: $f(x)=2^x,\ \ 2>0, 2\neq 1\ \ a=2$ $\qquad g(x)=(1/3)^x\ \ 1/3>0,\ \ 1/3\neq 1,\ \ a=1/3$

$h(x)=(-1)^x,\ \ -1>0$ false so not exponential, $\qquad$ z(x)= 2^{-x} $\ \ 2>0, 2\neq 1$, so it's exponential

reminder:

$x^{a/b}=\sqrt[b]{x^a}$

$a^{-n}=\dfrac{1}{a^n}$

example 2: $f(x)=2^x \Rightarrow f(-3.1)=2^{-3.1} \Rightarrow$ calculator work $\Rightarrow f(-3.1)=0.11$

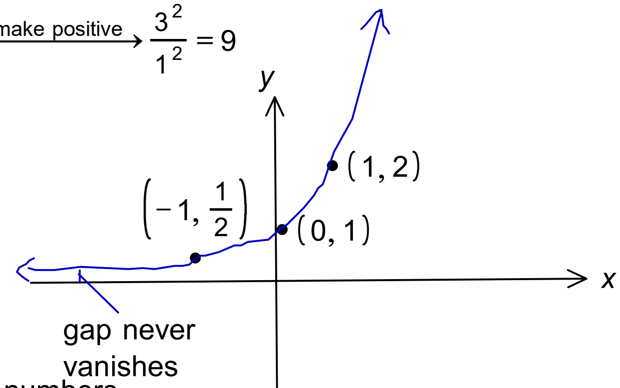
(example 1 in book) $g(x)=3^x \Rightarrow find\ g(2)=3^{2}=9$

$h(x)=0.6^x \xrightarrow{\text{find}} h(3/2)=0.6^{3/2}=(\sqrt{0.6})^3\approx 0.46$

$z(x)=(1/2)^x \xrightarrow{\text{find}} z(2)=\left(\dfrac{1}{2}\right)^2=\dfrac{1^2}{2^2}=\dfrac{1}{4}$

$p(x)=\left(\dfrac{1}{3}\right)^{-x} \xrightarrow{\text{find}} p(2)=\left(\dfrac{1}{3}\right)^{-2}=\dfrac{1}{\left(\frac{1}{3}\right)^2}=\dfrac{1}{\frac{1^2}{3^2}}=\dfrac{1}{\frac{1}{9}}=9$

$or\ p(2)=\left(\dfrac{1}{3}\right)^{-2}=\dfrac{1^{-2}}{3^{-2}} \xrightarrow{\text{flip and make positive}} \dfrac{3^2}{1^2}=9$



Example 2 (in book) Sketch y= 2^x , $2>0, 2\neq 1$

$x=-1:\ f(-1)=2^{-1}=\dfrac{1}{2}$ point=(-1,1/2)

$x=0:\ f(0)=2^0=1\ point=(0,1)$

$x=1:\ \ f(1)=2^1=2\ point=(1,2)$

domain..as we see above, positive and negative numbers can go in and any number can go in.. $(-\infty,\infty)$, all REAL numbers

$-\infty$ ... $\qquad$ ... ∞

$2^{-3}, 2^{0.25}, 2^{\sqrt{3}}, 2^6$ (examples x=-3, x=.25, x= $\sqrt{3}$, $x=6$ and so on)

imagine x= -10000: $\quad f(-10000)=2^{-10000}=\dfrac{1}{2^{10000}}$ very tiny number but NOT y=0! so gap between graph and x axis!

range: $Since$ y is never 0, range is $(0,\infty)$ $\qquad$ ... ∞

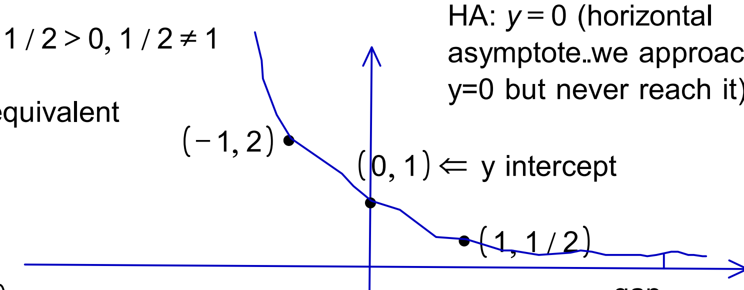
y axis taken out of picture

$y=0$

using this

Example 3/book: $F(x)=\overbrace{2^{-x}}^{\text{using this}}=2^{-1\cdot x}=(2^{-1})^x=\underbrace{\left(\dfrac{1}{2}\right)^x}_{\text{this is equivalent}},\ \ 1/2>0, 1/2\neq 1$

HA: $y=0$ (horizontal asymptote..we approach y=0 but never reach it)



$x=-1:\ \ F(-1)=2^{-(-1)}=2^1=2$, (-1,2)

x= 0: $\quad F(0)=2^{-0}=2^0=1\quad (0,1)$

$x=1:\ \ F(1)=2^{-1}=\dfrac{1}{2}\quad \left(1, \dfrac{1}{2}\right)$

domain: any values of x can go in, so domai is $(-\infty,\infty)$

range: (0, ∞)(gap says y is never equal to 0, so no [0...) (same pictures as above for domain and range)

gap never goes away

This function has not roots. So no x such that 2^x =0!

example 4: Solve $9=3^{x+1}$ $\qquad$ review: 2=2 exponentiate $3^2=3^2$

common base: $\mathbf{3}^2=\mathbf{3}^{x+1}$

set expos. equal: $2=x+1$

$x=1$

example 4b in book:

$\left(\dfrac{1}{2}\right)^x=8$ $\qquad$ -x=3 (set expos. equal)

x=-3

$\left(\dfrac{1}{2}\right)^x=2^3$

$(2^{-1})^x=2^3$

$2^{-x}=2^3$

Example 5 in book: Transformations of exponential functions:

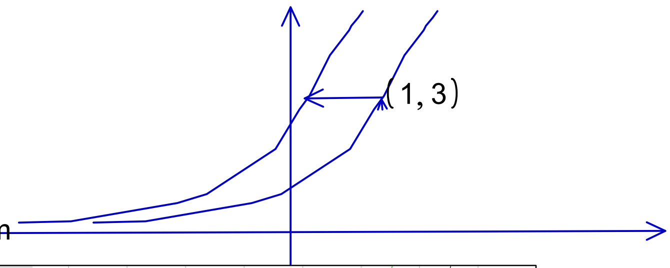
$g(x)=3^{x+1} \xrightarrow{\text{rewrite as}} 3^{x-(-1)}$ This says the graph of 3^{x+1} is the graph of 3^x shifted by 1 unit to the left b/c of the **-1** ..So make a picture of 3^x and and shift, like a rigid wire, by 1 unit left.

$f(x) = 3^x \Leftarrow$ parent graph

$x = -1: \quad f(-1) = 3^{-1} = 1/3, \; (-1, 1/3)$

$x = 0: \quad f(0) = 3^0 = 1 \quad (0, 1)$

$x = 1: \quad f(1) = 3^1 = 3 \quad (1, 3)$

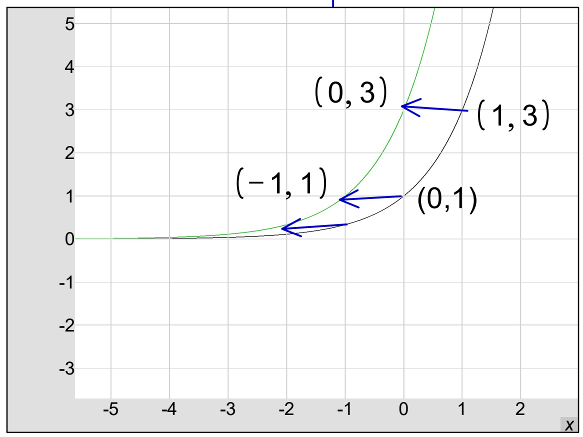


transformed version just means subtract 1 from each x and keep each y unchanged.

$(1,3) \xrightarrow{\text{subtract 1 from x}} (1-1, 3) = (0, 3)$

$(0,1) \xrightarrow{\text{subtract 1 from x}} (-1, 1)$

$(-1, 1/3) \xrightarrow{\text{subtract 1 from x}} (-2, 1/3)$



We're going from black to green:
domain is still $(-\infty, \infty)$
range is still $(0, \infty)$ $\ldots \infty$

HA: y=0
y=0 still cannot be reached, so (0..

$y=0$

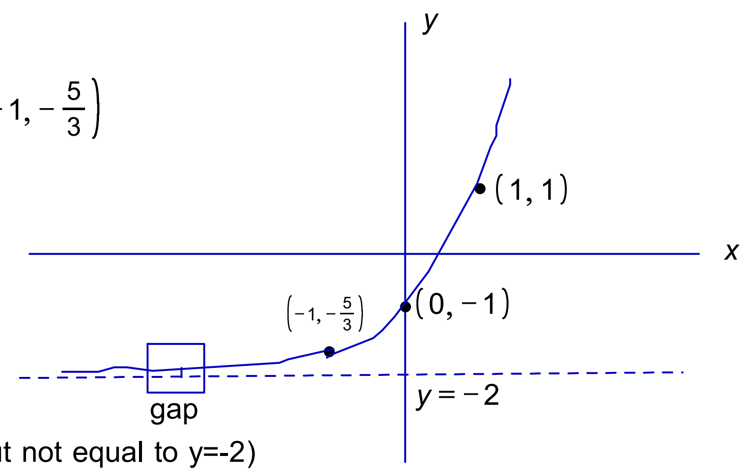
$f(x) = 3^x - 2$ (the -2 is on the level of the 3 and not the x, so subtract 2 from each y..leave x)

basic 3^x points:

$(-1, 1/3) \xrightarrow{\text{subtract 2 from each y}} \left(-1, \frac{1}{3} - 2\right) = \left(-1, \frac{1}{3} - \frac{6}{3}\right) = \left(-1, -\frac{5}{3}\right)$

$(0,1) \xrightarrow{\text{subtract 2 from y}} (0, 1-2) = (0, -1)$

$(1,3) \xrightarrow{\text{subtract 2 from y}} (1, 3-2) = (1, 1)$



Since this is just 3^x, pulled down by 2 units, keep the shape of 3^x
original HA for 3^x is y=0. So subtract 2 from this also to make 0-2=-2 new HA

check it with x= -10: $3^{-10} - 2$ (number very close to -2 but not equal to y=-2)

domain for 3^x -2: ..still can plug in any x: $(-\infty, \infty)$
Range is different : $(-2, \infty)$ (y=-2 is never reached, so (-2 and not [-2)

last example:

$8^{3x} = \left(\frac{1}{4}\right)^{x-3}$

$(2^3)^{3x} = (4^{-1})^{(x-3)}$

$2^{3 \cdot 3x} = ((2^2)^{-1})^{(x-3)}$

$2^{9x} = 2^{-2(x-3)}$

$9x = -2(x-3)$

$9x = -2x + 6$

$9x + 2x = 6$

$11x = 6$

$x = \frac{6}{11}$

$125^{3x} = \left(\frac{1}{25}\right)^{x-1}$

$(5^3)^{3x} = (25^{-1})^{(x-1)}$

$5^{3 \cdot 3x} = ((5^2)^{-1})^{(x-1)}$

$5^{9x} = 5^{-2(x-1)}$

$9x = -2(x-1)$

$9x = -2x + 2$

$9x + 2x = 2$

$11x = 2$

$x = \frac{2}{11}$