Take detailed notes. Make sure your face is visible **on camera.**
Good question to ask: how can this next year be the most successful year of our lifes?
Turn off your microphones please.
Section 5.2/Logarithmic functions and their graphs:
beginning at 9:40AM:
exponential was $f(x) = a^x$, $a > 0$ and $a \neq 1$, and x is a real number!
log functions:
$f(x) = log_a(x)$, $a =$ base, x=variable, log is short for logarithm, which just means exponent!

$\mathbf{3}^{2} = 9$ exponential form becomes $\mathbf{2} = log_{\mathbf{3}}(9)$ (log form)

$\mathbf{2}^{5} = 32$ $\quad base = 2$, exponent=5, value is 32

$5 = log_{\mathbf{2}}(32)$ log form

summary: If $\boldsymbol{a}^y = x$, then $\underbrace{log_{\boldsymbol{a}}(x)}_{\text{fancy for exponent!}} = y$

$log_2(4) = \mathbf{2} \xrightarrow{\text{becomes in exponential form}} 2^{\mathbf{2}} = 4$

example 1/page 391:
$f(x) = log_2(x)$

$f(4) = log_2(4) \Leftarrow$ 2 raised to WHAT EXPONENT is 4? 2 $\Leftarrow$ output! (4,2)

$f(16) = log_2(16) \Leftarrow$ 2 raised to what exponent is 16? 4 $\Leftarrow$ output (16,4)

$f(2) = log_2(2) \xrightarrow{\text{can rewrite as}} log_2(2^{\mathbf{1}}) = 1$ point is $(2, 1)$

logarithm=exponent!!!

$f(1) = log_2(1) = log_2(2^0) = 0$ point is (1,0)

$f(32) = log_2(32) \xrightarrow{\text{rewrite as } 2^5} log_2(2^5) = 5$ point is (32,5)

$g(x) = log_4(x)$

$g(4) = log_4(4) \xrightarrow{\text{rewrite as } 4^1} log_4(4^1) = 1$ $(4, 1)$

$g(16) = log_4(\mathbf{16}) \xrightarrow{\text{rewrite 16 as } 4^2} log_4(4^2) = 2$ point is (4,2)

When we have $log_{\boldsymbol{a}}(a^b) =$ b(*when* the input can be written as a^b, the answer is b)

$h(x) = log_{10}(x)$

$h(100) = log_{10}(100) \xrightarrow{\text{100 is not 10, but it is } 10^2} log_{10}(10^{\mathbf{2}}) = \mathbf{2}$, $(100, 2)$

$h\left(\frac{1}{100}\right) = log_{10}\left(\frac{1}{100}\right) = log_{10}(100^{-1}) = log_{10}((10^2)^{-1}) = \underbrace{log_{10}(10^{-2})}_{\text{bases match: 10 and 10}} = -2$, (1/100,-2)

doesn't work: $log_3(4^5)$ it's not 5 b/c 3 and 4 are different!

equations between different forms:
$y = log_2(x)$ (log form) $\Rightarrow$ exponential form $\Rightarrow 2^y = x$

$y = log_3(x) \Rightarrow$ becomes $\Rightarrow 3^y$ =x

example: $log_{10}(x)$ is usually written as $log(x) \Leftarrow$ base of 10 is dropped..on calcs. it's log (x)

Does log(-10) make sense? no..b/c this says $\log_{10}$ (-10) in exponential form: $10^? = -10$? can't have this!!

$log_2(-4)$ make sense? no b/c it's saying $2^{?}$ =-4 can't be done!

for $\log_a$ (x), domain is x>0. interval form: (0, ∞)

base is a..so true for any log function ~~$log(-10)$~~, ~~$log_3(-27)$~~...give errors

$log_2(4)$ works b/c $2^{\mathbf{2}}$ =4 ~~$log_2(-4)$ ...$2^?$ =-4~~

Properties of Logs:

$log_a(1) = log_a(a^0) = 0$ (not true for a=0,but true for everything else)

$log_2(1) = log_2(2^0) = 0$

$log_3(1) = log_3(3^0) = 0$

$log_{1/2}(1) = log_{1/2}(1/2)^0 = 0$

big rule: $log_a(a) = 1$ for any a

$log_{1/3}(1/3) = log_{1/3}(1/3)^1 = 1$

$log_{\sqrt{7}}(\sqrt{7})^1 = 1$ base is $\sqrt{7}$

big rule: $a^{log_a(x)} = x$

$2^{log_2(8)} = 2^{log_2(2^3)} = 2^3 = \mathbf{8}$

$3^{log_3(\mathbf{27})} = 3^{log_3(3^{\mathbf{3}})} = 3^3 = \mathbf{27}$

$$a^{log_a(\mathbf{x})} = \mathbf{x}$$

$6^{log_6(20)} = 20$

$\mathbf{10}^{log_{\mathbf{10}}(4.56)} = 4.56$ b/c both have 10 as the base!

If it's $2^{log_4(25)} \neq 25$ b/c 2 and 4 are not the same!!

Log functions and exponential functions are inverse functions:

$y = 2^x$ log form is $y = log_2(x)$

$x = -1$, $y = 2^{-1} = 1/2$, $(-1, 1/2)$ $y = log_2(1/2) = log_2(2^{-1}) = -1$

(x,y) are switched! $(1/2, -1)$

$x = 0:$ $y = 2^0 = 1, (0, 1)$ $y = log_2(1) = log_2(2^0) = 0$, (1,0)

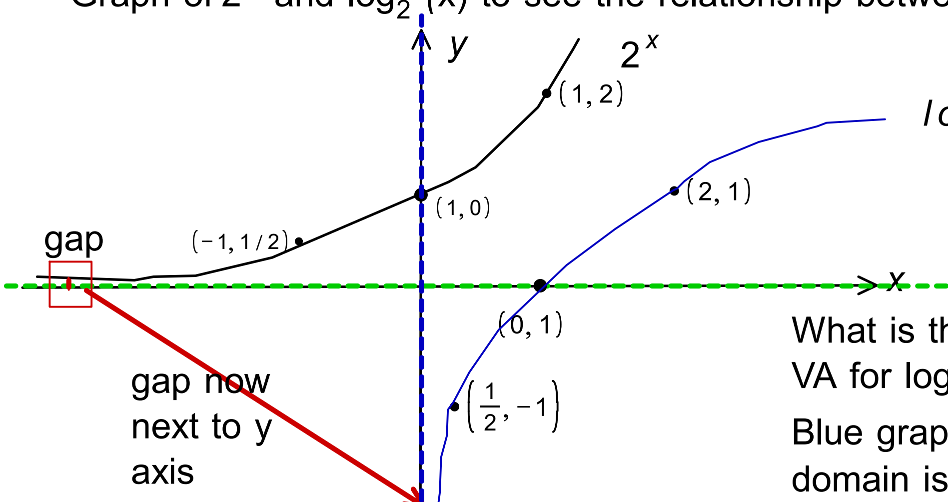
(x, y) are switched!

$x = 1:$ $y = 2^1 = 2, (1, 2)$ (x,y) are switched! $y = log_2(2) = log_2(2^1) = 1$

$(2, 1)$

So 2^x and log_2 (x) are inverse functions b/c the x and y coords. get flipped.

Graph of 2^x and log_2 (x) to see the relationship between them:



$y = 0$ is the HA for 2^x

What is the HA for 2^x becomes the VA for log_2 (x)

Blue graph for log_2 (x) has a gap, so domain is x>0.(not ~~$x \geq 0$~~)..in other words log_2 (0) is NOT defined!

range for 2^x is $(0, \infty)$
range for log_2 (x) is $(-\infty, \infty)$

$x = 0$ is now the vertical asymptote

Summary: same logic applies to 3^x and log_3 (x) or 4^x and log_4 (x) ...

summary again: 2^x has domain $(-\infty, \infty)$ and range $(0, \infty)$

$log_2(x)$ has domain $(0, \infty)$ and range $(-\infty, \infty)$

So points flip, asymptotes flip and domains and ranges flip.

We have bases like e, as in $y = e^x$, where e=2.718

For y=e^x, the inverse is $\log_e$ (x) but commonly this is written as $ln(x)$

$ln(x)$ read as "natural log function".

graphs of e^x and ln(x):

y=e^x

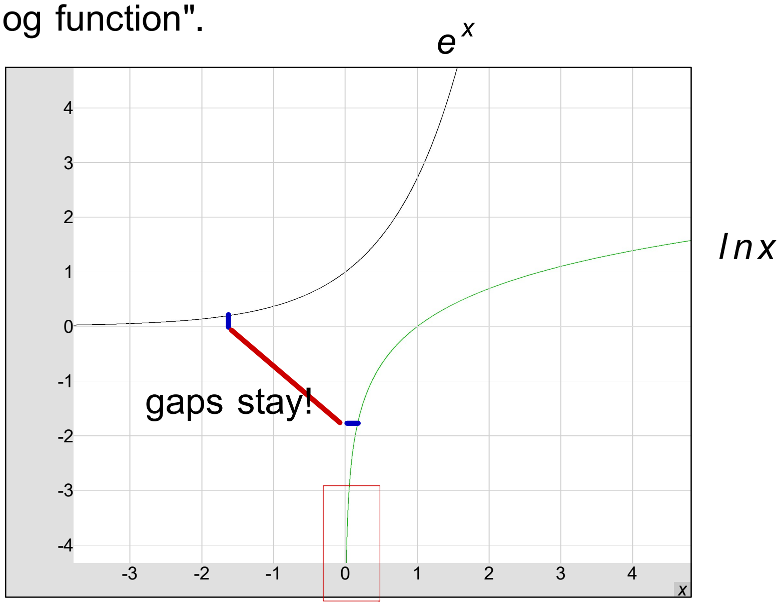
domain= $(-\infty, \infty)$

range= $(0, \infty)$

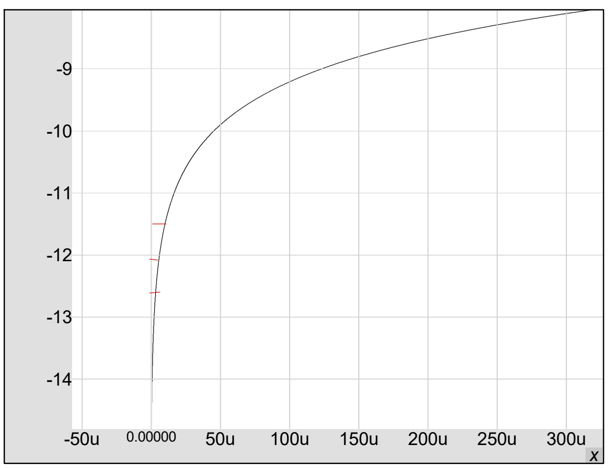
$y = ln(x)$

$range = (-\infty, \infty)$

domain= $(0, \infty)$



gap is there..just zoom in more!!



When we graph y=ln(x), do NOT cross the line x=0(y axis).

Transforming *the* ln function:

$y = ln(x-1)$ (-1 moves the graph of ln(x) 1 unit to the right, NOT left)

domain: ln(expression) , expression>0 *and* solve for variable:

$x - 1 > 0$

$x > 1$ interval form: $(1, \infty)$

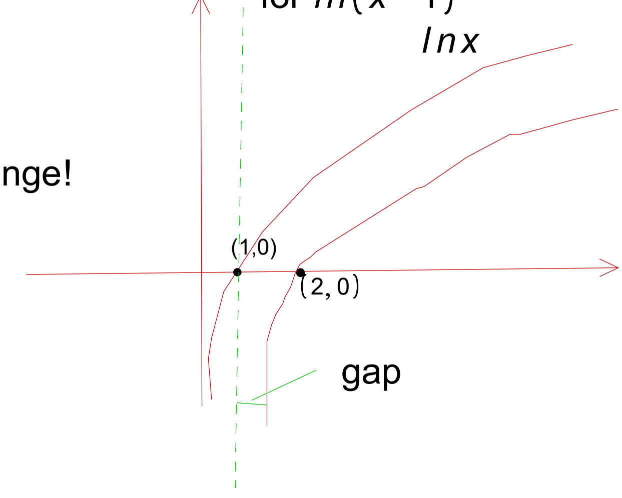
VA is x=1 b/c $ln(1-1) = ln(0) =$ undefined!

B/c we're doing a horizont shift, no range change!

just 1 to each x, so (1,0) becomes (2,0)

So VA of ln(x) gets shifted from x=0 to x=1 .

$x = 1$ is the VA for $ln(x-1)$



example: $ln(x+2)$ … $ln(0)$ IS NOT DEFINED!

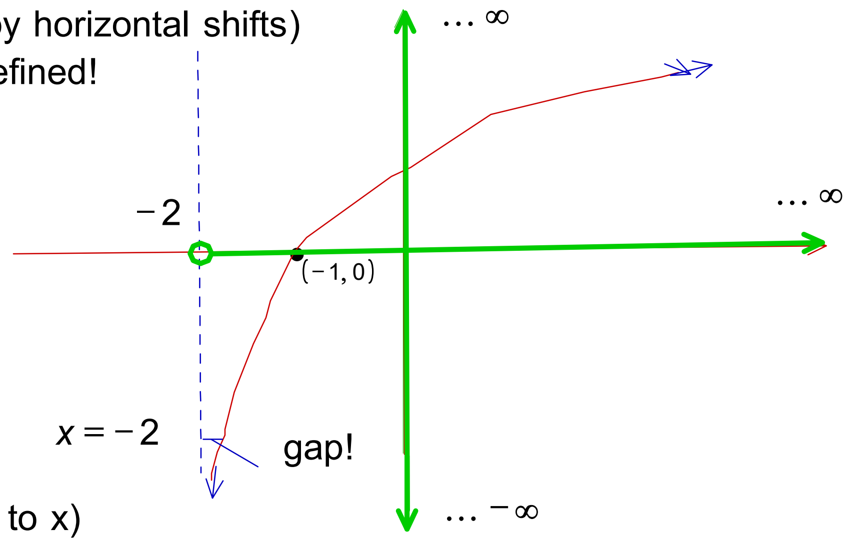
x+2 $>$ 0 and never ~~x+2 ≥ 0~~ can't have = in ≥.

$x > -2$…interval form $(-2, \infty)$ (changed relative to ln(x))

range=$(-\infty, \infty)$ (not affected by horizontal shifts)

VA: $ln(-2+2) = ln(0) =$ undefined!

$VA: x = -2$



$(1, 0)$ is a point on ln(x).

subtract 2 from x:

$(1-2, 0) = (-1, 0)$ on ln(x+2)

for ln(x+2), domain=$(-2, \infty)$

range=$(-\infty, \infty)$

$ln(x+2)$ it's really ln $(x-(-2))$

so subtract 2 from x(not add 2 to x)

on the other hand, ln(x-2) = $ln(x-(+2))$ (add 2 to each x)