Please be sure to take very detailed notes.

Section 5.5/Exponential and Logarithmic Models

exponential growth model: $y = a e^{bx}$, $b > 0$

exponential decay model: $y=ae^{-bx}$, $b>0$

Estimates of the amounts (in billions of dollars) of US online advertising spending from 2007 to 2011:

year	amount
2007	21.1
2008	23.6
2009	25.7
2010	28.5
2011	32.0

We feed this into a program and the result is

$S = \mathbf{10.33\, e^{0.1022 \cdot t}}$, $7 \le t \le 11$

$a = 10.33$, $b = 0.1022$, $e = 2.718 \ldots$

When will spending reach 40billion?

$t = ?$

$10.33\, e^{0.1022\, t} = 40$ (billions is not shown b/c it would cancel from both sides)

$\frac{10.33}{10.33} e^{0.1022\, t} = \frac{40}{10.33}$

$e^{0.1022\, t} = 40 / 10.33$ (will approximate at end)

$ln(e^{0.1022\, t}) = ln(40 / 10.33)$

$0.1022\, \mathbf{t}\, ln(e) = ln(40 / 10.33)$

$ln(e) = 1$ b/c it says $\log_e$ (e)

$0.1022\, \mathbf{t} = ln(40 / 10.33)$

$\frac{0.1022}{0.1022} \mathbf{t} = \frac{ln(40 / 10.33)}{0.1022}$

$\mathbf{t} = \frac{ln(40 / 10.33)}{0.1022} \xrightarrow{\text{calculator}} \approx 13.2$ years!

Assuming this model holds,(pattern continues)..t=13.2 *and* here t=7 represents 2007. So t=13.2 is 2013. Since t=7 means 2007, t=0 would be 200, t=13 would be 2013.

Example 2/Modeling Population Growth: In a research experiment, a population of fruit flies is increasing according to the law of exponnential growth. $P = \mathbf{a\, e^{bx}}$, $b > 0$. After 2 days there are 100 files, and after 4 days, there are 300 flies. How many flies will there be after 5 days?

Our model is $P = a e^{bx}$, $e = 2.718..$, we need to find a and b, x is the variable

use the first date point: $(2, 100)$ $\quad 100 = a e^{b \cdot 2}$

use the second data point: $(4, 300)$ $\quad 300 = a e^{b \cdot 4}$

$\Rightarrow$ divide equations bottom/top

$\frac{300}{100} = \frac{a e^{b4}}{a e^{2b}}$

$3 = \frac{e^{4b}}{e^{2b}} \xrightarrow{\text{rewrite}} 3 = e^{4b} e^{-2b} \xrightarrow{\text{combine}} 3 = e^{4b-2b}$

$\xrightarrow{\text{simplify}} 3 = e^{2b} \xrightarrow{\text{take ln}} ln3 = ln\, e^{2b} \xrightarrow{\text{2b down}} ln3 = \mathbf{2b}\, ln e$

$\xrightarrow{\text{lne=1}} ln3 = 2b \xrightarrow{\text{divide by 2}} \frac{ln3}{2} = b$

pick up here: now we have $b = ln3 / 2$:

Use it to find a using $100 = a e^{b \cdot 2}$

$100 = a e^{\frac{ln3}{2} \cdot 2} \xrightarrow{\text{cancel off 2}} 100 = a e^{ln3}$

$\xrightarrow{\text{recall e and ln are inverses, so they cancel}} 100 = a \cdot 3$

$\xrightarrow{\text{divide by 3}} \frac{100}{3} = a$

Our model is $P = \frac{100}{3} e^{\frac{ln3}{2} t}$

approximate $P = 33.3\, e^{0.5493 \cdot t}$

To use this mode, let t=5(how many flies in 5 days)

$P = 33.3\, e^{0.5493 \cdot 5} \approx 520$ flies.

Example 3/Carbon Dating: In living organic material, the ratio of the number of radioactive carbon isotopes (carbon 14) to the number of nonradioactive carbon isotopes(carbon 12) is about 1 to 10^{12}. WHen organic materil dies, its carbon 12 content remains fixed, whereas its radioactive carbon 14 maerial begins to decay with a half-life of about 5700 years Scientists use the following formula to estimate the age of dead organic material, which denotes the ratio of carbon 14 to carbon 12 present at any time t: $R = \frac{1}{10^{12}} e^{-t/8223}$ Question: **Estimate** the age of a newly discovered fossil in which the ratio of carbon 14 to carbon 12 is $R = \frac{1}{10^{13}}$.

$R =$ ratio $\quad \frac{1}{10^{13}} = \frac{1}{10^{12}} e^{-t/8223}$ $\Rightarrow$ goal is to get t, so multiply by 10^{12}: $\frac{10^{12}}{10^{13}} = \frac{10^{12}}{10^{12}} e^{-t/8223}$

$\Rightarrow \frac{1}{10} = e^{-t/8223} \Rightarrow$ take ln : $ln(1/10) = ln(e^{-t/8223}) \Rightarrow$ power rule $\Rightarrow ln(1/10) = -\mathbf{t}/8223 \cdot ln(e)$

$\Rightarrow ln(e) = 1 \Rightarrow ln(1/10) = -\mathbf{t}/8223 \Rightarrow$ multiply by 8223 $\Rightarrow ln(1/10) \cdot 8223 = -\mathbf{t} \Rightarrow -ln(1/10) \cdot 8223 = \mathbf{t}$

$\Rightarrow t = 18,934$ years.

Q1 homework: *A* worker's contract states that the hourly wage will start at 8.50 and will increase by r=5.5% annually, with a raise given every 12 months.

$S = P(1 + r/n)^{nt}$, P= 8.5, $r = 5.5\% = 0.055$, $n = 1$ (once per year)

exchange time for money or we make **profits!** — this will make you money rich!

The final hourly wage in 7 years will be ....?

$S = 8.5(1 + 0.055/1)^{1\cdot 7} = 8.5(1 + 0.055)^7 \Rightarrow$ calculator work $\Rightarrow$ 12.36 dollars

Question 2:

The half-life for thorium-227 is 18.72 days. The amount A(in grams..what is a gram? a gram is the amount of water in a 1 cm by 1cm by 1cm cube) of thorium 227 after t days for a 10-gram sample is given by $A(t) = 10 \cdot 0.5^{\frac{t}{18.72}}$, $a = 10, b = 1/18.72$ P=$a \cdot c^{b\cdot t}$, $a = 10, b = 1/18.72$ (not 18.72)

How long to get to 9 grams of thorium-227 in the sample?

$t = ?$

$10 \cdot 0.5^{\frac{t}{18.72}} = 9$

divide by 10: $0.5^{\frac{t}{18.72}} = 9/10$ (i will keep in this form)

take ln of both sides: $ln\left(0.5^{\frac{t}{18.72}}\right) = ln(9/10)$

bring t/18.72 donw: $\frac{t}{18.72} ln(0.5) = ln(9/10)$

multiply by $\frac{18.72}{\ln 0.5}$ $\frac{\cancel{18.72}}{\cancel{ln(0.5)}} \cdot t \cdot \frac{\cancel{\ln 0.5}}{\cancel{18.72}} = ln\left(\frac{9}{10}\right)\frac{18.72}{\ln 0.5}$

$t = 2.85$ days!

Question 3: Suppose a population of bacteria is modeled by f(t)= ab^t, where t is measured in days. Suppose after 2 days we have 95 little critters, and after 7 days we have 52 little critters.

Find a and b:

using $(2, 95): 95 = ab^2$

using $(7, 52): \ 52 = ab^7$

divide: $\frac{95}{52} = \frac{ab^2}{ab^7}$

$\frac{95}{52} = \frac{b^2}{b^7}$, $a/a = 1$

$\frac{95}{52} = \frac{1}{b^5}$

cross multiply: $95b^5 = 52$

divide by 95: $b^5 = \frac{52}{95}$

take the fifth root: $b = \sqrt[5]{\frac{52}{95}}$

$2 = 1 + 1$

$5 = 2 + 3$

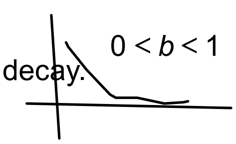
$\frac{2}{5} = \frac{1+1}{2+3}$

now find a:

$52 = a \bullet \left(\sqrt[5]{\frac{52}{95}}\right)^7$ (using (7,52) and b= $\sqrt[5]{\frac{52}{95}}$)

$\frac{52}{\left(\sqrt[5]{\frac{52}{95}}\right)^7} = a$

Is the population increasing or decreasing? $f(t) = ab^t$. Recall that when 0<b<1, we have decay.



Is our b between 0 and 1? $b = \sqrt[5]{\frac{52}{95}}$ approximate in calculator

$b = 0.8865$, which is such that $0 < 0.8865 < 1$.

So our population is decreasing.