Please take very detailed notes as usual. Make sure your **camera is on**. I will call attendance at some point. If you're not here to say "here", you will be marked absent. Turn off your microphones so we don't have background sound.

Section 5.3/Properties of Logs: Starting at 9:38..sent link around 9:27 am.

How to find things like $log_3(25)$ or $\log_4$ (18) or $\log_{1/2}$ (34) not easy to find!!

$3^x = 25$ $4^x = 18$ $\left(\frac{1}{2}\right)^x = 34$

ex1

change of base formula: most calculators have log(x) and ln(x).

$\log_a$ (x)= $\frac{ln(x)}{ln(a)}$ (simpler b/c ln is a key on most calcs)

$log_4(25)$ hard to type into a calculator..

$= \frac{ln25}{ln4}$ (these are easy to type in)

$= 2.3219$

b) $\log_2$ (12)(hard to type in on a calc.)

$= \frac{ln12}{ln2}$ (easy key presses)

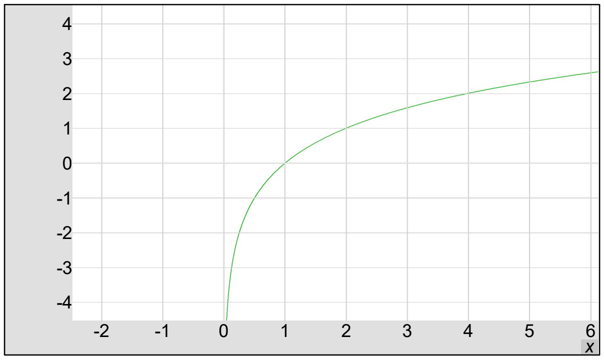
=3.5850

visual evidence for why this is true:

$log_3(36)$ (3^x =36? hard)

hard to type in

$log_3(36) = \frac{ln36}{ln3}$



$log_2(x)$

supposedly $\frac{\ln(x)}{ln(2)}$?

we have two graphs but they are overlapping perfectly!!

so $\log_2$ (x)= $\frac{lnx}{ln2}$

rule1

Properties of logs: $\log_a$ (u· v)=$\log_a$ *(*u)+$\log_a$ (v) (log of a product becomes the sum of the logs)

$log_a\left(\frac{u}{v}\right) = log_a(u) - log_a(v)$ (log of a quotient becomes the difference of the logs)

$log_a u^n = n \cdot log_a u$ (bring n down..power rule for logs)

These rules go from LHS to RHS **or** RHS to LHS, as needed! Rules are bidirectional.

example;   is $\log_2$ *(*2· 4) the same as $\log_2$ *(*2*)*+$\log_2$ *(*4*)*? (simple example of rule1)

$log_2(8)$ $1+2$ suggests that $log_2(2 \cdot 4) = log_2(2) + log_2(4)$

3   equal   **3**

example: ln6= $ln(2 \cdot 3) = ln(2) + ln(3)$ (book example)

$ln\left(\frac{2}{27}\right) = ln(2) - ln(27)$ quotient to difference rule

$= ln2 - ln(3^{\mathbf{3}})$ rewriting 27 as $3^{\mathbf{3}}$

$= ln2 - \mathbf{3}ln3$ $($*put* the 3 down in front of ln3 by power rule$)$

example 4/book: $log_{\mathbf{5}} \sqrt[\mathbf{3}]{5} \xrightarrow{\text{re write as } 5^{1/3}} log_5(5^{\mathbf{1/3}}) \xrightarrow{\text{bring 1/3 down by power rule}} \frac{1}{3} log_5(5)$

$\xrightarrow{\log_5 (5)=1} \frac{1}{3} \cdot 1 = \frac{1}{3}$ (answer)

recall: $log_e(x)$ = ln(x). $log_{\mathbf{e}}(\mathbf{e}) = \mathbf{1} \Leftarrow$ using ln now... $lne = 1$

$lne^6 \Rightarrow$ bring 6 down $\Rightarrow 6lne \Rightarrow$ lne is 1 $\Rightarrow 6 \cdot 1 = \mathbf{6}$ answer

$lne^6 - lne^2 = 6lne - 2lne = 6 \cdot 1 - 2 \cdot 1 = 6 - 2 = 4$ answer (red parts are the changed ones) *our* own examples (*more*) $In\left(\frac{2\cdot 3}{5}\right) \Rightarrow$ In(2· 3)-In(5) (apply quotient to difference)

$\Rightarrow In(2)+In(3)-In(5)$ (apply product to sum rule)

$In(\sqrt{x}\bullet z)=In\sqrt{x}+Inz$ (product to sum rule for logs)

$=Inx^{1/2}+Inz$ (rewrite $\sqrt{\ldots}$ as 1/2)

$=\frac{1}{2}Inx+Inz$ (bring 1/2 down)

expanded form

example 5 in book: $log_4(5x^3y)$ write as individual logs.(expand)

$=log_4(5\bullet x^3\bullet y)$ we have a product inside the parenthesis

$=log_4(5)+log_4(x^3)+log_4(y)$ multiplications become additions

$=log_4(5)+3log_4(x)+log_4(y)$ bring 3 down

5, *x* and y are as simple as possible now, so stop!

$(b)\ In\frac{\sqrt{3x-5}}{7}\xrightarrow{\text{convert to difference}}In(\sqrt{3x-5})-In(7)\xrightarrow{\text{rewrite root as 1/2}}In(3x-5)^{1/2}-In7$

$\xrightarrow{\text{bring 1/2 down}}\frac{1}{2}In(3x-5)-In7$ (stop)

stop means no other log rules are there to be applied!

these are NOT valid rules:

~~$In(x-y)$ might say $In(x)-In(y)$~~ (incorrect)

~~$In(x+y)$ might say In(x)+In(y)~~ incorrect!!

not valid: $In\left(\frac{x}{y}\right)$ might be tempted to do $\cancel{\frac{\text{In(x)}}{In(y)}}$ (incorrect)

example 6: condensing log. expressions:

product to sum, quotient to different, power rule..can all be applied in the reverse order!!
that's called "condensing"

$\frac{1}{2}Inx\Rightarrow$ put 1/2 back up $\Rightarrow Inx^{1/2}\Rightarrow$ rewrite as root $In\sqrt{x}$

2lnx+3lny $\Rightarrow$ put 2 and 3 in exponents $\Rightarrow Inx^3+Iny^{\mathbf{3}}\Rightarrow$ we have a sum, so make a product

$\Rightarrow In(x^3\bullet y^3)\Leftarrow$ condensend version

book: $\frac{1}{2}logx+3log(x+1)$

$\xrightarrow{\text{power rule backwards}}logx^{1/2}+log[(x+1)^{\mathbf{3}}]$

$\xrightarrow{\text{rewrite as root}}log\sqrt{x}+log(x+1)^3$

$\xrightarrow{\text{+ becomes } a \text{ single log}}log(\sqrt{x}\bullet(x+1)^3)$

book example:

$2In(x+2)-Inx$

$\xrightarrow{\text{power rule backwards on 2}}In(x+2)^2-Inx$

$\xrightarrow{\text{- becomes division}}In\left[\frac{(x+2)^2}{x}\right]$

stop..no other log rules left to apply!!

book example $\frac{1}{3}[log_2(x)+log_2(x+1)]$

addition to product first: $\frac{1}{3}[log_2(x(x+1))]$

put 1/3 in exponent: $log_2[x(x+1)]^{1/3}$

rewrite in radical form: $log_2\sqrt[3]{x(x+1)}$ (stop..no other rules left to apply)

$In(uv)=Inu+Inv$, $In\left(\frac{u}{v}\right)=Inu-Inv$, $Inu^n = n\cdot Inu$

change of base = $log_{a}(x)=\frac{lnx}{lna}$

These rules are BIDIRECTIONAL, so this means, as example, that $ln(uv)\Rightarrow lnu+lnv$ or , if needed, ln(u)+ln(v) $\Rightarrow ln(uv)$

recall : $x^{a}x^{b}=x^{a+b}$..it's not just from Left to RIGHT..it's from RIGHT To left also!!

two major ops: condensing..so making smaller
expanding..making bigger

rules apply to
log$_{a}$ (x), lnx= $log_{e}(x)$ where e=2.718

simple illustration of $ln\left(\frac{u}{v}\right)=ln(u)-ln(v)$ (u and v are just some expressions..)

$$\left.\begin{array}{ll} log_{2}\left(\frac{8}{2}\right)=? & log_{2}(8)-log_{2}(\mathbf{2}) \\ =log_{2}(4) & =3-1 \\ =\mathbf{2} & =\mathbf{2} \end{array}\right\}$$

Shows that $log_{2}\left(\frac{8}{2}\right)$ really is the same as $log_{2}(8)-log_{2}(2)$

question 24 in book: $log_{2}(4^{2}\bullet 3^{4})$

$log_{2}(4^{2})+log_{2}(3^{4})$ addition of two separate logs!

power rule: 2 log$_2$ 4+ 4 log$_2$ 3 bring 4 down on each

$2\cdot 2+4log_{2}(3)$

$\underbrace{4+4log_{2}(3)}_{\text{leave this}}$ (log$_2$ 3 is not a clean value..it's asking 2^{x} =3.. x is messy)

question 36: $\underbrace{log_{3}(-27)}_{\text{has no value!=DNE=does not exist=undefined!}}$..domain of any of the form log$_a$ (x) is x>0

If we have $ln(\text{expression})$
If ultimately expression ≤ 0, ln(expression) is NOT defined!
$ln(x-2), ln(\sqrt{x+4}),\ ln(x^{2}-5)$

condense: $\frac{1}{2}lnx-2ln(y)$ (notice the - in the middle)

carry the negative with the 2 and leave a + $\frac{1}{2}lnx+lny^{-2}$

apply power rule to 1/2: $lnx^{1/2}+lny^{-2}$

sum to product: $ln(x^{1/2}\bullet y^{-2})$

1/2 to root form: $ln(\sqrt{x}\cdot y^{-2})$

rewrite with a positive exponent: $ln\frac{\sqrt{x}}{y^{2}}$ $\Leftarrow$ this is NOT $\cancel{\frac{\ln\sqrt{x}}{lny^{2}}}$

this is equivlaent to $ln\left(\frac{\sqrt{x}}{y^{2}}\right)$

how to deal with negative in middle:

$lnx-2lny\Rightarrow$ carry the minus with the 2 $\Rightarrow lnx+lny^{-2}=ln(x\bullet y^{-2})=ln\left(\frac{x}{y^{2}}\right)$

or...leave the negative:

$lnx-2lny\Rightarrow$ carry only +ive 2 $\Rightarrow lnx-lny^{2}\Rightarrow$ apply difference to quotient $\Rightarrow ln\frac{x}{y^{2}}\Leftarrow$ same as above

caution: $lnx-lny^{2}$..don't turn into $\cancel{\frac{lnx}{lny^{2}}}$ (not valid!)