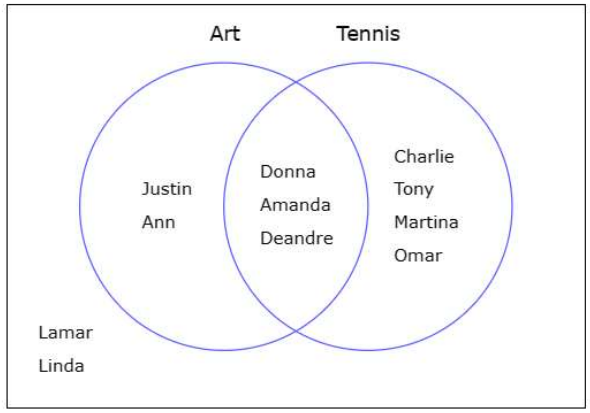
Using a Venn diagram to understand the addition rule for probability

The Venn diagram below shows the 11 students in Mr. Walker's class.
The diagram shows the memberships for the Art Club and the Tennis Club.

Note that "Lamar" and "Linda" are outside the circles since they are not members of either club.

One student from the class is randomly selected.
Let A denote the event "the student is in the Art Club."
Let B denote the event "the student is in the Tennis Club."



$P(A) = \frac{\text{favorable}}{total} = \frac{5}{11}$

justin, ann, donna, amanda, deandre (5 people)

$P(B) = \frac{\text{favorable}}{total} = \frac{7}{11}$

donna, amanda, deandre, charlie, tony, martina, omar

$P(A \text{ and } B) = \frac{\text{favorable}}{total} = \frac{3}{11}$

donna, amanda, deandre

P(A or B)= only art, or both or only tennis

$= \frac{\text{favorable}}{total} = \frac{9}{11}$

justin, ann, donna, amanda, deandre, charlie, tony, martina, omar

compute: $\underbrace{P(A)+P(B)-P(A \text{ and } B)}_{\text{formula}} = \frac{5}{11} + \frac{7}{11} - \frac{3}{11} = \frac{5+7-3}{11} = \frac{9}{11}$

Shows that
$P(A \text{ or } B) = P(A)+P(B)-P(A \text{ and } B)$
both produce the same value!

A tile is selected at random from 6 tiles. We put the tiles in a bag and shake it very well.
Possible outcomes : A , B, C ,D,E and F.

event X: selecting a letter from A to C: A or B or C $= \frac{\text{favorable}}{total} = \frac{3}{6} = \frac{1}{2}$

event Y: selecting a consonant sound (a non-vowel)$= \frac{\text{favorable}}{total} = \frac{4}{6} = \frac{2}{3}$ (B,C,D,F ...4 letters out of 6)

event Z :selected a letter from A to C and a consonant: (non-vowel):
A, B, C (letters from A to C)
and a consonant leaves B or C (A gets taken away) $= \frac{\text{favorable}}{total} = \frac{2}{6} = \frac{1}{3}$ } event Z is X *and* Y

event X or Y: X or Y or both: Selecting a letter that is from A to C **or** a consonant.

vowels: A,E,I,O and U.

favorable
fvrbl

A, B, C, D, F (E is not included because not a consosant and not in A to C)
A b/c it's in A to C,
B b/c it's in A to C and it's consonant
C b/c it's in A to C and it's a consonant
D b/c it's not in A to C but it is a consonant
F b/c it's not in A to C but it is a consonant
$\frac{5}{6}$

conclusion:
$P(X \text{ or } Y) = P(X)+P(Y)-P(X \text{ and } Y)$

Compute
$P(x)+P(Y)-P(X \text{ and } Y)$
$= \frac{3}{6} + \frac{4}{6} - \frac{2}{6} = \frac{3+4-2}{6} = \frac{7-2}{6} = \frac{5}{6}$

Here is a table showing all 52 cards in a standard deck.

J=jack, Q=queen, K=king

Color	Suit	Ace	Two	Three	Four	Five	Six	Seven	Eight	Nine	Ten	Jack (Face cards)	Queen (Face cards)	King (Face cards)		
Red	Hearts	A♥	2♥	3♥	4♥	5♥	6♥	7♥	8♥	9♥	10♥	J♥	Q♥	K♥	suit	13 cards
Red	Diamonds	A♦	2♦	3♦	4♦	5♦	6♦	7♦	8♦	9♦	10♦	J♦	Q♦	K♦	suit	13 cards
Black	Spades	A♠	2♠	3♠	4♠	5♠	6♠	7♠	8♠	9♠	10♠	J♠	Q♠	K♠	suit	13 cards
Black	Clubs	A♣	2♣	3♣	4♣	5♣	6♣	7♣	8♣	9♣	10♣	J♣	Q♣	K♣	suit	13 cards

A five-card hand is dealt at random, without replacement. This means number of cards gets decreased as we draw.

Probability of containing exactly one black card. $= \frac{\text{favorable}}{total}$. Total= $52\, C\, 5 = \frac{52!}{(52-5)!\,5!} = 2,598,960$ combinations

order doesn't matter: $\left.\begin{array}{l} A\,hearts, \text{2 hearts, 4clubs, Qspades, Adiamons} \\ \text{2 hearts, A hearts, 4clubs,Adiamons, Qspades} \end{array}\right\}$ same hands, so combinations!

favorable here? hands with exactly one black card: We have 26 black cards. We want to choose 1 of these for sure.
$26\ C\ \ 1 = 26!/(26-1)!\cdot 1! = 26!/(25!) = 26$

$26\ C\ 4 = \frac{26!}{(26-4)!\cdot 4!} = \frac{26!}{22!\,4!} = \frac{26\cdot 25\cdot 24\cdot 23\cdot \cancel{22!}}{\cancel{22!}\cdot 4\cdot 3\cdot 2\cdot 1} = \frac{26\cdot 25\cdot 24\cdot 23}{4\cdot 3\cdot 2\cdot 1} = 14950$

combos again b/c red cards: 10 *hearts*, 10 diamons, Qhearts, Qdiamons *or* 10diamons, 10 hearts, Qdis, Q hearts

the two above are considered the same, so combinations..no order

now we use counting principle: $26\cdot 14950 = 388700$

number of ways to get a black card $\cdot$ number of possible 4 card hands made of only red cards

$\frac{\text{favorable}}{total} = \frac{388700}{2598960} = 0.149$ or about 14.9%

in ALeks, it's written compactly as : $\frac{\mathbf{26}\,C\,1 \times \mathbf{26}\,C\,4}{\mathbf{52}\,C\,5}$ $\quad 4+1=5$ $\quad \mathbf{26}+\mathbf{26}=\mathbf{52}$

In Rita's bucket there are 10 brown worms and 8 red worms.
Rita is going to choose 10 worms at random.
Probability of 4 brown and 6 red? $\quad \frac{\text{favorable}}{total}$

b/c we have to different colors of worm, we need to use the counting principle and divide by total number of 10 worm batches we can get from 18 worms.

$= \frac{\mathbf{10}\,C\,4 \times \mathbf{8}\,C\,6}{\mathbf{18}\,C\,10}$ (6+4=10, **8+10=18**) in top it's $\times$ by the counting principle

$10\,C4 = \frac{10!}{(10-4)!\cdot 4!}$ (worms are named Bob, Jerry, Tom and Harry..then same as Jerry, Tom, Harry and Bob)

$= \frac{10!}{6!\cdot 4!} = \frac{10\cdot 9\cdot 8\cdot 7\cdot \cancel{6!}}{\cancel{6!}\cdot 4\cdot 3\cdot 2\cdot 1} = \frac{10\cdot 9\cdot 8\cdot 7}{4\cdot 3\cdot 2\cdot 1} = 210$ (210 hands with 4 brown worms out of 10 brown worms)

$8\ C\ 6 = \frac{8!}{(8-6)!\cdot 6!} = \frac{8\cdot 7\cdot \cancel{6!}}{2!\cdot \cancel{6!}} = \frac{8\cdot 7}{2\cdot 1} = 28$ (28 hands with 6 red worms drawn out of 8 red worms)

$18\ C\ 10 = \frac{18!}{(18-10)!\cdot 10!} = \frac{18!}{8!\cdot 10!} = \frac{18\cdot 17\cdot 16\cdot 15\cdot 14\cdot 13\cdot 12\cdot 11\cdot \cancel{10!}}{8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1\cdot \cancel{10!}} = 43758$

$\frac{\text{favorable}}{total} = \frac{210\cdot 28}{43758} = \frac{5880}{43758} = 0.134$