Doubling time:

An investment's value is given by A(t)= $9000\cdot e^{0.045t}$. form A(t)= Pe^{rt}., $P=9000$, $r=0.045$, $t=$ variable

What's the value at t=1: $A(1)=9000\cdot e^{0.045\cdot 1} \xrightarrow{\text{in words..how much money in 1 year}} 9414.25$

How long for the investment to double? P goes from 9000 to 18000.

$18000=9000\cdot e^{0.045\cdot t}$

divide by 9000: $\frac{18000}{9000}=\frac{9000}{9000}e^{0.045(t)}$

$2=e^{0.045\cdot t}$

take ln of both sides: $ln2=ln(e^{0.045\cdot t})$

power rule for logs: $ln2=0.045t\cdot lne$

$lne=1$ $\quad ln2=0.045t$

divide by 0.045: $\frac{ln2}{0.045}=t$

calculator work.... $t=$ finish up on calculator on your own!

more generally:

model:

If P turns into 2P, we get :

repalce A with 2P: $2P=Pe^{rt}$

divide P away: $2\frac{P}{P}=\frac{P}{P}e^{rt}$

simplify: 2=e^{rt}

solve for t:

take ln: $ln2=lne^{rt}$

bring rt down: $ln2=rt\cdot lne$

$lne=1$ $\quad ln2=rt$

⇐ t is not special ..could solve for r

divide by r: $\frac{ln2}{r}=t$

template..just plug numbers in

q2: amount of money in an investment is modeled by A(t)= $500(1.0291)^t$

What is the doubling time for the investment?

500=P, 1000=2P

replace LHS wiht 2P: $1000=500\cdot 1.0291^t$

divide by 500: $\frac{1000}{500}=\frac{500}{500}\cdot 1.0291^t$

simplify $\quad 2=1.0291^t$

take ln: $\quad ln2=ln(1.0291^t)$

bring t down: $ln2=t\cdot ln(1.0291)$

divide by ln..: $\frac{ln2}{ln(1.0291)}=t$

$t\approx\frac{\ln 2}{\ln 1.0291}=24.16$ years!!

start investing now because things take a long time ..

financial independence!!

template:

We have P become 2P:

$2P=Pa^t$, a=base, P=investment,

2P is double the investment

$2P=Pa^t$

divide by P: $2\frac{P}{P}=\frac{P}{P}a^t$

simplify: $2=a^t$

take ln: $ln2=lna^t$

bring t down: $ln2=t\cdot lna$

divide by lna : $\frac{ln2}{lna}=t$

most general form:

It's not just P becoming 2P. Let's say P turns into bP. b is some number that multiplies P . b=1/2, b=.25, b=7

$bP=Pa^t$

divide by P: $b\cdot\frac{P}{P}=\frac{P}{P}a^t$

simplify: $b=a^t$

take ln: $lnb=lna^t$

bring t down: $lnb=tlna$

divide by lna: $\frac{lnb}{lna}=t$

b=3 (3 times P)

$a=$ base = 1.5

t= $\frac{\ln 3}{\ln 1.5}=2.71$

q3: You deposit 2600 in an account earning 7% interest with quarterly compounding. How long will it take this investment to triple?

$A = P(1 + r/n)^{nt}$ (discrete compounding formula)

P=principal, r=rate, n=number of times we compound per year, r/n=tax rate at end of each period, nt=total number of times we compound

$P = 2600$, triple means 3P= $3 \cdot 2600$ (leave in this form so it's easier to cancel)

quarterly means n=4, r= 0.07. we don't know t !

$$3 \cdot \mathbf{2600} = \mathbf{2600}\left(1 + \frac{0.07}{4}\right)^{4 \cdot t}$$

cancel off 2600: $3 \cdot \frac{2600}{2600} = \frac{2600}{2600}\left(1 + \frac{0.07}{4}\right)^{4 \cdot t}$

$$3 = \left(1 + \frac{0.07}{4}\right)^{4 \cdot t}$$

abc (product of three things)

(ab)c

a(bc)

$ac \cdot b$

$bc \cdot a$

take ln of both sides: $\ln 3 = \ln\left(1 + \frac{0.07}{4}\right)^{4t}$

$$\ln 3 = 4t \cdot \ln\left(1 + \frac{0.07}{4}\right)$$

divide by $\ln(1 + 0.07/4) \cdot 4$: $\frac{\ln 3}{4\ln(1 + 0.07/4)} = t$

$$t = \frac{\ln 3}{4 \cdot \ln(1 + 0.07/4)} = 15.8 \text{ years!!}$$

more generally:

We have *our* amount P become bP. (b=2, b=3, b=4,...)

P becomes bP

replace A with bP: $bP = P\left(1 + \frac{0.07}{4}\right)^{4 \cdot t}$

divide P away: $b\frac{P}{P} = \frac{P}{P}\left(1 + \frac{0.07}{4}\right)^{4 \cdot t}$

$$b = \left(1 + \frac{0.07}{4}\right)^{4 \cdot t}$$

solve for t: $\ln(b) = \ln\left(1 + \frac{0.07}{4}\right)^{4t}$

bring 4t down by power rule for logs: $\ln b = 4t \cdot \ln(1 + 0.07/4)$

regroup 4 with ln... $\ln b = t \cdot 4\ln(1 + 0.07/4)$

divide by 4ln(1+.07/4) $\frac{\ln b}{4\ln(1 + 1.07/4)} = t$

most general form: P becomes bP, we keep r, keep n, keep P, and solve for t:

$bP = P\left(1 + \frac{r}{n}\right)^{n \cdot t}$ solve for t

cancel off P: $b\frac{P}{P} = \frac{P}{P}\left(1 + \frac{r}{n}\right)^{n \cdot t}$

$$b = (1 + r/n)^{n \cdot t}$$

take ln: $\ln b = \ln[1 + r/n]^{nt}$

bring nt down: $\ln b = \mathbf{n} \cdot \mathbf{t} \cdot \mathbf{\ln}(1 + r/n)$

divide by $\mathbf{n} \cdot \mathbf{\ln}(1 + r/n)$: $\frac{\ln b}{n \cdot \ln(1 + r/n)} = t$

derivation of formula ..template!

say b=4 (quadrupling money), n=12 , r=9%=.09:

$$t = \frac{\ln 4}{12 \cdot \ln(1 + 0.09/12)} = 15.46 \text{ years}$$

discrete compounding: $A = P(1 + r/n)^{nt}$
cont. compounding: $A = Pe^{rt}$, e is derived from letting n go to infinity in

$$\mathbf{1}\left(1 + \frac{\mathbf{1}}{n}\right)^{n \cdot \mathbf{1}} \qquad P = 1, r = 1, t = 1$$

annunity formula: $(book)$ $A = \dfrac{P((1 + i/1)^n - 1)}{i/1}$, i =interest rate, n =number of payments,