Math 200 notes 11/13/2023. Please put away computers and phones and take detailed notes.
Section 6.1/Interest:

Review of percents/decimal and fractions:

$1\% = \frac{1}{100} = 0.01$, $12\% = \frac{12}{100} = .12$, $0.3\% = \frac{0.3}{100} = 0.003$

$0.005 \rightarrow$ five one-thousdanths $\rightarrow \frac{5}{1000} = \frac{\cancel{5}}{\cancel{5}\cdot 200} = \frac{1}{200} = \frac{1}{2\cdot 100} = \frac{0.5}{100} = 0.5\%$

$\% = 1\% = \frac{1}{100}$ $\quad 0.35 \xrightarrow{\text{35 one-hundreths}} \frac{35}{100} = 35\%$ $\quad 1.25 \xrightarrow{\text{one and 25 one hundreths}} \frac{125}{100} = 125\%$

$0.0025 = \frac{25}{10000} = \frac{1}{400} = \frac{1}{4\cdot 100} = \frac{0.25}{100} = 0.25\%$ $\quad 0.0025 \Rightarrow$ float decimal 2 places right and percent symbol $\Rightarrow 0.25\%$

example 1/page 294: (a) Find **12% of** 80
$x = \mathbf{0.12} \bullet 80 = 9.6$

(b) What percent of 40 **is** 18?
$x \cdot 40 = 18$
$x = \frac{18}{40} = 0.45 = 45\%$
or $\frac{18}{40} = \frac{9}{20}\cdot\frac{5}{5} = \frac{45}{100}$

(c) 8 is **15% of** what number?
$8 = \mathbf{0.15}\cdot x$
$\frac{8}{0.15} = x$
$53.33 = x$

Example 2: Computing state income tax: Illinois guy has a base income , after adjustments for deductions, of 18000. State income tax on this is 3%. What tax is due?
tax= $3\% \, of \, 18000 = 0.03\cdot 18000 = \$540$
Think and Grow Rich, Napoleon Hill

Defintion: Simplest interest: $I = Prt$ $\quad P = principal,\ r = rate, t = time$
Simple interest is interest computed on the principal for the entire preiod it is borrowed.
Total amount= Principal + Interest= $P\cdot 1 + Prt = P(1+rt) \Leftarrow$ future value

Example 3: Computing Interest and the Amount Due on a Loan: A loan of 250 is made for 9 months at a simple rate of 10% per annum. What is the interest charge? What amount is due after 9 months?
The Intelligent Investor
$I = Prt = 250\cdot 0.1\cdot \frac{\mathbf{9}}{\mathbf{12}} = \$18.75$. Total to pay back= $250 + 18.75 = 268.75$

Example 4: A person borrow \$1000 for a period of 6 months. What simple interest rate is being charged if the amount A that must be repaid after 6 months is 1045? $A = P + Prt$
$A = 1045$, $P = 1000$, $t = 6/12$ (not 6..fraction of a year..out of 12), $r = ?$

$1045 = 1000 + 1000\cdot \mathbf{r}\cdot \frac{6}{12}$ (reminder $abc = (ab)c = a(bc)$)

$1045 - 1000 = 1000\cdot r\cdot \frac{1}{2}$

$45 = 1000\cdot r\cdot \frac{1}{2}$ $\rightarrow$ $45 = \frac{1000}{2} r$

$45 = 500r$

$\frac{45}{500} = r$

$0.09 = r \Rightarrow$ we have $\frac{9}{100} = 9\%$

9% means for every 100 dollars borrowed, we have to return the 100 original and 9 more.

Example 5/Computing the Amount Due on a Loan: A company borrows 1,000,000 for 1 month at a simple interest rate of 9% per annum. How much must the company pay back at the end of 1 month?
Company has to pay back the borrowed amount of 1,000,000 and 7,500 more in interest.

Stock market, over looooong periods of time, pays an average of 9 to 11 percent.
Two ways to make money: exchange your time for money or PROFIT!

$A = P + Prt$
$A = P(1+rt)$
$A = 1000000\left(1 + 0.09\cdot \frac{1}{12}\right)$
$A = 1000000\left(1 + \frac{\mathbf{0.09}}{\mathbf{12}}\right)$ $\quad \frac{0.09}{12} = 0.0075$ (effective rate)
$A = 1000000(1 + 0.0075)$
$A = \$1,007,500$

Example 6: A borrower signs a note for a discounted loan and agrees to pay the lender \$1000 in 9 months at a rate of 10%. How much does the borrower receive? r= per annum rate of interest, t=time in years, L=amount of the loan
R= proceeds $\quad L = 1000, r =$ 10%= 0.1, 9 months= 9/12
R= $L - Lrt = L(1 - rt)$
$R = 1000\left(1 - 0.1\cdot \frac{9}{12}\right) = 1000(1 - 0.075) = 1000(0.925) = \$925$
Summary: Give the borrower \$925 today.

Example 7: What simple interest is the borrower in example 6 paying on the 925 that was borrowed for 9 months and paid back in the amount of 1000? $A = P + Prt$
$1000 = 925 + 925\mathbf{r}\cdot \frac{9}{12} \Rightarrow 1000 - 925 = 925\cdot r\cdot \frac{9}{12} \Rightarrow 75 = \frac{925\cdot 9}{12} r \Rightarrow 75 = 693.75r \Rightarrow \frac{75}{693.75} = r \Rightarrow r = 0.108108 \Rightarrow r = 10.81\%$

Example 8: Treasury Bills (T-Bills) are short-term securities issued by the Federal Reserve. The bills do not specify a rate of interest. The are sold at public auction with financial institutions making competitive bids. For example, a financial institution may bid 982,400 for a 3-month, $1 million treasury bill. At the end of the three months the institution receives $ 1 million and the cost of the T-Bill. This is an example of a discounted loan.

How much should a bank bid on a 6-month, 500,000 (500k, k=kilo) treasury bill if it wants a .25% discounted rate of interest?

R= $L(1-rt)$

$$0.25\% = \frac{0.25}{100} = 0.0025$$

$$R = 500000\left(1 - 0.0025 \cdot \frac{6}{12}\right), \quad L = 500000,\ r = 0.25\%,\ t = \frac{6}{12}$$

financial instruments

$$= 500000(1 - 0.00125)$$

$$= 500000(0.99875)$$

$= 499,375$ The bank should bid $499,375 if it wants to earn a rate of .25%. (book says ~~.0025%~~)