Please be sure to take detailed notes. The same rules apply as always.
Perhaps a good question: how can the next year of your life be the most successful and amazing ever ?
Please **put your cameras on so I can be sure you're in class.**

Section 6.2/Compund Interest:

A= $P\left(1+\frac{r}{n}\right)^{nt}$ ⇐ discrete compounding formula

$A=Pe^{rt}$ ⇐ continuous compounding formula

$A=\mathbf{P}(1+r/n)^{nt}$, $P=$ principal now

divide both sides by $(1+r/n)^{nt}$

$$\frac{A}{(1+r/n)^{nt}}=P\frac{(1+r/n)^{nt}}{(1+r/n)^{nt}}$$

recall that $a^{-n}=\frac{1}{a^n}$

$$\frac{A}{(1+r/n)^{nt}}=P$$

$A(1+r/n)^{-nt}=P$

present value formula

Present Value: "time value of money" refers to the present value of money. the **present value** of A dollars to be received at a future date is the principal thta you would need to invest now so as to have A dollars in the future at time t.

$a^{-n}=\frac{1}{a^n}$

$a^{-n}\xrightarrow{\text{go to}}\frac{1}{a^n}$

$\frac{1}{a^n}\xrightarrow{\text{go to}}a^{-n}$

cont. compound interest:
$A=Pe^{rt}$
divide by e^{rt}

$$\frac{A}{e^{rt}}=P\frac{e^{rt}}{e^{rt}}$$

$$\frac{A}{e^{rt}}=P$$

$Ae^{-rt}=P$

present value

example 6/book: present value of 10,000:
to get 10,000 in 2 years at 4% per annum, how much to invest
(a) annual compounding: n=1:

$$P=10000\left(1+\frac{0.04}{1}\right)^{-2\cdot 1}=10000(1+0.04)^{-2}=10000(1.04)^{-2}=\$9245.56$$

To get 10,000 in 2 years at 4% with annual compounding, invest $9245.56 today.

(c) daily compounding: $n=365$ (calculate interest at end of each day)

$$P=10000(1+0.04/365)^{-365\cdot 2}=10000(1+0.04/365)^{-730}=\$9231.20$$

Invest $9231.20 today to get 10,000 in 2 years using the values above!

(d) conti. compounding:

$$P=Ae^{-rt}=10,000\,e^{-0.04\cdot 2}=\$9231.16$$

We're going backwards in values...now Ae^{-rt} gives us the SMALLEST amount to be invested today.

9231.16 < 9231.20 < 9245.56 (best investing option is cont. compounding)

Example 7: Computing the value of a zero-coupon bond:
A zero-coupon(non-interest bearing) bond can be redeemed in 10 years for $1000. How much should you be willing to pay for it now if you want a return of
(a) 5% compounding monthly:
what is the present value of $1000?
$r=5\%=0.05$, $t=10$, $n=12$

$$P=A\left(1+\frac{r}{n}\right)^{-nt}=1000\left(1+\frac{0.05}{12}\right)^{-10\cdot 12}=1000\left(1+\frac{0.05}{12}\right)^{-120}=\$607.16$$

You should be willing to pay $607.16 for the bond. (company bonds and government bonds)
BOND= fancy for "I owe you".

(b) cont. case: $A=\$1000$, $r=0.04$, $t=10$: (book says use 4%)
$P=1000\,e^{-0.04\cdot 10}=\$670.32$

$(c)\,P=1000\,e^{-0.05\cdot 10}=606.53$

Comparing (a) an (c) says (c) is better in that it's slighly less we have to invest today. Comparing (a) and (c) b/c they have the same values.

example 8: Rate of interest required to double an investment:
What annual rate of interest compounded annually should you seek if you want to double your investment in 5 years?

$A = P\left(1+\frac{r}{n}\right)^{nt}$ (not using -nt here)

n= 1, $t=5$, double your investment means P becomes 2P=A

$2P = P\left(1+\frac{r}{1}\right)^{1\cdot 5}$ (replace A with 2P, n with 1, t with 5)

divide P away:

$2\frac{P}{P} = \frac{P}{P}(1+r)^5$

$2 = (1+r)^5$

opposite of raising to the 5th is to take the fifth root: $\sqrt[5]{2} = \sqrt[5]{(1+r)^5}$ $\qquad \sqrt[a]{\text{expression}^a} = \text{expression}$

$\sqrt[5]{2} = 1+r$

$\sqrt[5]{2} - 1 = r$

$r = 1.148698 - 1 = 0.148698$

So r $\approx$ 14.87% (high rate in real terms..hard in real life)

Example 9: Time required to double an investment:
How long will it take for an investment to double in value if it earns 5% compounded annually?

math 111 review: logs... $x = y^t$ and we want t: (t is in the exponent)

use logs: take natural log of both sides:

$ln(x) = ln(y^t)$

recall power rule for logs: ln (expression^a)=a ln(expression)

$ln(x) = t\cdot ln(y)$ (bring t down)

$\frac{\ln(x)}{ln(y)} = t$ divide by lny

$A = P(1+r/n)^{nt}$ (not -nt) "how long..." means use t as the variable

double means A=2P

$2P = P = \left(1+\frac{0.05}{1}\right)^{t\cdot 1}$

$2\frac{P}{P} = \frac{P}{P}(1+0.05)^t \Leftarrow$ t in exponent!

18/19..start investing today b/c things take a very long time!

$2 = (1.05)^t$

take the natural log of both sides(math 111 or 113...)

$ln2 = ln(1.05^t)$

bring t down: $ln2 = t\cdot ln(1.05)$

divide by ln1.05: $\frac{ln2}{ln(1.05)} = t \xrightarrow{\text{calculator work}} t = 14.207$ years.

(b) If P is the initial investment and we want P to double using conti. compounding, we get...

$A = Pe^{rt}$
$A = 2P$ (double P)
$r = 0.05, t = ?$

plug in:

$2P = Pe^{0.05\cdot t}$

$2\frac{P}{P} = \frac{P}{P}e^{0.05\cdot t}$

$2 = e^{0.05\cdot t}$

natural log of both sides , $ln(2) = ln(e^{0.05\cdot t})$

$ln(2) = 0.05\cdot t$ (power rule for logs and recall $ln(e) = 1$)

$\frac{ln(2)}{0.05} = t \Rightarrow$ so t= 13.86 years!!

most advanced math exercise today:

$A = P\left(1 + \frac{r}{n}\right)^{nt}$ nt=number of times we compound, r/n=tax rate at end of each period

solve for r:

r is inside parenthesis..

$\frac{A}{P} = \frac{P}{P}\left(1 + \frac{r}{n}\right)^{nt}$ (divide by P)

$\frac{A}{P} = \left(1 + \frac{r}{n}\right)^{nt}$ (we have nt in the top...)

to simplify things..set t=1 (for no reason..but to simplify)

$\frac{A}{P} = \left(1 + \frac{r}{n}\right)^{n}$ (set t=1 to simplify..)

both sides are raised to the n..opposite of raising to the n is raising to the 1/n:

$\left(\frac{A}{P}\right)^{1/n} = \left(1 + \frac{r}{n}\right)^{n(1/n)}$

$\left(\frac{A}{P}\right)^{1/n} = \left(1 + \frac{r}{n}\right)^{n/n}$

$\left(\frac{A}{P}\right)^{1/n} = \left(1 + \frac{r}{n}\right)^{1}$

pick up here

drop 1 and parenthesis on RHS:

$\left(\frac{A}{P}\right)^{1/n} = 1 + \frac{r}{n}$

subtract 1: $\left(\frac{A}{P}\right)^{1/n} - 1 = \frac{r}{n}$

multiply by n *and* on LHS two terms, so brackets:

$n\left[\left(\frac{A}{P}\right)^{1/n} - 1\right] = \frac{r}{n} \cdot n$

$n\left[\left(\frac{A}{P}\right)^{1/n} - 1\right] = r$

What interest rate compounded quarterly is equivalent to an effective interest rate of 6.25%?

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Using simplest $A = P + Prt = P(1 + rt)$

compound interest formula: $A = P(1 + r/n)^{nt}$

$A = P(1 + r)$ (b/c t=1)

A= $P(1 + r/4)^{4 \cdot 1}$ (n=4, t=1)

$A = P(1 + 0.0625)$, $A = P(1 + r/4)^4$

divide equations:

$$\frac{A}{A} = \frac{P(1 + 0.0625)}{P(1 + r/4)^4}$$

$$1 = \frac{1.0625}{(1 + r/4)^4}$$

cross multiply: $1(1 + r/4)^4 = 1.0625$

take the 4th root: $\sqrt[4]{(1 + r/4)^4} = \sqrt[4]{1.0625}$

$$1 + r/4 = \sqrt[4]{1.0625}$$

$$\frac{r}{4} = \sqrt[4]{1.0625} - 1$$

$$4 \cdot \frac{r}{4} = 4[\sqrt[4]{1.0625} - 1]$$

$$r = 4[\sqrt[4]{1.0625} - 1]$$

$= 6.109\%$ to get P to become A after n=4 , we have to use 6.109% rate.

quarterly means n=4

t=1

P= not given..so it has to vanish

effective interest =6.25%=.0625

"..interest rate compounded quarterly...keep the r in $(1+r/4)^4$..r=?

example..dividing equation:

$2 + 4 = 6$

$3 + 4 = 7$

divide them:

$\frac{2 + 4}{3 + 4} = \frac{6}{7}$ true!

To get the same using quarterly compounding, we use r=6.109% .