Please take **meticulous** notes as usual. We will begin at 8:00 AM.
Please be sure your cameras are on and microphones are off so we don't have background sounds.
I will check attendance at some point. section 6.3

An annunity is a sequence of periodic deposits. The periodic deposits can be annual, semiannual, quarterly, monthly or any other fixed length of time. When the deposits are made at the same time the interest is credited, the annunity is called **ordinary.** The amount of an annunity is the sum of all deposits made plus all interest accumulated.

Example 1/book: Find the amount of an annunity after 5 deposits if each deposit is equal to $100 and is made on an annual basis at an interest rate of 4% per annum compounded annually.

After 5 deposits the first $100 deposit will have accumulated interest compounded annually at 4% for 4 years.

$A_1 = 100(1+0.04)^4 = 100(1.04)^4 \xrightarrow{\text{calculator}} \$116.99$

The second deposit of 100, made 1 year after the 1st deposit, will accumulate interest for 3 years.

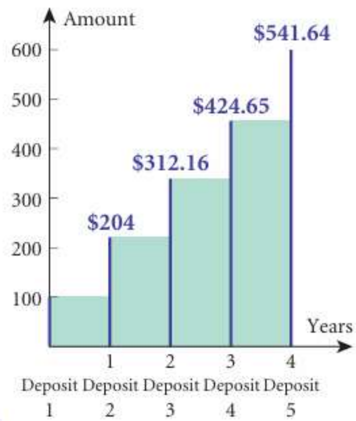
$A_2 = 100(1+0.04)^3 = \$112.49$

Third deposit: $A_3 = 100(1+0.04)^2 = 100(1.04)^2 = \$108.16$

fourth deposit: $A_4 = 100(1+0.04)^1 = 100(1.04) = \$104.00$

fifth deposit: $A_5 = 100$

total value of annunity= $116.99+112.49+108.16+104.00+100 = \$541.64$



skipping some very technical steps...
Amount of an Annunity Formula:

$A = P\left[\frac{(1+i)^n - 1}{i}\right]$, P=deposit, i=interest rate, n=number of deposits

Example 2: Finding the Amount of an Annuity: Find amount of annunity after 5 deposits if a deposit of $100 is amde each year, at 4% compounded annually. How much interest is earned?

$A = P\left[\frac{(1+i)^n - 1}{i}\right] = 100\left[\frac{(1+0.04)^5 - 1}{0.04}\right] = \$100(5.416323) = \$541.63$

Interest earned= $A - P = 541.63 - 500 = \$41.63$. i=.04 *and* it's annual, so it's just .04 in bottom.

Example 3: Mary decides to put aside $100 every month in a credit union that pays 5% compounded monthly. After making 8 deposits , how much money does Mary have?

$P = 100, n = 8, i = \frac{0.05}{12}$ = interest rate at each of each period! (not 5%...divide by 12)

A= $100\left[\frac{(1+0.05/12)^8 - 1}{0.05/12}\right] = 100(8.117644) = \$811.76$. Mary has $811.76 after 8 deposits.

Interest= $A - P = 811.76 - 800 = 11.76$.

Example 4/Savings for College: To save for her daugther's college education, Ms. Miranda decides to put $50 aside every month in a bank guaranteed-interest account paying 4% interest compounded monthly. She begins this savings program when her daughter is 3 years old. How much **will she have saved** by the time she makes the 180th deposit? How old is her daugther at this time?

P=50, n=180 deposits , i= $\frac{0.04}{12}$ (divide rate by 12 ...don't use .04)

$A = 50\left[\frac{(1+0.04/12)^{180} - 1}{0.04/12}\right] = 50(246.090488) = \$12,304.52$ (nice tidy sum!!)

What if she did just 100 per month?

There are 12 deposits per year. When the 180th deposit is made, it's 180/12=15 years. 3+15=18..So daugther is 18 years old!

$A = 100\left[\frac{(1+0.04/12)^{180} - 1}{0.04/12}\right] = \$24609..$

Example 5: /Funding an IRA=Individual Retirement Account: Joe, at age 35, decides to place 2000 into an IRA for each of the next 30 years. (Don't wait unitl you're 35.Begin now!!) What will the value of the IRA be when Joe makes his 30th deposit? Assume that the rate of return of the IRA is 4% per annum compounded annually.

answer: Since Joe is making regular payments, it's an annunity. P=2000, $i = 0.04/1 = 0.04$ (annual). n=30 deposits.

$A = P\left[\frac{(1+0.04/1)^{30} - 1}{0.04/1}\right] = 2000(56.084938) = \$112,169.88$. $30 \cdot 2000$=60,000 *input* but getting 112,169.88

Example 6: If Joe had begun his IRA at age 25 (..best to begin at age 18 if you can...) , and made 40 deposits (65-25=40 deposits) , what would his IRA be worth at age 65? i=.04 annual compounding

$$A = 2000\left[\frac{(1+0.04/\mathbf{1})^{40}-1}{0.04/\mathbf{1}}\right] = \$2000(95.025516) = \$190,051.03$$

Joe inputs 40 · 2000=80000, but getting \$190,051.03

Let's say Joe is super ambitious and thinks **very long term** and starts at age 18 $65-18=47$ deposits

$$A = 2000\left[\frac{(1+0.04/1)^{47}-1}{0.04/1}\right] = 265,890.78$$

$2000 \cdot 47 = 94000$ over the 47 years, but gets 265,980.78.

$I = 265980.78 - 94000 = \mathbf{\$171,980} \Leftarrow$ this money is generated by compound interest..NOT constant work!

This examples illustrate the power of compound interest!!

example 7: Finding the time it takes to reach a certain savings goal:

How long does it take to save 500,000 if you place \$500 per month in an account paying 6% compounded monthly?

$A = P\left[\frac{(1+i)^n-1}{i}\right]$. We want A=500,000. , P=500, i=.06 but monthly compounding means use .06/12 in formula.

$$500000 = 500\left[\frac{(1+0.06/12)^{n}-1}{0.06/12}\right]$$ (n=number deposits is not known, **n**)

divide by 500: $$\frac{500000}{500} = \frac{500}{500}\left[\frac{(1+0.06/12)^n-1}{0.06/12}\right]$$

$$1000 = \frac{(1+0.06/12)^{n}-1}{0.06/12}$$

$$1000 = \frac{(1+0.06/12)^{n}-1}{0.005}$$

multiply by .005: $$0.005 \cdot 1000 = \cancel{0.005} \cdot \frac{(1+0.005)^{n}-1}{\cancel{0.005}}$$

$$5 = (1.005)^{n} - 1$$

$$6 = 1.005^{n}$$ $\Leftarrow$ since n is in exponent, apply ln to both sides!

recall last class..we had a review of logs...or recall math 113 or 111:

take the ln of both sides: $ln6 = ln(1.005^n)$

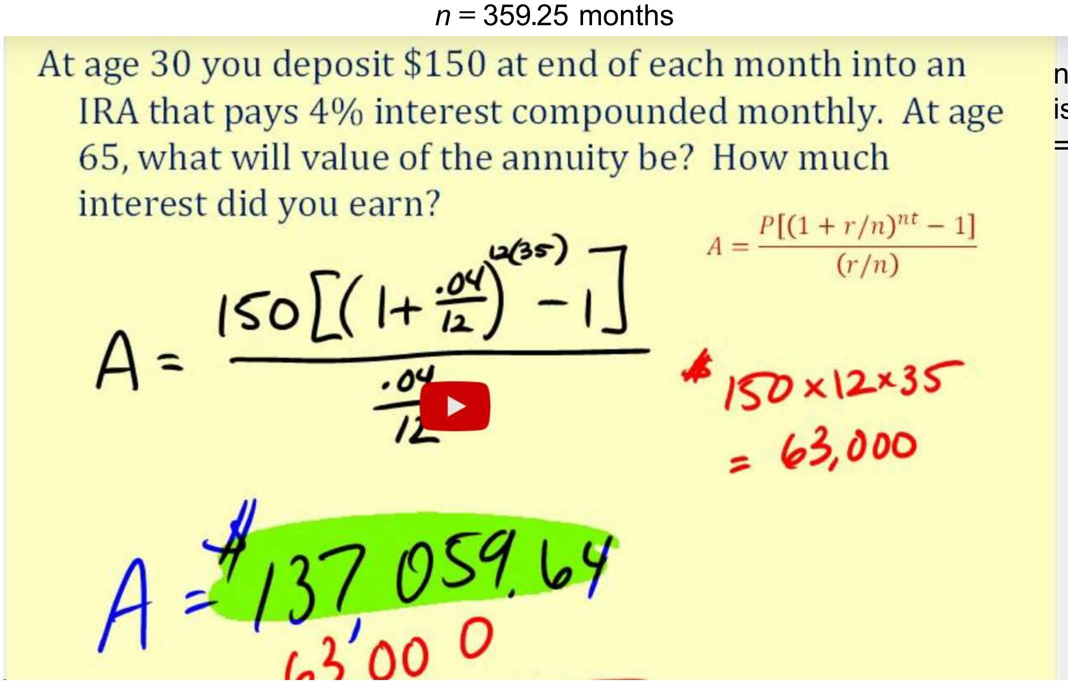
$$ln6 = n \cdot ln(1.005)$$

$$\frac{ln6}{ln(1.005)} = n$$

$$n = 359.25 \text{ months}$$

convert to years:

$359.25\ \cancel{\text{months}} \cdot \frac{1\text{ year}}{12\ \cancel{\text{months}}}$ = almost 30 years!



number of times we deposit is 12 times per year · 35 years = 420

$$A = P\left[\frac{(1+r/n)^{n \cdot t}-1}{r/n}\right]$$

r instead of i!

n is number of times per year. (n in our notes and book is total number of deposits)here in MOM we have to write nt as the exponent !.