Section 6.4/Present value of an annunity:

present value=amount of money needed now to obtain an amount A in the future.

Suppose we want to withdraw 10,000 per year each year for the next 5 years from a retirement account that earns 5% compunded annually. How much money is needed initially in this account for this to happen? The total amount is sum of the present values of each of the 10,000 withdrawals.
Present value of an annuity= sum of the present values of the withdrawals.

Example 1: Compute the amount of money to pay out 10,000 per year for 5 years at 5% compounded annually.

$A=P(1+r/n)^{nt} \xrightarrow{\text{divide both sides}} \frac{A}{(1+r/n)^{nt}}=P \Rightarrow A(1+r/n)^{-nt}=P, \qquad t=5, n=1, r=0.05$

$V_1=10000(1+0.05/1)^{-1}=10000(0.95238095)=\$9523.81$ (-1 in exponent) (money now to get 10,000 in 1 year)

$V_2=10000(1+0.05/1)^{-2}=10000(0.90702948)=\$9070.29$ (money now to get 10,000 in 2 years)

$V_3=10000(1+0.05/1)^{-3}=\ldots$ cal work= \$8638.38 (money now to get 10000 in three years)

$V_4=10000(1+0.05/1)^{-4}=\$8227.02$ (money now to get 10000 in 4 years)

$V_5 = 10000(1+.05)^{-5}$ = \$7835.26 (money now to get 10000 in 5 years)

present value= $V_1+V_2+V_3+V_4+V_5=9523.81+9070.29+8638.38+8227.02+7835.26=43,294.75$.

A person would need 43, 294.75 now, invested at 5% per annum, in order to withdraw \$10,000 per year for the next 5 years so no money is left over. Big idea is that we start with 43,294.75 but withdraw 50,000 over 5 years, so we make some free money.

Formula in general: $V_1=P(1+i)^{-1}$, i=rate, -1 means first year , n=1

$V_2=P(1+i)^{-2}$

$V_3=P(1+i)^{-3}$ and in general $V_n=P(1+i)^{-n}, i=rate, n=$ year

$V=V_1+V_2+\ldots+V_n=P(1+i)^{-1}+P(1+i)^{-2}+\ldots+P(1+i)^{-n}$.

$=P[(1+i)^{-1}+(1+i)^{-2}+\ldots(1+i)^{-n}]$

$$= \text{...some more details...} = \underbrace{P\left[\frac{1-(1+i)^{-n}}{i}\right]}_{\text{present value of an annunity}} \Leftarrow \text{formula.}$$

Suppose an annuity earns interest at a rate of i per payment period. If we make n withdrawals of \$P at each payment period, the amount V required now is

$P\left[\frac{1-(1+i)^{-n}}{i}\right]$.

Example 2: Getting By in College:

A student needs \$200 each month for the next 10 months to cover expenses. A money market fund will pay her interest of 2% per annum compounded monthly. How much should she ask for from her parents now so that she can withdraw 200 each moneh for the next 10 months?

$P=200$, $rate=$ i/12= $0.02/12$ (rate at end of month), n=10 withdrawals

$200\left[\frac{1-(1+0.02/12)^{-10}}{0.02/12}\right]=\$1981.79$. Student should ask \$1981.79 from parents.

Example 3: Determining the cost of a car: A man agrees to pay \$300 per month for 48 months to pay off a used car loan with no money down. If interest of 12% per annum is charged monthly, how much did the car originally cost? How much interest was paid?

Jim Rohn

$V=P\left[\frac{1-(1+i)^{-n}}{i}\right]=300\left[\frac{1-(1+0.12/12)^{-48}}{0.12/12}\right]=\$11,392.19.$

$300\left[\frac{1-(1+0.12/12)^{-4\cdot 12}}{0.012/12}\right]$

total payment= $300\cdot 48=14,400$.

Interest paid= $14400-11392.19=3007.81$

Example 4: A company may obtain a copying machine either by leasing it for 4 years(useful life) at an annual cost 1000 or by purchasing the machine for 3000.

(a) lease for 4 years, or buy now for 3000.

$P=1000$, annual so it's rate=.05, $n=4$ payments

$V=1000\left[\frac{1-(1+0.05)^{-4}}{0.05}\right]=\$3545.95$ From a purely financial view, it's better to pay 3000 today to avoid interest.

(b) Repeat with rate=7% per annum? r=.07 (per annum means we keep the rate as .07 and don't divide it)

$V = P\left[\frac{1-(1+i)^{-n}}{i}\right] = 1000\left[\frac{1-(1+0.07)^{-4}}{0.07}\right] = 3387.21$ Still better to purchase today for 3000 to avoid interest of 387.21.

another version of this formula:

$P\left[\frac{1-(1+r/n)^{-nt}}{r/n}\right]$, $r = rate$, n=number of times we compound per year, t=time

example: You want to be able to withdraw 45,000 each year for 30 years from your annunity.

how much do we need now?

$P_0 = V = 45000\left(\frac{1-(1+0.04/1)^{-1\cdot 30}}{0.04/1}\right) = 778,141.50$ (figure out how to save this amount over your working life)

retire at 55 and live to be 85...so by 55 must have 778,141.50 .

altruism

math of keeping a big perspective in life:

college is 4 years, you live to be 80

$\frac{4}{80} = 0.05 = 5\%$...college is 5% of your life..if you divide your life in 100 equal time periods, college is only 5 of these periods, so you have 95 other periods of life.