Please put away everything that's a distraction and take very detailed notes.

Section 4.1/Linear Programming (there is no programming..it's just a name that's stuck...)

Def: A linear programming problem in two variables x and y , consists of maximizing or minimizing an objective function $z = Ax + By$ subject to a set of constraints expressed as inequalities.

Andrew Huberman

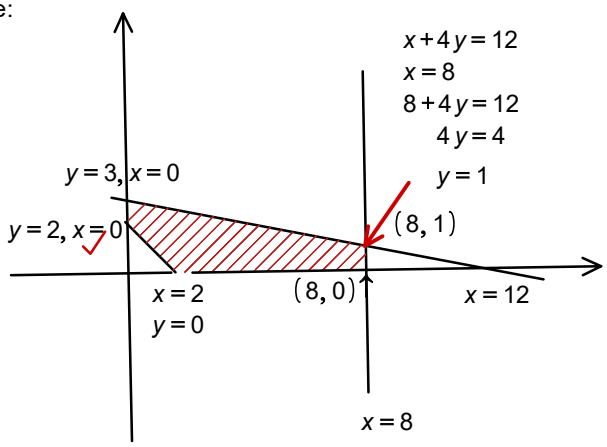
Major Step 1: Write an expression for the quantity to be maximized or minimized (objective function)
Step 2: Determine all constraints and graph the set of feasible points.
Step 3. List the corner points of the set of feasible points.
Step 4. Determine the value of the objective function at each corner point.
Step 5. Select the maximum or minimum value of the objective function.

Example 1: Maximize and minimize the objective function: $z = x + 5y$

set of constraints:

$x + 4y \le 12$ ← $x+4y=12$, $x=0 \to y=3$ ... $y=0 \to x=12$
$x \le 8$ ← $x \le 8$, solid line at x=8, shade left b/c of <
$x + y \ge 2$ ← $x+y \ge 2 \to x+y=2$, *interce* pts: $y=2, x=2$
$x \ge 0$ ← line throgh x=0 and shade right
$y \ge 0$ ← line through y=0 and shade above

picture:



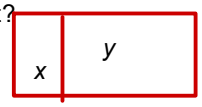
check z=x+5y at each corner point: $(0,2)$: $z = 0 + 5 \cdot 2 = 10$
$(0,3)$: $z = 0 + 5 \cdot 3 = 15$
$(8,1)$: $z = 8 + 5 \cdot 1 = 13$
(8,0): $z = 8 + 5 \cdot 0 = 8$
$(2,0)$: $z = 2 + 5 \cdot 0 = 2$

max value is 15
min value is 2

example of HWORK 1: A farmer is going to divide her 30 acre farm between two crops. Seed for crop A costs $10 per acre, and seed for crop B costs $20 per acre. The farmer can spend at most $500 on seed. If crop B bring in a profit of $100 per acre, and crop A brings in a profit of $120 per acre, how many acres of each crop should the farmer plant to maximize her profit?

constraints(inequalities)
$x \ge 0$ (land seeded with A)
$y \ge 0$ (land seeded with B)

Usually the FEASIBLE region is the one trapped between the x-axis, the y axis, and the other lines.



constraint on available land: $x + y \le 30$ (30 acress at most)

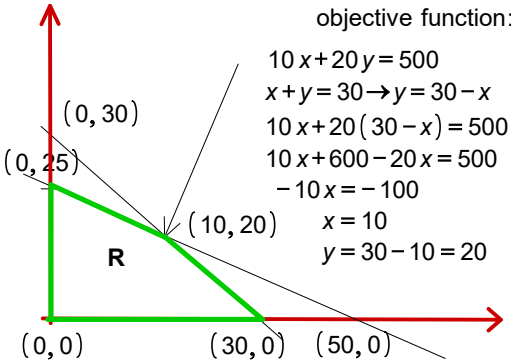
constraint on money: $10 \cdot x + 20 \cdot y \le 500$ (money for x+money for y <=500)

$10x$ = 10 dollars per acre · x acres (for seed A)

money

objective function: $z = 120x + 100y$

$120 \frac{\text{dollars}}{\cancel{\text{acre}}} \cdot x \cancel{\text{acres}} = 120x$ dollars ..likewise for 100y



$10x + 20y = 500$
$x + y = 30 \to y = 30 - x$
$10x + 20(30 - x) = 500$
$10x + 600 - 20x = 500$
$-10x = -100$
$x = 10$
$y = 30 - 10 = 20$

$10x + 20y \le 500$ boundary
$10x + 20y = 500$
x=0, 20y=500 $\to y = 500/20 = 25$
$y = 0$, $10x = 500 \to y = 500/10 = 50$
check with (0,0): $10 \cdot 0 + 20 \cdot 0 \le 50$ true
shade where (0,0) is located
solid line b/c of the = part of <=

$x \ge 0$ means line through x=0 and shade right
$y \ge 0$ means line through y=0 and shade above
$x + y \le 30$
boundary line is x+y=30
x=0, y=30..
y=0, x=30..
b/c <=, use solid line
shade where (0,0) is located..or check as before!

Check corner points: $(0,0)$: $z = 120 \cdot 0 + 100 \cdot 0 = \$0$ profit
$(0,25)$: $z = 120 \cdot 0 + 25 \cdot 100 = \$2500$ profit
$(10,20)$: $z = 120 \cdot 10 + 20 \cdot 100 = \$3200$ profit
$(30,0)$: $z = 30 \cdot 120 + 0 \cdot 100 = \$3600$ profit

Business Summary: Plant 30 acres of seed A and no acres of seed B and earn 3600 in profit.

A farmer is going to divide her 40 acre lot between two crops. Seed A cots $10 per acre, and seed B costs $5 per acre. Her budget is $250. Seed B brings in $90 profit per acre. Seed A brings in $160 profit per acre. x=number of crop A(acres), and y=number of crop B(acres)

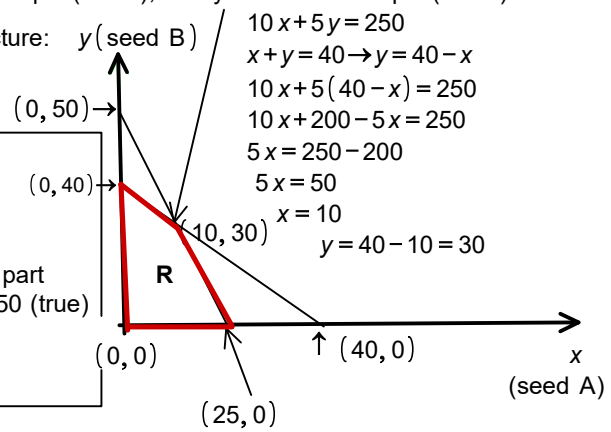
$x \ge 0, y \ge 0$ ($acres$ can't be negative)
$x + y \le 40$ (total land is <=40 acres)
$10 \cdot x + 5 \cdot y \le 250$ (money for seed A+money for seed B<=250)
Objective function: $z = 160x + 90y$ (money earned from A+money earned from seed B)

$x \ge 0$ (line through x=0 and shade to the right b/c of the >)
$y \ge 0$ (line through y=0 *and* shade above)

x+y ≤ 40 boundary: $x + y = 40$
$x = 0, y = 40$ (A)
$y = 0$, $x = 40$ (B)
shade where (0,0) is located
from $0 + 0 \le 40$
solid b/c of the = part of <=

$10x + 5y \le 250$
equation form: $10x + 5y = 250$
$x = 0, 5y = 250 \to y = 50$ (C)
$y = 0, 10x = 250 \to x = 25$ (D)
≤ means solid line b/c of the = part
check with (0,0): $10 \cdot 0 + 5 \cdot 0 \le 250$ (true)
shade where (0,0) is located

picture: y(seed B)



$10x + 5y = 250$
$x + y = 40 \to y = 40 - x$
$10x + 5(40 - x) = 250$
$10x + 200 - 5x = 250$
$5x = 250 - 200$
$5x = 50$
$x = 10$
$y = 40 - 10 = 30$

check object function at each corner point: $(0,0)$: $z = 160 \cdot 0 + 90 \cdot 0 = \$0$ profit

Business Summary: Plant 10 acres of A and 30 acres of B to make a max. profit of $4300.

$(0, 40): z = 160 \cdot 0 + 90 \cdot 40 = \$3600$ profit

$(10, 30): z = 160 \cdot 10 + 90 \cdot 30 = \$4300$ profit

$(25, 0): z = 160 \cdot 25 + 90 \cdot 0 = \$4000$ profit