Math 111 Notes 11/6/2023. Take DETAILED notes and put away all phones and computers while taking notes.

Applications of Rational Functoions:

Example 4/Page 335:

A utility company (ConED) burns coal to make electricity. The cost C(in dollars) of removing p% of the smokestack pollutants is given by $C=\frac{80000p}{100-p}$, where $0\le p<100$. This is a rational function. R(x)= $\frac{N(x)}{D(x)}$, $N(p)=80000p$

D(p)= $100-p$

p is the independent variable

$R(p)=\frac{N(p)}{D(p)}$

p comes from "pollutant"

at p=100: $C(100)=\frac{80000\cdot 100}{100-100}=\frac{80000\cdot 100}{0}\Leftarrow$ not defined for p=100..domain: $0\le p<100$

If p=100, we'd be saying we can remove all the pollutants, which is probably not realistic.

$If\ p=0$, then we get $C(0)=\frac{80000\cdot 0}{100-0}=\frac{0}{100}=0\Leftarrow$ To remove 0% of pollutans will cost you 0dollars.

C(85)= $\frac{80000\cdot 85}{100-85}\xrightarrow{\text{remove 85\% of pollutans}}\approx \$453,333.$ $C(90)=\frac{80000\cdot 90}{100-90}=\$720,000.$

What's the additional cost to go from 85% to 90%: $C(90)-C(85)=720,000-453,333=\$266,667.$

Example 5: Ultraviolet radiation:

For a person with sensitive skin, the amount of time T (in hours) the person can be exposed to the sun iwth minimal burning can be modeled by $T=\frac{0.37s+23.8}{s}$, $0<s\le 120$, s= Sunsor Scale reading (Sunsor Inc.)

$s=10:\ T(10)=\frac{0.37\cdot 10+23.8}{10}=2.75$ hrs

$T(s)=\frac{0.37s+23.8}{s}$ rational function

$s=25:\ T(25)=\frac{0.37\cdot 25+23.8}{25}=1.32hrs$

$N(s)=0.37s+23.8$

$s=100:\ T(100)=\frac{0.37\cdot 100+23.8}{100}=0.61$ hrs

$D(s)=s$

$s=120(last$ value from the domain of s b/c it says < or $=):T(120)=\frac{0.37\cdot 120+23.8}{120}=0.57$ hrs

Imagine s=1000: $T(1000)=\frac{0.37\cdot 1000+23.8}{1000}=0.39$

Imagein $s=10000(not$ real$)$: T(10000)= $\frac{0.37\cdot 10000+23.8}{10000}=0.37$

Horizontal Asymptote is $y=0.37$

Shortest possible exposure time with minimal burning is .37 hrs.

. 3. (Hwork): Mr. Ewald decides to make and sell Left Handed Smoke Shifters (made up whatever). The function below gives the average cost (in dollars) per Smoke Shifter when x Smoke Shifters are produced.

$A(x)=\frac{3x+450}{x}$, rational function N(x)= $3x+450$, $D(x)=x$ Average Cost= $\frac{\text{total cost}}{\text{number of units}}$

Determine A(15).. Average cost to produce 15 smoke shifters.

$A(15)=\frac{3\cdot 15+450}{15}=\frac{45+450}{15}=\frac{495}{15}=33$

How many smoke shifters must be produced to reduce the **average cost to $12 each?**

$x=?$ A

$\frac{3x+450}{x}=12\leftarrow$ given

$\frac{3x+450}{x}=\frac{12}{1}\xrightarrow{\text{any number is a i sreally a/1}}$

$1(3x+450)=12\cdot x$ (cross multiplication)

$3x+450=12x$ (linear equation b/c x^1)

$450=12x-3x$

$450=9x$

$\frac{450}{9}=x\to x=50$ We have to make 50 smoke shifters to ensure the average cost is 9 dollars.

Find the average cost to make 1000 smoke shifters:

$A(1000)=\frac{3\cdot 1000+450}{1000}=$ **3.45**

$A(10000)=\frac{3\cdot 10000+450}{10000}=$ **3.045**

(in MOM..input only two decimal places..3.05)

Horizontal asymptote:

$As x\to\infty, y\to 3$

$y=3$ If we made a hugggge number of smoke shifters, the average cost would be 3 per smoke shifter.

last example: Human Memory Model:

$P(n)=\frac{0.5+0.9(n-1)}{1+0.9(n-1)}, n>0$ $n=$ number of trials of some task(how many times did we do this task)

n	1	2	3	4	5
P					

$P(100)=\frac{0.5+0.9(100-1)}{1+0.9(100-1)}=1.0$

1 means 100% correct

$P=$ fraction of correct responses

Summary: As $n\to\infty$, $P\to 1$. We get better and better simply by repeating the task. No genius required!

$P(1)=\frac{0.5+0.9(1-1)}{1+0.9(1-1)}=\frac{0.5+0.9(0)}{1+0.9(0)}=\frac{0.5}{1}=0.5$ (do something once..50% chance it's correct)

$P(2)=\frac{0.5+0.9(2-1)}{1+0.9(2-1)}=0.74$ (tasks we can control at each stage)

$P(3)=\frac{0.5+0.9(3-1)}{1+0.9(3-1)}=0.82$ ... make n=10: $P(10)=\frac{0.5+0.9(10-1)}{1+0.9(10-1)}=0.95$