$\frac{5x-13}{(x-3)(x-2)}$ (two distinct linear factors on bottom)

$\frac{5x-13}{(x-3)(x-2)} = \frac{a}{x-3} + \frac{b}{x-2}$ $\qquad (x-2)^{1}, (x-3)^{1}$

$$5x-13 = \frac{a}{\cancel{x-3}}\cancel{(x-3)}(x-2) + \frac{b}{\cancel{(x-2)}}(x-3)\cancel{(x-2)}$$

$5x-13 = a(x-2)+b(x-3)$

$x=2: \quad 5\cdot 2-13 = a(2-2)+b(2-3)$

$-3 = a0+b(-1)$

$-3 = -b$

$b = 3$

$x=3: 5\cdot 3-13 = a(3-2)+b(3-3)$

$2 = a(1)$

$2 = a$

$5x-13 = ax-2a+bx-3b$

$5x-13 = ax+bx-2a-3b$

$\mathbf{5}x-13 = x(\boldsymbol{a}+\boldsymbol{b})-2a-3b$

$a+b=5$

$-2a-3b=-13$

$$\frac{5x-13}{(x-3)(x-2)} = \frac{2}{x-3}+\frac{3}{x-2}$$

$\frac{x+4}{(x+1)^2}$ ⇐ repeating linear factor (x+1)(x+1)

$\frac{x+4}{(x+1)^2} = \frac{a}{x+1}+\frac{b}{(x+1)^2}$

$\cancel{(x+1)^2}\frac{(x+4)}{\cancel{(x+1)^2}} = \frac{a}{(x+1)^{1}}(x+1)^{\mathbf{2}}+\frac{b}{(x+1)^2}(x+1)^2$

$x+4 = a(x+1)+b$

$x+4 = ax+a+b$

$1x+\mathbf{4} = ax+\boldsymbol{a}+\boldsymbol{b}$

$a=1, a+b=4$

$1+b=4$

$b=4-1=3$

⇐ finishing up

$\frac{x+4}{(x+1)^2} = \frac{1}{x+1}+\frac{3}{(x+1)^2}$ answer!

$\frac{2x+2}{x^2-2x+1} = \frac{2x+2}{(x-1)^2}$

$\frac{2x+2}{(x-1)^2} = \frac{a}{x-1}+\frac{b}{(x-1)^2}$

$\cancel{(x-1)^2}\frac{(2x+2)}{\cancel{(x-1)^2}} = \frac{a}{(x-1)}(x-1)^2+\frac{b}{(x-1)^2}(x-1)^2$

$2x+2 = a(x-1)+b$

$2x+\mathbf{2} = ax-\boldsymbol{a}+\boldsymbol{b}$

$a=2, \quad -a+b=2$

$-2+b=2$

$b=4$

$x^2+2xy+y^2$ ⇐ perfect square trinomial

$(1x)^2+2(1x)(-1)+(-1)^2$

$(x-1)^2 \qquad x^2-2x+1$

$\frac{2x+2}{x^2-2x+1} = \frac{2}{x-1}+\frac{4}{(x-1)^2}$

$x=1:$

$2+2 = a(0)+b$

$b=4$

$\frac{z+1}{z^2(z-1)^1}$ z repeating and z-1 is distinct

$\frac{z+1}{z^2(z-1)} = \frac{a}{z}+\frac{b}{z^2}+\frac{c}{z-1}$

answer: $\frac{-2}{z}+\frac{-1}{z^2}+\frac{2}{z-1}$

multiply by ~~bottom of LHS:~~

$$z^2(z-1)\frac{(z+1)}{\cancel{z^2(z-1)}} = \frac{a}{z}z^2(z-1)+\frac{b}{z^2}(z^2)(z-1)+\frac{c}{z-1}z^2(z-1)$$

$1z+\mathbf{1} = az(z-1)+b(z-1)+c(z^2)$

$0z^2+1z+1 = az^2-az+bz-b+cz^2$

$= (a+c)z^2+z(-a+b)-\boldsymbol{b}$

$0=a+c$

$-a+b=1$

$-b=1$

$b=-1$

$-a-1=1$

$-a=2$

$a=-2$

$0=-2+c$

$c=2$

$z=0: \quad 0+1=a0(0-1)+b(-1)+c(0^2)$

$1=-b \Rightarrow b=-1$

$z=1: \; 2=a\cdot 1(0)+b(0)+c(1^2)$

$2=c$

$z=2:$ (i set b equal to -1, c=2, and z=2)

$1\cdot 2+1=a\cdot 2(2-1)-1(2-1)+2(2^2)$

$3=2a-1+8$

$3-8=2a-1$

$-5=2a-1$

$-4=2a$

$a=-2$

$$\frac{z}{z^3-z^2-6z}=\frac{z\cdot 1}{z(z^2-z-6)}=\frac{1}{z^2-z-6}=\frac{1}{(z-3)(z+2)}$$

two distinct linear factors in b0ttom

$$\cancel{(z-3)(z+2)}\cdot\frac{1}{\cancel{(z-3)(z+2)}}=\frac{a}{\cancel{z-3}}\cancel{(z-3)}(z+2)+\frac{b}{\cancel{z+2}}(z-3)\cancel{(z+2)}$$

$1=a(z+2)+b(z-3)$

$z=-2: \; 1=a(0)+b(-5)$

$1=-5b \Rightarrow b=-1/5$

$z=3: \; 1=a(5)+b(0)$

$a=1/5$

$$\frac{z}{z^3-z^2-6z}=\frac{1/5}{z-3}-\frac{1/5}{z+2}=\frac{1}{5}\cdot\frac{1}{(z-3)}-\frac{1}{5}\left(\frac{1}{z+2}\right)$$

$\frac{t^2+8}{t^2-5t+6}$ $\quad t^2$ and t^2 (top expo. to be smaller..don't have this)

$$t^2-5t+6\,\overset{1}{\overline{\big)\,t^2+0t+8}}$$

$t^2/t^2=1 \leftarrow$ goes in top

$\underline{-t^2+5t-6}$ (flip signs)

$5t+2$

$$5\,\overset{4}{\overline{\big)\,24}}$$

$\underline{-20}$

4

$$\frac{t^2+8}{t^2-5t+6}=1+\frac{5t+2}{t^2-5t+6}$$

t in top vs. t^2 in bottom

$\frac{5t+2}{t^2-5t+6}$ factor bottom $\quad \frac{5t+2}{(t-3)(t-2)}=\frac{a}{t-3}+\frac{b}{t-2}$ (distinct linear factors)

$5t+2=a(t-2)+b(t-3)$

$t=2: 12=a(0)+b(-1)\Rightarrow b=-12$

$t=3: \quad 17=a(3-2)+b(0)\Rightarrow a=17$

$\int f(x)+g(x)\,dx$

$\int f(x)\,dx + \int g(x)\,dx$

answer: $1+\frac{17}{t-3}-\frac{12}{t-2}$

$\frac{x+4}{x^2+5x-6}$ $\quad x^1$ vs x^2, so good to go!

$$\int\frac{x+4}{x^2+5x-6}\,dx=\int\frac{2/7}{x+6}\,dx+\int\frac{5/7}{x-1}\,dx$$

$$=\frac{2}{7}\,ln|x+6|+\frac{5}{7}\,ln|x-1|+C$$

$$=\frac{1}{7}\,ln|(x+6)^2|+\frac{1}{7}\,ln|(x-1)^5|+C$$

$$=\frac{1}{7}\,ln|(x+6)^2(x-1)^5|+C$$

$\frac{x+4}{(x+6)(x-1)}=\frac{a}{x+6}+\frac{b}{x-1}$

$x+4=a(x-1)+b(x+6)$

$x=1: \quad 5=a(0)+b(7)\Rightarrow b=5/7$

$x=-6: \quad -2=a(-7)+b(0)\Rightarrow a=2/7$

power rule
for logs:
bidirectional:
$a\,ln(x)=ln\,x^a$

$$\int \frac{dt}{t^3-t^2-2t} = \int \frac{dt}{t(t^2-t-2)} = \int \frac{dt}{t(t+2)(t-1)}$$

$$\frac{1}{t(t+2)(t-1)} = \frac{a}{t} + \frac{b}{t+2} + \frac{c}{t-1}$$

$$1 = a(t+2)(t-1) + b(t)(t-1) + c(t)(t+2)$$

$$t=0: \quad 1 = a(2)(-1) + b(0)(-1) + c(0)(2) \Rightarrow 1 = -2a \Rightarrow a = -1/2$$

$$t=1: \quad 1 = a(3)(0) + b(1)(0) + c(1)(3) \Rightarrow 3c = 1 \Rightarrow c = 1/3$$

$$t=-2: \quad b = \ldots ? b = 1/6$$

integral back....

$$\int \frac{-1/2}{t}\,dt + \int \frac{1/6}{t+2}\,dt + \int \frac{1/3}{t-1}\,dt$$

$$-\frac{1}{2} ln|t| + \frac{1}{6} ln|t+2| + \frac{1}{3}|t-1| + C$$ (answer)

$$\frac{x^3}{x^2+2x+1}$$ 3 > 2 in bottom on x

$$\begin{array}{r} x-2 \\ x^2+2x+1\,\overline{\big)\,x^3+0x^2+0x+0} \\ \underline{-x^3-2x^2-x} \\ -2x^2-x-0 \\ \underline{+2x^2+4x+2} \\ 3x+2 \end{array}$$

flip signs

$$\frac{-2x^2}{x^2} = -2$$

flip signs here

$$\frac{x^3}{x^2+2x+1} = x-2+\frac{3x+2}{x^2+2x+1}$$ $3x$ vs x^2, 1<2 in expos!

$$\frac{3x+2}{x^2+2x+1} = \frac{(3x+2)}{(x+1)^2} = \frac{a}{x+1} + \frac{b}{(x+1)^2}$$

$$3x+2 = a(x+1)+b$$

$$3x+2 = ax+a+b$$

$$a = 3, \quad 2 = a+b \rightarrow b = -1$$

$$\int x^1\,dx = \frac{x^{1+1}}{1+1} + C = \frac{x^2}{2} + C$$

$$\int_0^1 x-2+\frac{3}{x+1}-\frac{1}{(x+1)^2}\,dx$$

$$\int_0^1 x-2\,dx + 3\int_0^1 \frac{1}{x+1}\,dx - \int_0^1 (x+1)^{-2}\,dx$$

$$\left(\frac{1}{2}x^2-2x\right)\Bigg|_0^1 + 3\;ln(x+1)\Bigg|_0^1 + \frac{1}{x+1}\Bigg|_0^1$$

$$= \frac{1}{2} - 2 - \cancel{\left(\frac{1}{2}\cdot 0 - 2\cdot 0\right)} + 3ln(2) - 3ln(1) + \frac{1}{2} - \frac{1}{1}$$

$$= \frac{1}{2} - 2 + 3ln2 - 3ln1 + \frac{1}{2} - 1$$

$$= \frac{1}{2} - 2 + 3ln2 - 0 + \frac{1}{2} - 1$$

$$= -2 + 3ln2$$

$$\frac{a+b}{c}=\frac{a}{c}+\frac{b}{c}$$

$$\int_0^1 \frac{dx}{(x+1)(x^2+1)}$$

$$\frac{1}{(x+1)(x^2+1)}=\frac{a}{x+1}+\frac{bx+c}{x^2+1}$$

$$1=a(x^2+1)+(bx+c)(x+1) \qquad a, b, c$$

$$1=ax^2+a+bx^2+bx+cx+c$$

$$1=(a+b)x^2+(b+c)x+a+c$$

$$a+b=0, \quad b+c=0 \quad a+c=1$$

$$-a=b \qquad -b=c$$

$$a=1-c$$

$$a=-b \qquad a=1+b$$

$$1+b=-b$$

$$a=1+(-1/2)=\mathbf{1/2}$$

$$1=-2b$$

$$c=-(-1/2)=1/2$$

$$b=-1/2$$

$$u=x^2+1$$

$$du=2xdx$$

$$\frac{du}{2}=xdx$$

$$u(0)=0^2+1=1$$

$$u(1)=2$$

$$1=a(x^2+1)+(bx+c)(x+1)$$

$$x=0: \quad 1=a(1)+(c)(1)$$

$$1=a+c$$

$$x=-1: \quad 1=a(2)\Rightarrow \mathbf{a=1/2}$$

$$1=1/2+c$$

$$c=1/2$$

$$\int_0^1 \frac{dx}{(x+1)(x^2+1)}=\int_0^1 \frac{1/2}{x+1}+\int_0^1 \frac{\frac{-1}{2}x+\frac{1}{2}}{x^2+1}dx$$

$$=\frac{1}{2}(ln(1+1)-ln(0+1))-\frac{1}{2}\int_0^1 \frac{x-1}{x^2+1}dx$$

$$=\frac{1}{2}(ln2-ln1)-\frac{1}{2}\left[\int_0^1 \frac{x}{x^2+1}-\frac{1}{x^2+1}dx\right]$$

$$=\frac{1}{2}(ln2)-\frac{1}{2}\left[\int_1^2 \frac{1}{u}\frac{du}{2}-\int_0^1 \frac{1}{x^2+1}dx\right]$$

$$=\frac{1}{2}ln2-\frac{1}{2}\left[\frac{1}{2}(ln2-ln1)-(tan^{-1}(1)-tan^{-1}(0))\right]$$

$$=\frac{1}{2}ln2-\frac{1}{2}\left[\frac{ln2}{2}-0-\left(\frac{\pi}{4}-0\right)\right] \qquad tan^{-1}\left(\frac{0}{1}\right)$$

$$=\frac{1}{2}ln2-\frac{1}{2}\left[\frac{ln2}{2}-\frac{\pi}{4}\right]$$

$y=0, x=1$

$(1,0)$ angle=0

$$=\frac{1}{2}ln2-\frac{ln2}{4}+\frac{\pi}{8}$$

$$=\frac{4}{8}ln2-\frac{2ln2}{8}+\frac{\pi}{8}=\frac{4ln2-2ln2+\pi}{8}=\frac{2ln2+\pi}{8}$$

29. $\int \frac{x^2}{x^4-1}dx$

$$\frac{x^2}{x^4-1}=\frac{x^2}{(x^2+1)(x^2-1)}=\frac{x^2}{(x^2+1)(x+1)(x-1)}$$ (two linear, 1 irreducib..)

x^2 vs. x^4

$$\frac{x^2}{(x^2+1)(x-1)(x+1)}=\frac{ax+b}{x^2+1}+\frac{c}{x-1}+\frac{d}{x+1} \qquad b=1/2, c=1/4, d=-1/4$$

$$1x^2=(ax+b)(x-1)(x+1)+c(x+1)(x^2+1)+d(x^2+1)(x-1)$$

$$x=1: \quad 1=(a+b)(0)(2)+c(2)(2)+d(2)(0)\Rightarrow c=1/4$$

$$x=-1: \quad 1=(-a+b)(-2)(0)+c(0)(2)+d(2)(-2)\Rightarrow d=-1/4$$

$$x^2=(ax+b)(x-1)(x+1)+1/4(x+1)(x^2+1)-1/4(x^2+1)(x-1)$$

$$x=0: \quad 0^2=(a0+b)(-1)(1)+1/4(1)(1)-1/4(1)(-1)$$

$$0=-b+1/4+1/4\Rightarrow b=2/4=1/2$$ (get rid of a with x=0)

$$x=? \qquad x^2=(ax+1/2)(x-1)(x+1)+1/4(x+1)(x^2+1)-1/4(x^2+1)(x-1)$$

can't use x=1...(1-1) on (ax+1/2), $x=0$ b/c this would wipe away (ax+1/2)

$$x=2: \quad 4=(2a^{1}+1/2)(1)(3)+1/4\ (3)(5)-1/4(5)(1)$$

$a = 0!\;!$ at end

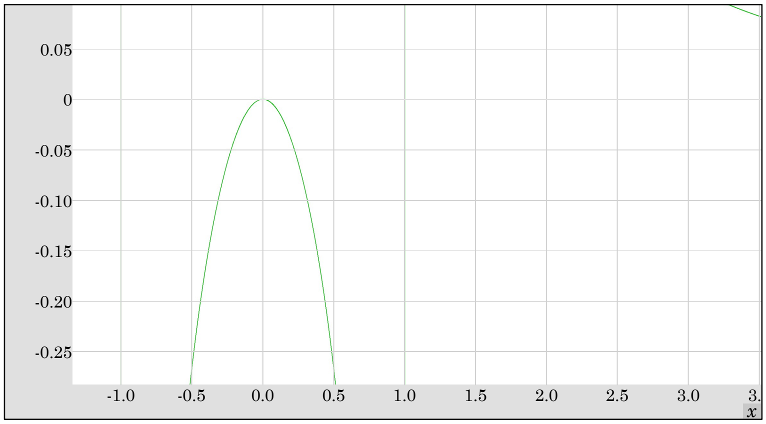
$$4 = (2a + 1/2)(3) + 15/4 - 5/4$$

$$4 = (2a + 1/2)\cdot 3 + 10/4$$

$$4 = 6a + 3/2 + 10/4$$

$$0 = 6a \rightarrow a = 0$$

$$\int \frac{x^2}{x^4-1}dx = \int \frac{0x+1/2}{x^2+1} + \frac{1/4}{x-1} - \frac{1/4}{x+1}dx = \frac{1}{2}tan^{-1}(x) + \frac{1}{4}ln|x-1| - \frac{1}{4}ln|x+1| + C$$



$\Leftarrow \frac{x^2}{x^4-1}$ has the identital graph to

$$\frac{1/2}{x^2+1} + \frac{1/4}{x-1} - \frac{1}{4}\left(\frac{1}{x+1}\right)$$

$$\int \frac{cosy}{sin^2y + siny - 6}dy \quad , siny = t, cosydy = dt$$

$$\int \frac{cosydy}{(siny)^2 + 1sin(y) - 6} \rightarrow \int \frac{dt}{t^2+t-6} = \int \frac{dt}{(t-2)(t+3)}$$

partial fractions

$x^2 + 5x + 6$

$$\frac{1}{(t-2)(t+3)} = \frac{a}{t-2} + \frac{b}{t+3}$$

$$= \frac{1}{5}\int \frac{1}{t-2} - \frac{1}{t+3} \quad dt$$

$$b = -1/5, a = 1/5$$

$$= \frac{1}{5}[ln|t-2| - ln|t+3|]$$

$$= \frac{1}{5}ln\left|\frac{t-2}{t+3}\right| \Rightarrow \frac{1}{5}ln\left|\frac{siny-2}{siny+3}\right| + C$$