Bridging the Gap to High School Calculus (6 Weeks) 0629PC7H 6/29/2021 - 8/5/2021 (3 students)

Sessions (12 total)

~~Tue, Jun 29, 2021 8:00 PM-9:00 PM EDT~~
~~Thu, Jul 1, 2021 8:00 PM-9:00 PM EDT~~
~~Tue, Jul 6, 2021 8:00 PM-9:00 PM EDT~~
~~Thu, Jul 8, 2021 8:00 PM-9:00 PM EDT~~
~~Tue, Jul 13, 2021 8:00 PM-9:00 PM EDT~~
~~Thu, Jul 15, 2021 8:00 PM-9:00 PM EDT~~
~~Tue, Jul 20, 2021 8:00 PM-9:00 PM EDT~~
~~Thu, Jul 22, 2021 8:00 PM-9:00 PM EDT~~
~~Tue, Jul 27, 2021 8:00 PM-9:00 PM EDT~~
~~Thu, Jul 29, 2021 8:00 PM-9:00 PM EDT~~
Tue, Aug 3, 2021 8:00 PM-9:00 PM EDT
Thu, Aug 5, 2021 8:00 PM-9:00 PM EDT

873896

4 Ways To Represent A Function:

$f(x) = 2x + 4 + \sqrt{x} + \left(\frac{1}{x}\right)$

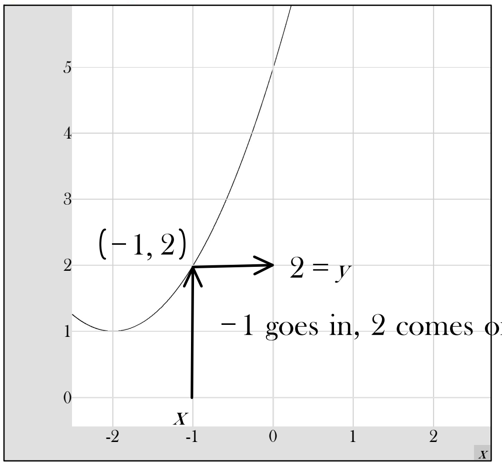
"f of x", f is the name of the function,
x is the input, f(x) is the output
f(x)=y
We can write points as (x, y), $(x, f(x))$

$f(x) = x^2 + 4x + 5$

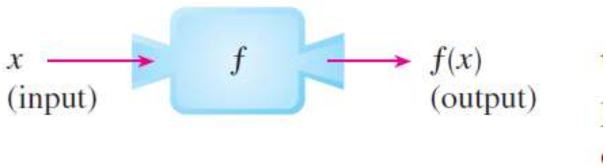
$f(1) = 1^2 + 4(1) + 5 = 1 + 4 + 5 = 10$

$(\text{input,output}) = (1, 10) = (1, f(1))$

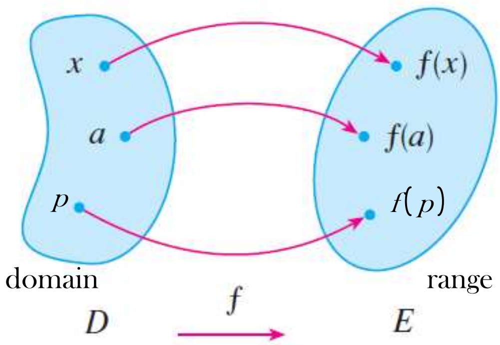
At it's core, a function is an input/output machine.



−1 goes in, 2 comes on the vertical axis



flow diagram
for functions



The values we input are called the domain.
The values we output form the range.

Important Detail About Functions:
For a function, all the points are such that
the first coordiante of each point is unique.

f= $\{(1, 2), (2, 3), (4, 5), (6, 7)\}$ points: (x, y)

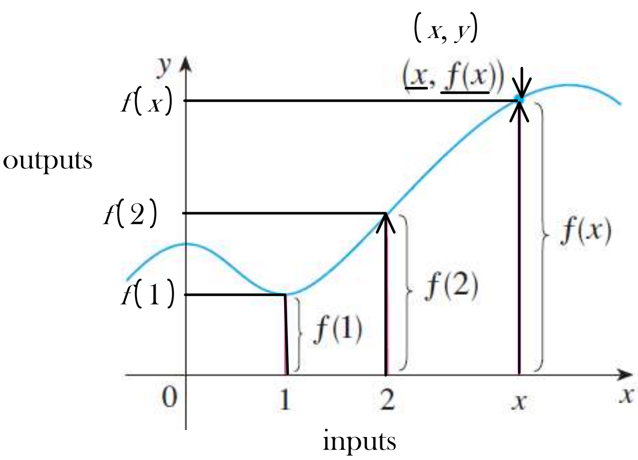
For the set of points to make a function, each
x coordiante has to be unique.

$1 \neq 2 \neq 4 \neq 6$ $\neq$ is read as "not equal to"

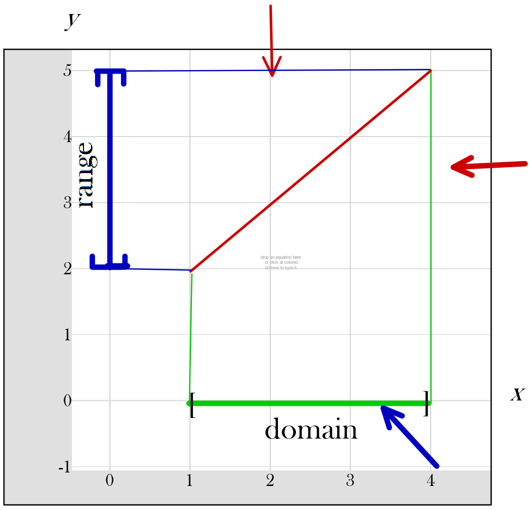
Because the x-coordiantes are all different, the set
$\{(1, 2), (2, 3), (4, 5), (6, 7)\}$ is a function.

A more mathematical way to write
all the points that belong to a function.
$\{(x, f(x)) \mid x \in \mathbf{R}\}$
R stands for all real numbrers.
$-5, 3, 10, \pi, 0.236, 3.29$

Is the following set a function?
$g = \{(1, 2), (2, 3), (4, 5), (1, 7)\}$
Is the set above a function or not?
It's not a function because x=1 repeats in (1,2) and (1,7).



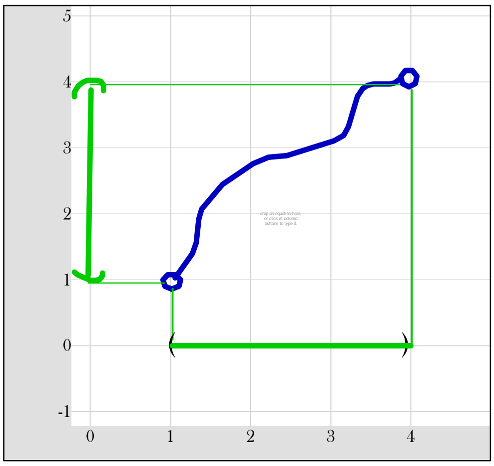
so this graph generates the following points:
$(1, f(1)), (2, f(2)), (x, f(x))$



find the domain and range:

domain first:
imagine the shadow of the graph on the x axis

Interval Notation:
domain in interval notation: $[1, 4]$
range in interval notation:[2,5]



Holes in graphs indicate that the function is not defined there.

Parenthesis indicate that the values 1 and 4 are not part of the domain. The domain , then , written in interval notation is
$(1, 4)$ ← this is not a point , though it looks like one

$2 \in (1, 4)$ "2 is an element of the set from 1 to 4"

"2 belongs to the set from 1 to 4"

What's the range in interval notation?

$(1, 4)$

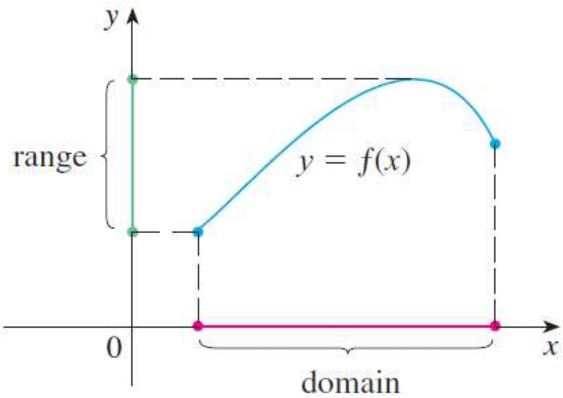
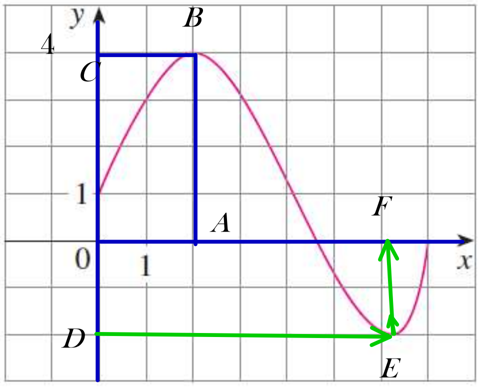


FIGURE 5



What's the domain in interval form?
[0,7]
What's the range in interval form?
[-2,4]
What's f(2)?
f(2)=4 as the blue lines indicate. ABC
What's the x that gives a value of -2? →

about 6 as the green arrows indicate DEF

Inequality Notation:
the interval form $[2, 4]$ becomes
, assuming x is the variable, $2 \leq x \leq 4$
because of the brackets, use $\leq$, not just $<$

The interval $(1, 2)$ becomes the inequality
$1 < x < 2$ (this says x can be any value from 1 to 2, but NOT 1 or 2)

Major Forms of Notation for Different Sets:
interval notation, inequality notation, set builder notation

$[1, 3]$ $1 \leq x \leq 3$ $\{x \mid 1 \leq x \leq 3\}$

"the set of all x such that the inequality is true" interval notation:

$(-\pi, 4)$ write the inequality notation: $-\pi < x < 4 \rightarrow$ write the set builder notation: $\{x \mid -\pi < x < 4\}$

mixed version of this:

$(1,4]$ *write* the inequality notation: $1 < x \leq 4 \rightarrow$ write the set builder notaton: $\{x \mid 1 < x \leq 4\}$

"set of all x such that 1 is less than x , which is less than or equal to 4"

$(1, 4)$ → $1 < x < 4$

$[1, 4]$ → $1 \leq x \leq 4$

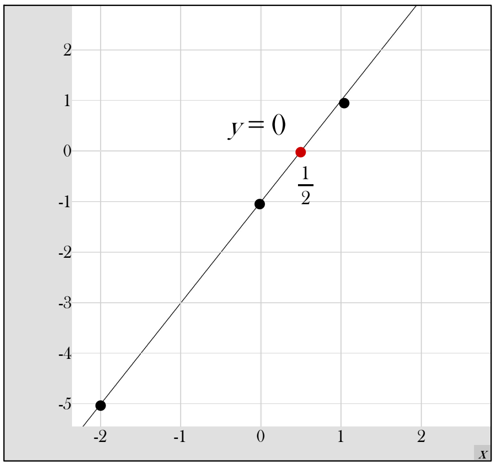
Sketching graphs: $x = 1 \cdot x^1$

$f(x) = 2x - 1$

when it's x, it's really x^1, so f(x)=2x-1 reprsesents a linear function.

x	$f(x) = 2x - 1$	(x, y)
-2	$f(-2) = -5$	$(-2, -5)$
0	$f(0) = -1$	$(0, -1)$
1	$f(1) = 1$	$(1, 1)$

$f(x) = 2x - 1$



Puzzle:

$f(x) = 0$?

Where is the y coordinate equal to 0?

$x = \frac{1}{2}$

solve for it exactly:

$2x - 1 = 0$

$2x - 1 + 1 = 0 + 1$

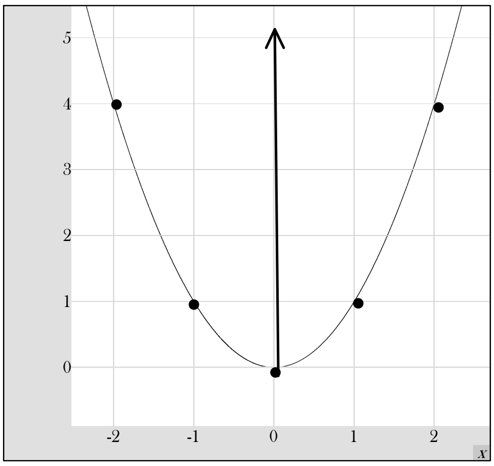
$2x = 1$

$\frac{2x}{2} = \frac{1}{2}$

$x = \frac{1}{2}$ is the solution

$f(x) = x^2$ (anything squared is 0 or positive, never negative)

x	$f(x) = x^2$	(x, y)
-2	$f(-2) = 4$	$(-2, 4)$
-1	$f(-1) = 1$	$(-1, 1)$
0	$f(0) = 0$	$\boxed{(0, 0)}$
1	$f(1) = 1$	$(1, 1)$
2	$f(2) = 4$	$(2, 4)$



domain: (are there any restrictions on the inputs?)

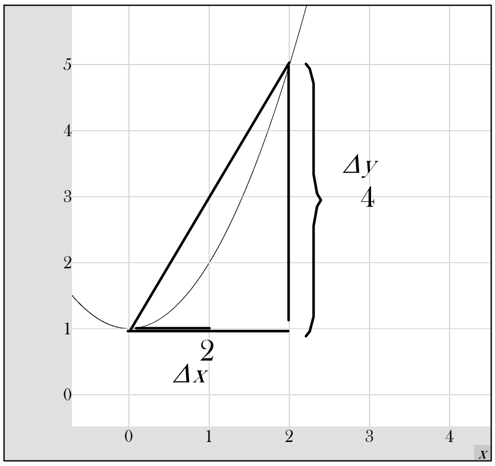
In this, case there are no restrictions. So any x can be plugged. Thus, the domain is **R** (this means the real number line, any number whatsoever)

In interval notatoin, we say domain= $(-\infty, \infty)$

Range : What's the lowest value of y? 0, y coordinates do not have a finite value and can go up forever.

$[0, \infty)$

The Difference Quotient (this is really the key to calculus for you later on)



The goal is to find the slope of the line .

$$\text{slope}=\frac{y_2-y_1}{x_2-x_1}=\frac{\Delta y}{\Delta x}=\frac{\text{change in y}}{\text{change in x}}$$

$$=\frac{5-1}{2-0}=\frac{4}{2}=2 \text{ slope of the}$$

line in the graph.

Same concept as slope, but it uses different symbols.

Δy "delta y"

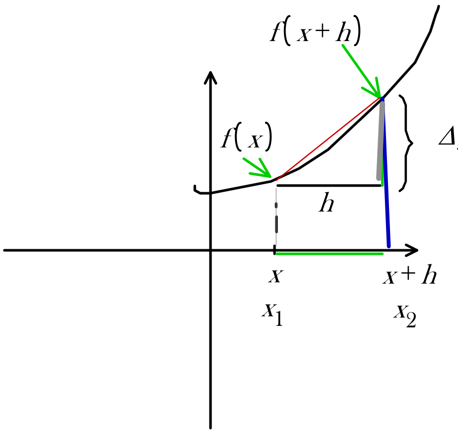
Δx "delta x"

doesn't mean $\Delta \cdot y$ or $\Delta \cdot x$

Δy is just one unit

using functoin notation:

$f(x)$, h, $f(x+h)$



$\Delta y = f(x+h)-f(x)$ (just another way of writing y_2-y_1)

relationship:

$$\frac{y_2-y_1}{x_2-x_1}=\frac{f(x+h)-f(x)}{\cancel{x}+h-\cancel{x}}=\frac{f(x+h)-f(x)}{h}=\frac{y_2-y_1}{x_2-x_1}$$

$\dfrac{f(x+h)-f(x)}{h}$ is called the difference quotient.

Find the differece quotient for $f(x)=3x$

$$\frac{f(x+h)-f(x)}{h}$$

$f(x+h)=3(x+h)$

$f(x)=3x$

$g(x)=x^2+4$

$g(2)=2^2+4$

compute the difference quotient $\dfrac{{\color{red}3}(x+h)-3x}{h}=\dfrac{\cancel{3x}+3h-\cancel{3x}}{h}=\dfrac{3\cancel{h}}{\cancel{h}}=3$ is the slope of the line $f(x)=3x$

(slope)

The meaning of 3 is that is tells you the slope of the line.

FOIL:

$$(x+4)^2=(x+4)(x+4)=x\cdot x+4x+4\cdot x+4\cdot 4=x^2+4x+4x+16=x^2+8x+16$$

$$(a+b)^2=(a+b)(a+b)=a\cdot a+a\cdot b+a\cdot b+b\cdot b=a^2+2ab+b^2$$

Find the difference quotient for $f(x)=x^2$

$f(x+h)=(x+h)^2$

$f(x)=x^2$

h

$$\frac{(x+h)^2-x^2}{h}=\frac{(x+h)(x+h)-x^2}{h}=\frac{\cancel{x^2}+xh+hx+h^2-\cancel{x^2}}{h}=\frac{2xh+h^2}{h}=\frac{\cancel{h}(2x+h)}{\cancel{h}}$$

$=2x+h$

the difference quotient for x^2 is the expression $2x+h$

2x+h will become very important in finding what is called the derivative.

this is the difference quotient

$f(x)=x^3$

$f(x+h)=(x+h)^3$

$g(x) = -3x - 4$

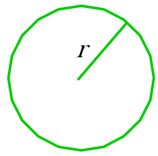
$g(\boxed{x+h}) = -3(\boxed{x+h}) - 4 = -3x - 3h - 4$ (nothing else here b/c terms are unlike)

$g(x+h) - g(x) = -3x - 3h - 4 - 1[-3x - 4] = -3x - \cancel{3h} - 4 + 3\cancel{x} + \cancel{4} = \cancel{-3h}$

$$\frac{g(x+h) - g(x)}{h} = \frac{-3h}{h} = -3$$

Different Ways to Represent Functions:
verbally (words), numerically (table of values)
visually (graph) , algebraically (formulas)

Area of a circle. Write a function for the area.

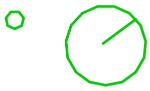


$Area = \pi r^2$

in function notation, this is written as

$A(r) = \pi r^2$ "A of r"

r is the independent variable. It's the one we can control.



$A(1) = \pi \cdot 1^2 = \pi \cdot 1 = \pi$

$A(4) = \pi \cdot 4^2 = \pi \cdot 16 = 16\pi$ (units^2, cm^2, in^2, ft^2)

domain for $A(r)$, r=0 there is no circle

$A(0) = \pi \cdot 0^2 = \pi \cdot 0 = 0$ (no circle)

domain: r>0 we get circles we can actually draw.

$r = -4$? (in this world it doesn't make sense)

tive radius, the domain is $\{r \mid r > 0\} = (0, \infty)$, and the range is also $(0, \infty)$.

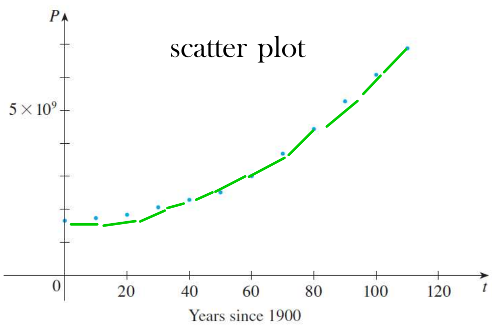
B. We are given a description of the function in words: $P(t)$ is the human population of the world at time t. Let's measure t so that $t = 0$ corresponds to the year 1900. The table of values of world population provides a convenient representation of this function. If we plot these values, we get the graph (called a *scatter plot*) in Figure 9. It too is a useful representation; the graph allows us to absorb all the data at once. What about a formula? Of course, it's impossible to devise an explicit formula that gives the exact human population $P(t)$ at any time t. But it is possible to find an expression for a function that *approximates* $P(t)$. In fact, using methods explained in Section 1.2, we obtain the approximation

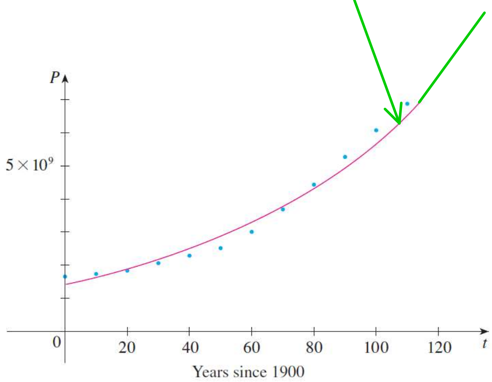
t (years since 1900)	Population (millions)
0	1650
10	1750
20	1860
30	2070
40	2300
50	2560
60	3040
70	3710
80	4450
90	5280
100	6080
110	6870

$$P(t) \approx f(t) = \boxed{(1.43653 \times 10^9) \cdot (1.01395)^t}$$

Then this function we have generated can be potentially used to make predictions about the future.

scatter plot





A function defined by a table of values is called a *tabular* function.

w (ounces)	$C(w)$ (dollars)
$0 < w \leqslant 1$	0.98
$1 < w \leqslant 2$	1.19
$2 < w \leqslant 3$	1.40
$3 < w \leqslant 4$	1.61
$4 < w \leqslant 5$	1.82
⋮	

w is the independent variable. It's the one whose values we can control . C(w) represent the corresponding cost in dollars.

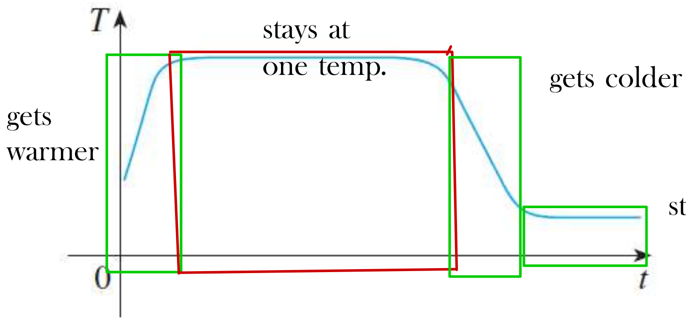
$C(0.5) = 0.98$ since $0 < 0.5 \le 1$

$C(4.9999) = 1.82$ since $4 < 4.99999 \le 5$

to perform the operations of calculus on such a function.

C. Again the function is described in words: Let $C(w)$ be the cost of mailing a large envelope with weight w. The rule that the US Postal Service used as of 2015 is as follows: The cost is 98 cents for up to 1 oz, plus 21 cents for each additional ounce (or less) up to 13 oz. The table of values shown in the margin is the most convenient representation for this function, though it is possible to sketch a graph (see Example 10).

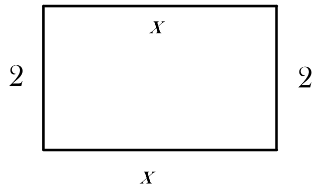
Graphical Display of a Function:



stays at that colder temperature

EXAMPLE 4 When you turn on a hot-water faucet, the temperature T of the water depends on how long the water has been running. Draw a rough graph of T as a function of the time t that has elapsed since the faucet was turned on.

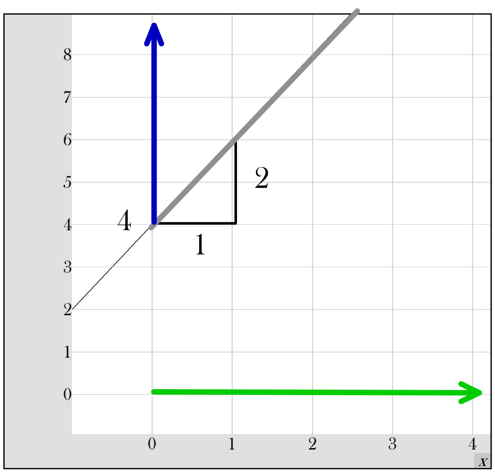
Let's build some functions:



Build a function to represent the perimeter:
add up all the sides: $x + 2 + x + 2 = 2x + 4$
Write this in function notation: $P(x) = 2x + 4$
$P(1) = 2 \cdot 1 + 4 = 2 + 4 = 6$ units(like inches, cm, meters, miles)
$P(0) = 2 \cdot 0 + 4 = 4$

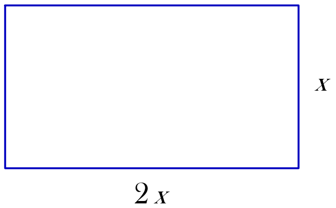
We can graph this function:

Domain:
$x = -3$?
$x = -3$ can't work since
it's a side length.

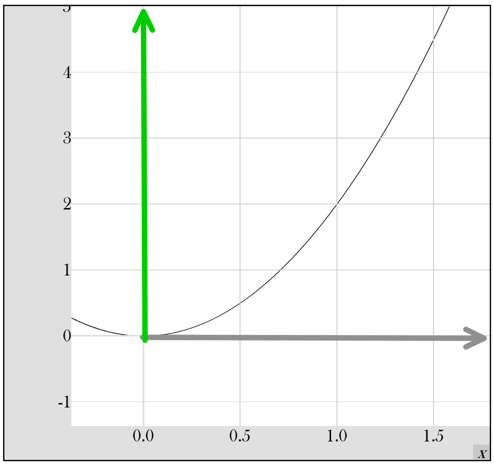


range:
What's the lowest y coordainte for the domain?
$[4, \infty)$

domain
$[0, \infty)$



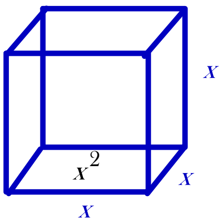
Area Function:
(area is flat space contained by a figure)
$A(x)$, "area as a function of x, x is the independent variable".
$A(x) = 2x \cdot x = 2x^2$



Domain: (allowed x values)
All real numbers, can x be -3?
[0, ∞)
$A(0) = 2 \cdot 0^2 = 2 \cdot 0 = 0$

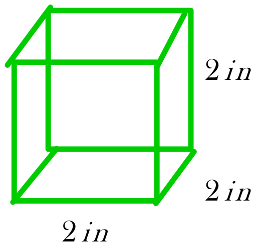
Range:
[0, ∞)
$A(3) = 2 \cdot 3^2 = 2 \cdot 9 = 18 \text{ units}^2$

examples of area units:
units^2: cm^2, in^2, mi^2



Create a function that represents the volume of the cube:
$V(x)$, "volume as a function of x".
$V(x) = x \cdot x \cdot x = x^1\, x^1\, x^1 = x^{1+1+1} = x^3$
What are the units of volume ? (1,2,3)?

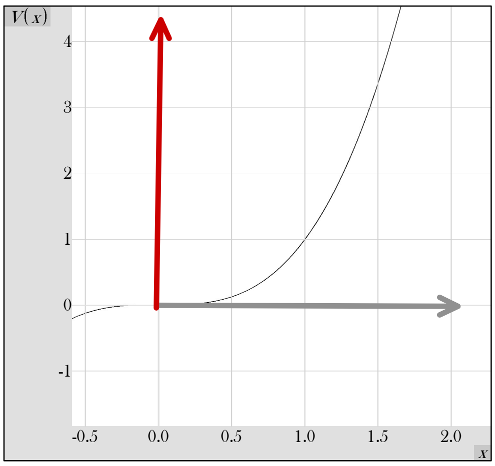
Volume is 3D space occupied by the shape.



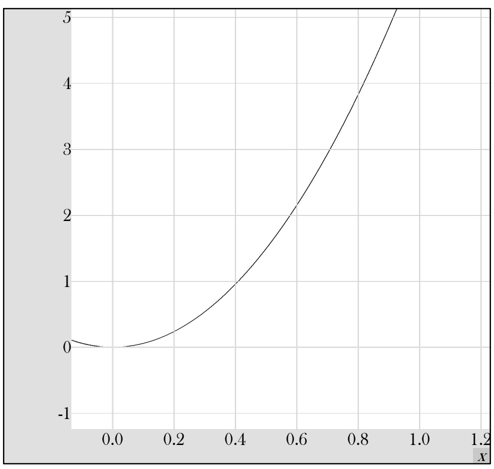
$V = 2\,in \cdot 2\,in \cdot 2\,in = 8\,in^3$

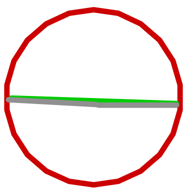
What's the domain?
Volume should be positive or 0.
domain: [0, ∞)
$range$: [0, ∞)

$V(10) = 10^3 = 1000 \text{ units}^3$

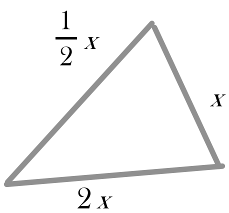


Creating a Function for the Surface area of a cube:
1. How many sides does a cube have? 6
2. What's the area of one face? x^2
3 What's the total area: $6 \cdot x^2$ = surface area
4. $S(x) = 6x^2$





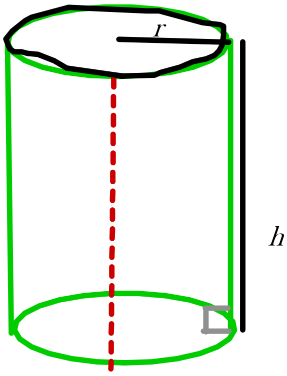
circle:
A(r)= πr^2
$C(r) = 2\pi r$
$C(D) = \pi \cdot D$, because $2r = D$



$P = a + b + c$

$P(x) = x + 2x + \frac{1}{2}x = 3x + \frac{1}{2}x$
$= \frac{6}{2}x + \frac{1}{2}x$
$= \frac{7}{2}x$ perimeter

as a function of x.

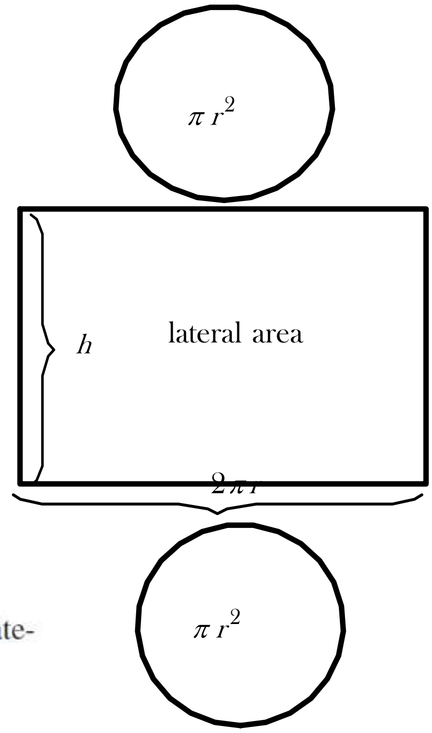


This is called a circular cylinder.
What's the volume of this shape?

Volume, $Volume = \pi r^2 h$

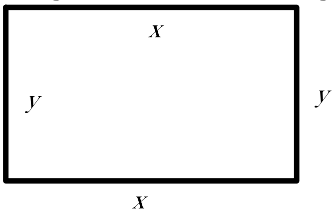
Surface Area= $\pi r^2 + \pi r^2 + h \cdot 2\pi r = 2\pi r^2 + 2\pi r h$

Surface Area:



EXAMPLE 5 A rectangular storage container with an open top has a volume of 10 m^3. The length of its base is twice its width. Material for the base costs \$10 per square meter; material for the sides costs \$6 per square meter. Express the cost of materials as a function of the width of the base.

Imagine we have a rectangle give by x as one side and y as the other side.



$Area = xy$

Perimeter= $2x + 2y$

Imagine a rectangle whose Area is 100 units^2 . Write a function for the perimeter in terms of x:

1. $xy = 100$ somehow use this equation

Perimeter= $2x + 2y$ 2.

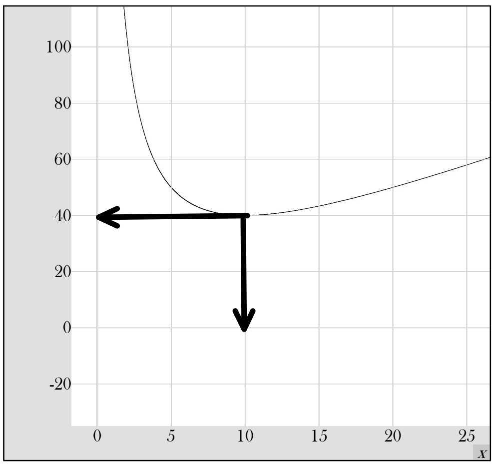
solve 1. for y: $\frac{xy}{x} = \frac{100}{x}$

$$y = \frac{100}{x}$$

replace y in 2. above with 100/x: Perimeter= $2x + 2\left(\frac{100}{x}\right)$ this has only x in it

$P(x) = 2x + \frac{200}{x}$ (function for perimeter only in terms of x)

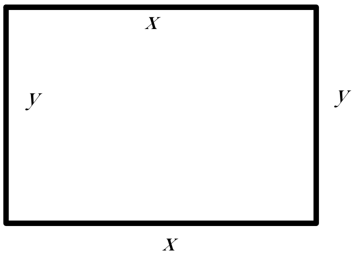
$P(0) = 2 \cdot 0 + \frac{200}{0}$ Is this allowed? Not allowed because division by 0 is not defined.



What's the domain? $(0, \infty)$ Notice here 0 is not included because the area has a constraint, which is that 100 units^2 should always be the value of the area.

What's the minimum value of the perimeter? The graph suggests that 40 is the lowest value of the perimeter and it occurs at x=10.

Imagine a rectangle whose perimeter is always to be 300 units.
Find a function for the area in terms of x only.



1 $2x+2y=300$ we have to make use of this equation

2 $area=x\cdot y \leftarrow$ has both x and y, we want only x

How do we get a function for the area in terms of x only?

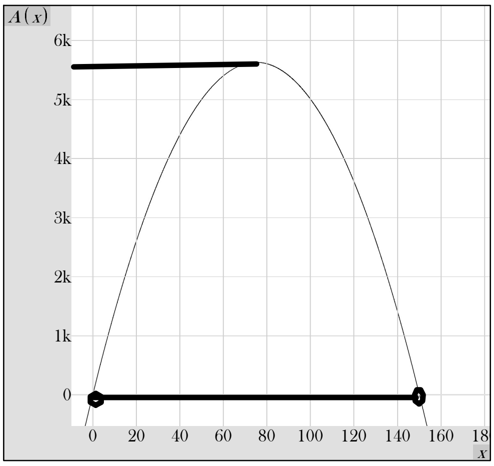
in 1 above, solve for y:

$2x+2y=300$

$2y=300-2x$

$y=\frac{300-2x}{2}$ $\qquad A(x)=x\left(\frac{300-2x}{2}\right)$

SO now we have a function of x only. In other words, x controls the area.



This graph represents the area for any x we input.
Can area be negative?
like $-50\,in^2$
We don't allow area of 0.
What's the domain?
(0,150)
What's the range?
(0,5500) (possible values) of the area range from 0 to 5500.

Imagine we have a cylinder with a volume of 100 inches^3.

$\pi r^2 h=100$ (For a cylinder the volume is V= $\pi r^2 h$)

Find a function in terms h for the surface area of the cylinder.

surface area= $2\pi r^2+2\pi rh \leftarrow$ this contains two variables, r and h.

S(h), "S of h"

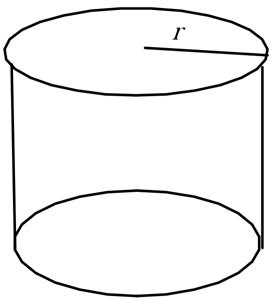
S(r), "S of r", this would be the surface area in terms of r only.

$\pi r^2 h=100$ $\qquad r\cdot\frac{1}{r^2}=\cancel{r}\cdot\frac{1}{r\cdot\cancel{r}}=\frac{1}{r}$

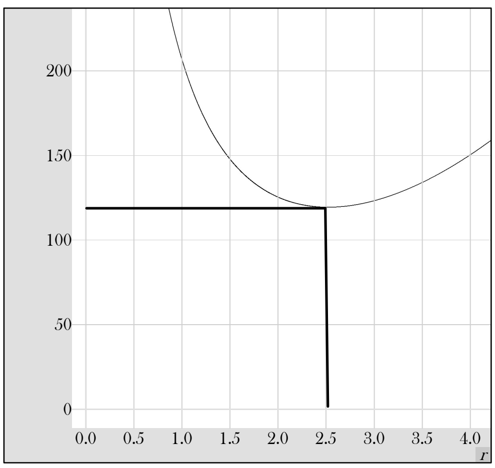
solve this for h: $h=\frac{100}{\pi r^2}$

$S(r)=2\pi r^2+2\pi \cancel{r}\left(\frac{100}{\cancel{\pi r^2}}\right)=2\pi r^2+\frac{2\cdot 100}{r}=2\pi r^2+\frac{200}{r}$

This shows the surface area in terms of r, which is the radius of the cylinder.



Here, the volume has a constraint. In others, no matter the choice of r and h, the volume has to be always equal to 100 inches^3.



What's the domain of S(r)?

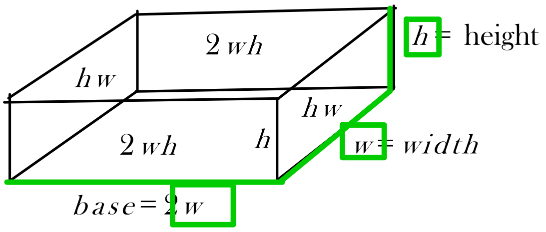
$S(r)=2\pi r^2+\frac{200}{r}$

$(0,\infty)$

$S(0)=2\pi 0^2+\frac{200}{0}=0+\frac{200}{0}$ (division by 0)

What's a question we can ask about the surface area based on the graph?
(once I was offered a job doing this)
What's the minimum surface area?
A radius of about 2.5 units gives a minimum surface area.
IN calculus, you do the same things, but find the minimum differently.

EXAMPLE 5 A rectangular storage container with an open top has a volume of 10 m³. The length of its base is twice its width. Material for the base costs \$10 per square meter; material for the sides costs \$6 per square meter. Express the cost of materials as a function of the width of the base.

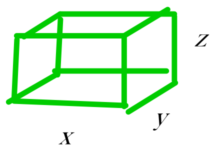


1. no top
2. how many sides in all? 5 4 verticals, 1 bottom
3. What's the area of the bottom piece? $2w\cdot w = 2w^2$
4. What' s the cost for the bottom piece? $10\cdot 2w^2 = 20w^2$
5. How many areas are there that are h · w? 2
6. The left and right sides have a total area of $2hw$
10. What' s the cost for the left and right sides? 6 · 2hw= $12hw$
11. What's the cost of the front and back faces? 6 · (2 · 2wh)

$24wh$

What's the total cost for all the faces ? $20w^2 + 12hw + 24wh = 20w^2 + 36\boxed{hw}$

cost is in terms of w and h



$V = xyz$

There is one equation here to write: Volume = $10\,m^3$

$$2w\cdot w\cdot h = 10$$

$$2w^2\cdot h = 10 \rightarrow h = \frac{10}{2w^2} = \frac{5}{w^2}$$

Express the cost of materials as a function of the w(width) only? $C(w) = 20w^2 + 36\left(\frac{5}{w^2}\right)w$

$$= 20w^2 + 36\cdot\frac{5}{w}$$

$$= 20w^2 + \frac{180}{w}$$

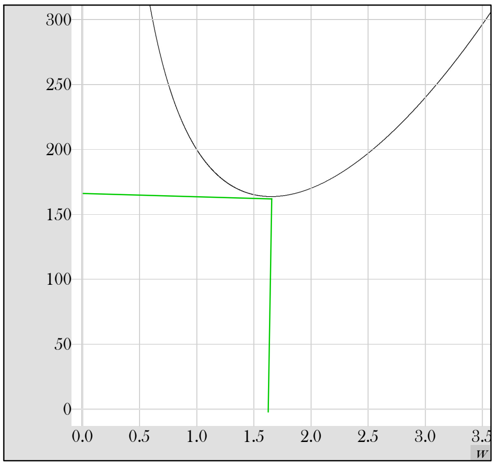
$36\cdot 5 = 150 + 30$

$(30+6)\cdot 5 = 180$

What's the domain of $C(W)$? $(0, \infty)$

$C(0) = 20\cdot 0^2 + \frac{180}{0} = \frac{180}{0} =$ undefined

width is 0.



This graph represents the cost.
What's a question we might ask about it?
What's the minimum cost?

Seems like about 1.6 as the value of w gives a minimum cost for the box.

Now let's go back to finding domains of just some random functions:

$f(x) = \sqrt{x+1}$

$f(-4) = \sqrt{-4+1} = \sqrt{-3}$ issue is that this is not defined here. (i)

$f(0) = \sqrt{0+1} = \sqrt{\boxed{1}} = 1$

$f(3) = \sqrt{3+1} = \sqrt{\boxed{4}} = 2$

$f(8) = \sqrt{8+1} = \sqrt{\boxed{9}} = 3$

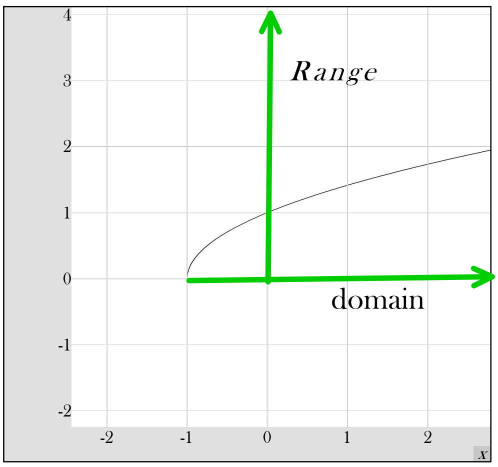
The values whose root is to be taken have to 0 or more ultimately.

$x + 1 \geq 0$

$x + 1 - 1 \geq 0 - 1$

$x \geq -1$ (inequality version of the domain)

Make this into interval notation: [-1, ∞)



Review of some factoring methods:

$x^2 - x = x\cdot x - x\cdot \boxed{1} = x(x-1)$ To factor means to write as a product: a · b

$2x^2 - 4x = 2\cdot x\cdot x - 2\cdot 2\cdot x = 2x(x-2)$

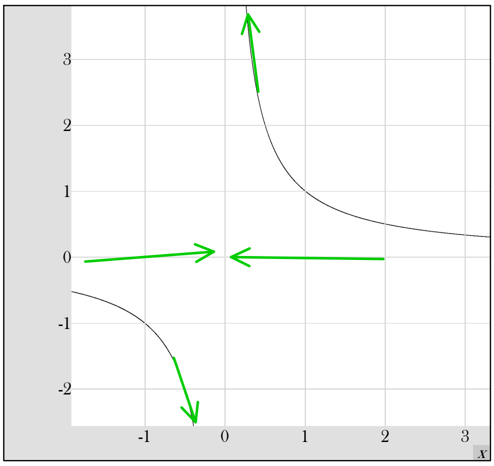
what's common to the two terms

$x^2 - a^2$ (difference of squares pattern)

$(x+1)(x-1) = \text{FOIL} = x^2 - 1x + 1x - 1 = x^2 - 1$

$x^2 - 4 = x^2 - 2^2 = (x-2)(x+2)$ factored form of the expression.

Big idea: we do not divide by 0.



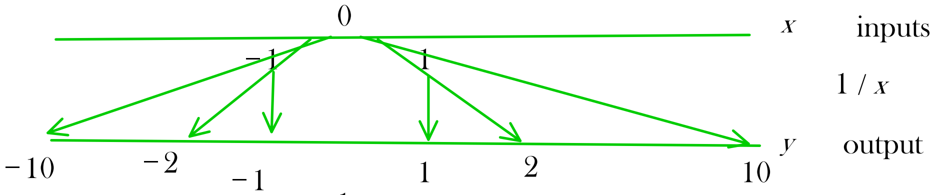
$f(x) = \frac{1}{x}$

$f(1) = \frac{1}{1} = 1$

$f(-1) = \frac{1}{-1} = -1$

0.5

$f(0) = \frac{1}{0}$ there is no value we can assign to this.



$\frac{1}{0}$ = undefined

$\frac{1}{0.5} = 2, \frac{1}{-0.5} = -2$

FOIL

$(x+a)(x+b) = x\cdot x + b\cdot x + a\cdot x + a\cdot b = x^2 + bx + ax + ab = x^2 + (a+b)x + ab$

Factoring

$x^2 + 7x + 10 = x^2 + (5+2)x + 5\cdot 2 = (x+5)(x+2)$

$x^2 + 7x + 12 = x^2 + (3+4)x + 3\cdot 4 = (x+3)(x+4)$

$x^2 + 8x + 15 = (x+3)(x+5)$

two numbers that multiply to 15 and add up to 8:

Find the domain of the function given by : $f(x) = \frac{1}{x^2 - x\cdot 1} = \frac{1}{x\cdot x - x\cdot 1} = \frac{1}{x(x-1)}$, bottom is factored now

Exclude from the domain(set of allowed inputs) the numbers that lead to divsion by 0.

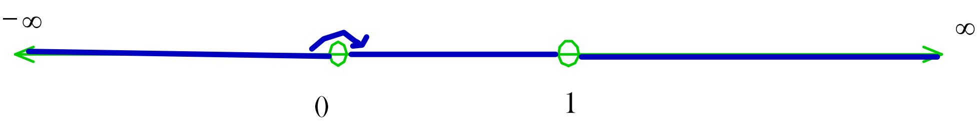
$f(0) = \frac{1}{0(0-1)} = \frac{1}{0(0)} = \frac{1}{0}$, so 0 is excluded

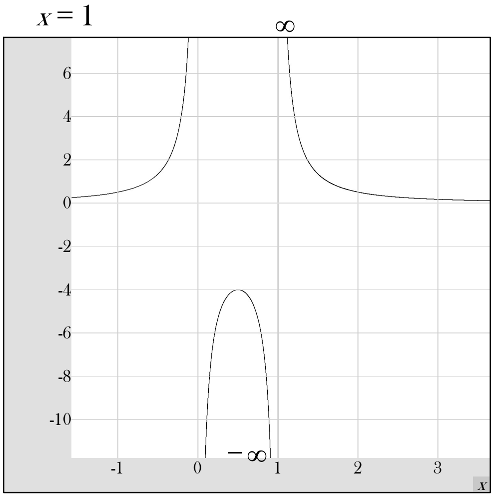
$f(1) = \frac{1}{1(1-1)} = \frac{1}{1\cdot 0} = \frac{1}{0}$, undefined when x=1 goes in.

$x(x-1) = 0$

$x = 0,\ x - 1 = 0$

$x = 1$





mathematical form of this: $(-\infty, 0) \cup (0,1) \cup (1, \infty)$

read from left to right:

we want the interval notation of the domain

$f(0.99999) = \frac{1}{0.99999^2 - 0.99999} = -100001$

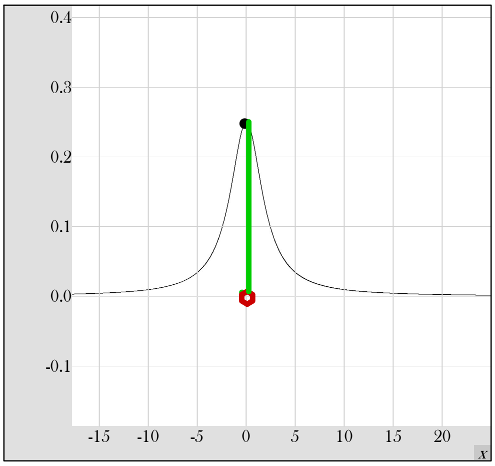
$f(1.0001) = \frac{1}{1.0001^2 - 1.0001} = 9999$

set builder notation: $\{x \mid x \neq 0, x \neq 1\}$

$g(x) = \dfrac{1}{x^2+4}$

Is the expression in the bottom ever equal to 0?
notice :4 is always positive, x^2 is always 0 or positive, so x^2 +4 is always positive.

For the function g(x) = $\dfrac{1}{x^2+4}$ there is no domain restriction.



Since x^2 +4>0, we're able to get a number out of $\dfrac{1}{x^2+4}$ for every input value of x.

domain would be written as $(-\infty, \infty)$
in set builder form: $\{x \mid -\infty < x < \infty\}$

We write a single symbol for infinity, like ∞, but it's not a number. It's a concept.

Range of this graph: (0, 0.25]

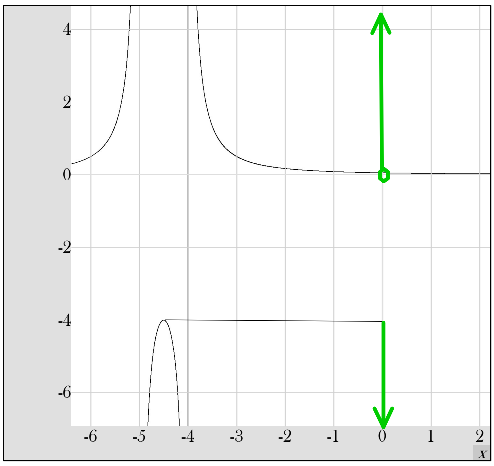
$x = 100000 : \dfrac{1}{100000^2+4} \neq 0$

(input, output), (x,f(x))=(x, y)

$h(x) = \dfrac{1}{x^2+9x+20} = \dfrac{1}{(x+5)(x+4)}$

Find the domain:
$x+5 \neq 0$ $\quad$ $x+4 \neq 0$
$x \neq -5$ $\quad$ $x \neq -4$

Express the domain in interval form:
$(-\infty, -5) \cup (-5, -4) \cup (-4, \infty)$



Range:

What's the first symbol when writing the range?
$(-\infty, -4\,] \cup\ (0, \infty\ )$

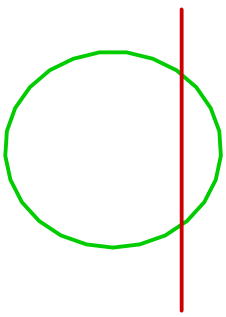
h(x) never reaches the value 0.

$h(x = 100000) = \dfrac{1}{100000^2+9(100000)+20} \neq 0$

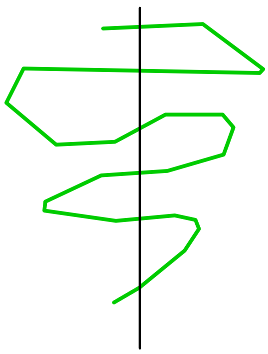
the y coordinates on the graph are never equal to 0.

The vertical line test helps to check whether a graph is a function or not. When a vertical crosses a graph only once, the graph is that of a function.

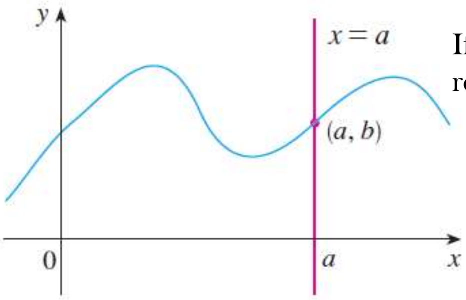
this is a function



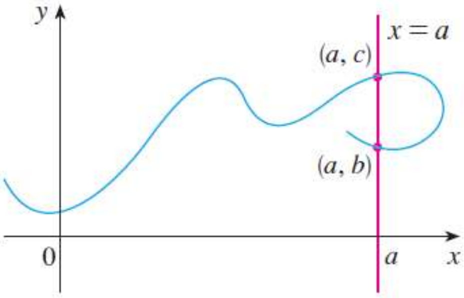
this is not a function



also not a function graph

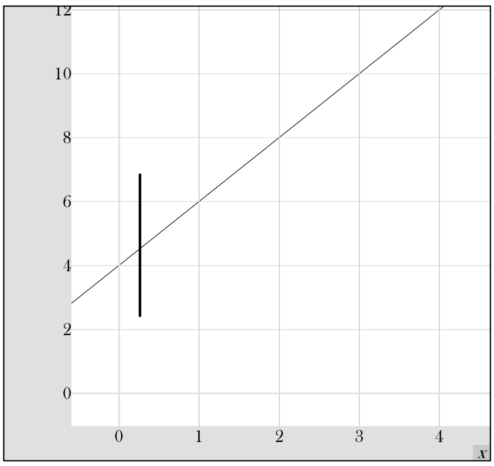


If we wrote down all the points, a would occur only once.a would not repeat. (a,b)



This is not a function because (a,c) and (a,b) both have a as the first coordinate. In other words, a repeats in two different points. The graph fails the vertical line test.

Is y(x)=2x+4 a function or not?
slope of 2, y intercept of 4



So it's a function b/c it passes the vertical line test.

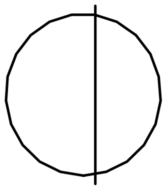
Is this relationship a function or not?

$y(x) = \pm\sqrt{x^2-4}$

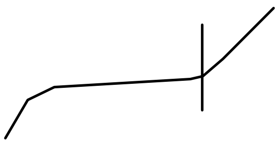
$y(5) = \pm\sqrt{5^2-4} = \pm\sqrt{25-4} = \pm\sqrt{21}$

How many output(s) are there? 2, $+\sqrt{21}$, $-\sqrt{21}$
So we get two points from a single input.
$(5, \sqrt{21}), (5, -\sqrt{21})$ This means that $y(x) = \pm\sqrt{x^2-4}$ is not a function because the x-coordinate repeats for every input.

A graph/set of points is called a relation. When the relation has the condition that each x is unique, the relation is called a function.

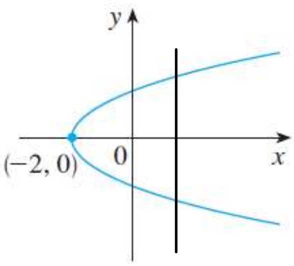


relation
but not a function



this graph is a relation and
it's also a function.

dent variable) and the parabola



14 (a) $x = y^2 - 2$

this relation is not a function

How can we make this relation into a function?

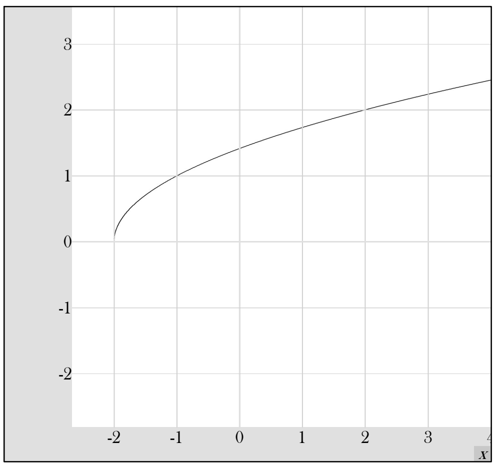
$x = y^2 - 2$

solve for y:

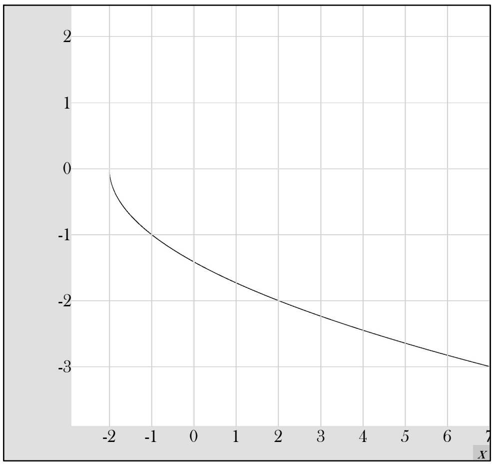
$x+2=y^2$

$\pm\sqrt{x+2} = \sqrt{y^2}$

$\pm\sqrt{x+2} = y$



$+\sqrt{x+2}$ is a function



$-\sqrt{x+2} = y$ also represents a function.

Piecewise Defined Function:

$$f(x) = \begin{cases} 1-x, & x \le -1 \\ x^2, & x > 1 \end{cases} \qquad f(2) = \begin{cases} 1-2, & 2 \le -1 \\ \boxed{2^2}, & \boxed{2 > 1} \end{cases} \qquad f(2) = 4$$

output

$(2, 4)$

$$g(t) = \begin{cases} t^2+1, & t \le 0 \\ 2t, & t > 0 \end{cases} \qquad g(-1) = \begin{cases} \boxed{(-1)^2+1}, & \boxed{-1 \le 0} \\ 2(-1), & -1 > 0 \end{cases}$$

$g(-1) = (-1)^2 + 1 = 1 + 1 = 2$

as a point, this is $(-1, 2)$

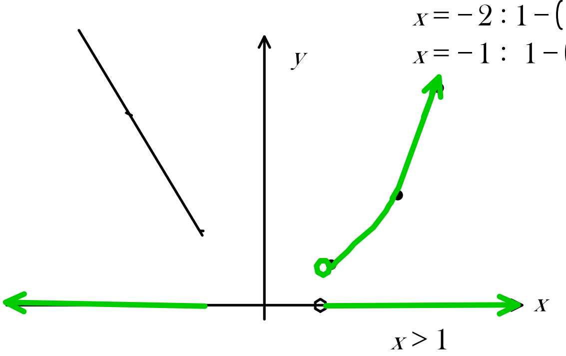
$$f(x) = \begin{cases} 1-x, & x \le -1 \\ x^2, & x > 1 \end{cases}$$

Focus on each piece independently:

$1-x,\ x \le -1$

$x = -2 : 1-(-2) = 3 \Rightarrow (-2, 3)$

$x = -1 :\ 1-(-1) = 2 \Rightarrow (-1, 2)$



$x^2,\ x > 1$

$x = 3 : 3^2 = 9$

$(3, 0)$

$x = 2 : 2^2 = 4$

$(2, 4)$

$x = 1.001 : (1.001)^2 = 1.002$

$(1.001, 1.002)$

$$g(t) = \begin{cases} 2t, & t \ge 0 \\ \boxed{-t, \quad t < 0} \end{cases}$$

$y = 2t,\ t \ge 0$

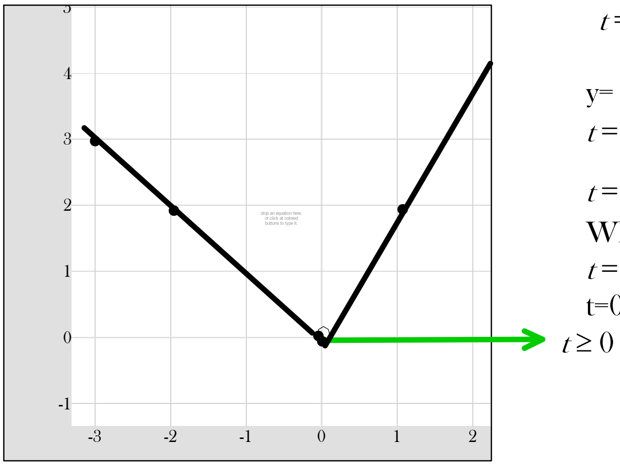
$t = 0 : 2(0) = 0 \Rightarrow (t = 0, y = 0)$

$t = 1 :\ 2(1) = 2 \Rightarrow (t = 1, y = 2)$

logical comparison operators

$<, >, \le, \ge$

$2 < 3$, returns a true or false.



$y = -t, \quad t < 0$

$t = -2 < 0$ is true: $-(-2) = 2 \Rightarrow (t = -2, y = 2)$

$t = -3 < 0$ is true: $-(-3) = 3 \Rightarrow (t = -3, y = 3)$

What's a value close to 0 but to the left of it?

$t = -0.1 < 0$ so it's true: $-(-0.1) = 0.1 \Rightarrow (-0.1, 0.1)$

t=0 is not part of the domain for y=-t 0 <0 is false.

The absolute value function:

$|x| = \begin{cases} x, x \ge 0 \\ -x\ , x < 0 \end{cases}$ official defintion of the absolute value function.

$|3| = \begin{cases} \boxed{3}, \boxed{3 \ge 0} \\ -3, \quad 3 < 0 \end{cases}$ $\qquad |3| = 3$

$|-3| = \begin{cases} -3\ , -3 \ge 0 \\ -(-3), \quad \boxed{\cancel{-3<0}} \end{cases}$ $\qquad |-3| = -(-3) = 3$

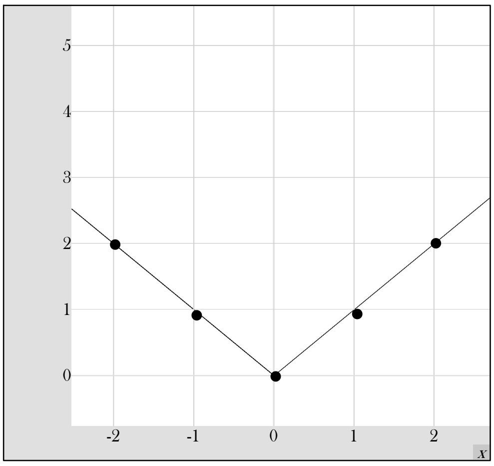
$O \qquad T$

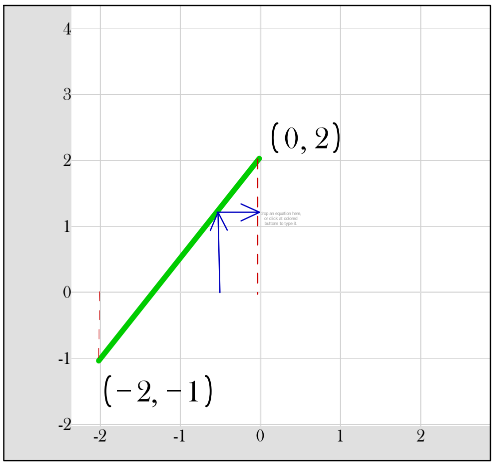
Absolute value outputs 0 or a positive number, regardless of the input.

$|0|=0, |10|=10, |-10|=10$

Grap of the $f(x) = |x|$

x	$y = \|x\|$	(x, y)
-2	$\|-2\| = 2$	$(-2, 2)$
-1	$\|-1\| = 1$	$(-1, 1)$
0	$\|0\|=0$	(0,0)
1	$\|1\| = 1$	$(1, 1)$
2	$\|2\| = 2$	$(2, 2)$





Represent the segment with a function.

f(x)=......., domain=......

domain: [-2,0]

range:[-1,2]

f(x)= some expression

slope of the segment:

$\frac{y_2 - y_1}{x_2 - x_1} = \frac{2-(-1)}{0-(-2)} = \frac{2+1}{+2} = \frac{3}{2}$

y=mx+b (this is the equation of a line with slope m and y intercept b)

$m = 3/2$, $b =$ y intercept = y coordinate where graph hits y axis=2

$y = \frac{3}{2}x + 2$, this is valid when $-2 \le x \le 0$

$\boxed{y(x) = \frac{3}{2}x + 2, \ -2 \le x \le 0}$

$y(-3), -2 \le -3 \le 0$ false

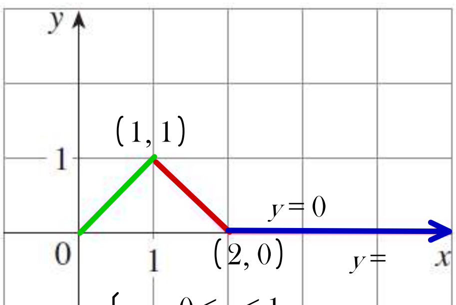
y(-3)=undefined=does not exist=DNE

$y(1), -2 \le 1 \le 0$ false

y(1)= undefined=does not exist=DNE

$y(\underline{-0.5}) = \frac{3}{2}(-0.5) + 2 = 1.25$

EXAMPLE 9 Find a formula for the function f graphed in Figure 17.



$f(x) = \begin{cases} x, \quad 0 \le x \le 1 \\ -1x + 2, 1 < x \le 2 \\ 0, x > 2 \end{cases}$

Make a piecewise defined function from the graph.

1. How many pieces to the function? 3

For the green segment, what's the domain? $0 \le x \le 1$

slope of the green segment: $\frac{1}{1} = 1$, What's the y intercept ? y=0

y=mx+b: $y = 1x + 0 \Rightarrow y = x$, valid when $0 \le x \le 1$ $\boxed{}$

for red piece: slope is $\frac{0-1}{2-1} = \frac{-1}{1} = -1$

domain of the red segment: $1 < \boxed{x \le 2}$

$y = mx + b$: $(1, 1)$ is a point on the red segment, $1 = -1(1) + b$

$1 = -1 + b \Rightarrow b = 2$

Symmetry makes sketching graphs go faster.
Here is a property that has been found to be useful:
$f(-x)=f(x)$

exponent review

$(ab)^p = a^p\, b^p$

$3^2 = 3\cdot 3 = 9$

$(2x)^3 = 2^3\, x^3 = 8\,x^3$

$3^2 \neq 3\cdot 2$

y coordinate when -x goes in is the same as the y coordinate when +x goes into the function.
$f(x)=x^2$

$f(1)=1,\ f(-1)=(-1)^2=1 \Leftarrow$ both outputs are the same, but the inputs are negated.

$f(2)=4,\ f(-2)=(-2)^2=4 \Leftarrow$ both outputs are the same, but the inputs are negated.

$f(x)=x^2$ (this gives a point of the form (x ,x^2), (input, output)

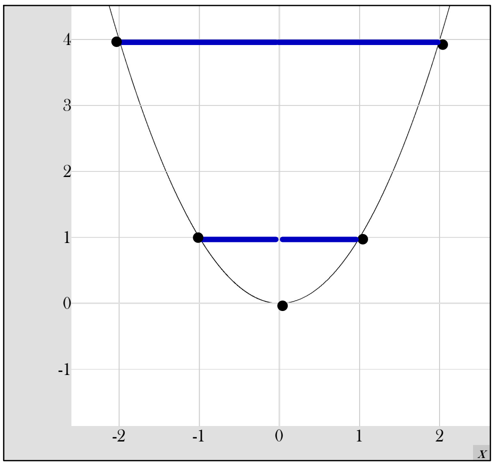
$f(-x)=(-x)^2=(-1\cdot x)^2=(-1)^2(x^2)=(-1)(-1)\,x^2=1\,x^2=x^2$ (-x, x^2) (input, output)

When a function has the property or characteristic that $f(-x)=f(x)$, we call the function an even function. This kind of function is said to have y-axis symmetry.

x	y	(x,y)
-2	4	$(-2,4)$
-1	1	$(-1,1)$
0	0	$(0,0)$
1	$1^2=1$	$(1,1)$
2	$2^2=4$	$(2,4)$

Since $f(x)=x^2$ is even, once we calculate f(1)=1, we know that f(-1)=1 *also*.

$f(x)=x^2,\ f(-x)=x^2$



Half on the right is the same as the half on the left.

Another Kind of Symmetry:
$f(x)=x^3$
$f(1)=1^3=1$ point (1, 1)
f(-1)=$(-1)^3=(-1)(-1)(-1)=1(-1)=-1$ point (-1,-1)

when 1 goes in , 1 comes out.
when -1 goes in , -1 comes out.
outputs are different in the sign.

$f(2)=2^3=8$ gives the point $(2,8)$
$f(-2)=(-2)^3=-8$ gives the point $(-2,-8)$

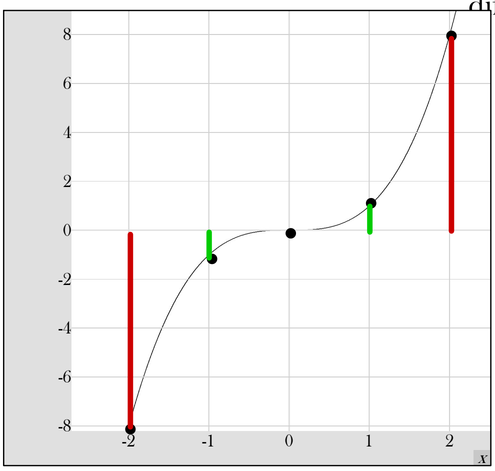
the output is the in absolute value but the signs are different.

So for $f(x)=x^3$, negating the input also negates the output.
More generally, $f(x)=x^3$ gives the point (x, x^3).
$f(-x)=-x^3$ so the point is $(-x,-x^3)$

signs of inputs differe and signs of outputs differ

$f(3)=27,\ f(-3)=-27$

$f(x=0)=0$



A function having the property that $f(-x)=-f(x)$ is called an odd function.
(our example: $f(-x)=-x^3=-f(x)$
It's said to have origin-symmetry.
In terms of points, this means that if a point (x,y) is on the graph, so is the point (-x,-y).

$\boxed{f(x)} = \boxed{x^2 + x^4}$ original definition

Both exponents are even.

$f(1) = 1^2 + 1^4 = 1 + 1 = 2$

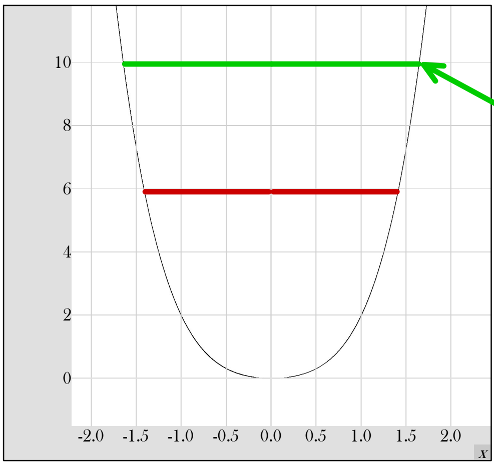
$f(-1) = (-1)^2 + (-1)^4 = 1 + 1 = 2$

$f(-x) = (-x)^2 + (-x)^4 = x^2 + x^4 = f(x)$ $\Rightarrow f(-x) = f(x)$

$(-x)^2 = (-x)(-x) = x^2$

$(-x)^4 = (-x)(-x)(-x)(-x) = x^4$

We can say that the function $f(x) = x^2 + x^4$ is even. In other words, it has y-axis symmetry.



We see the y-coordinates are the same, but the x values are negatives of each other.

When a function has terms with even exponents, the function is even b/c even exponents get rid of negatives.

$f(x) = x^6 + x^8 + x^{10}$, (6, 8, 10 each is an even number.)

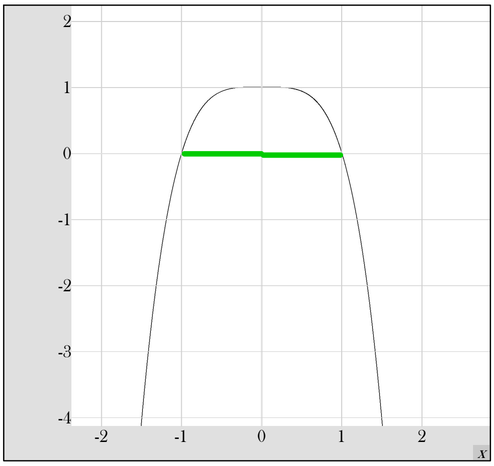
$f(-x) = x^6 + x^8 + x^{10} \Leftarrow$ Same expression produced by $f(-x)$ because each even exponent gets rids of negatives)

$f(x) = 1 - x^4$

Even or odd ?

$f(-x) = 1 - (-x)^4 = 1 - x^4 = f(x)$

$(-x)^4 = x^4$



$f(-x) = f(x)$

The graph and the check with -x indicate that f(x)= $1 - x^4$ is an even function.

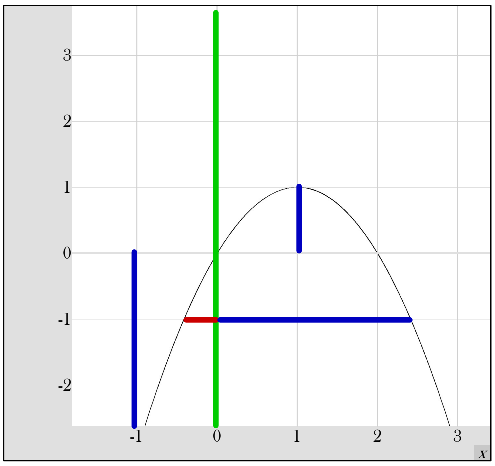
$h(x) = 2x^1 - x^2$

1. what are the exponents? 2,1

 1 is an odd number, 2 is an even number.

 So we have a mixture of different types of numbers in the exponents.

 Is this even? no because both exponents are not even.



Def. not symmetric about the y axis because the red segment is of different length compared to the horizontal blue segment.

Does it have origin symmetry?

There is no origin symmetry.

For example, f(1)=2(1)-1^2 = $2 - 1 = 1$ so the point is $(1, 1)$

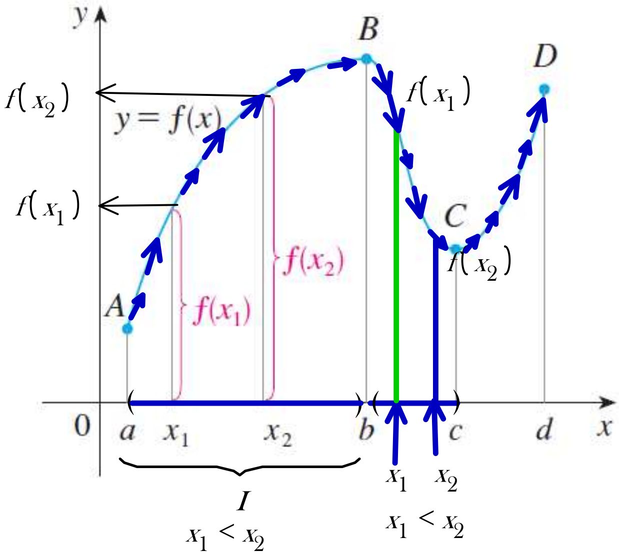
$f(-1) = 2(-1) - (-1)^2 = -2 - (1) = -3$ point is $(-1, -3)$

Notice, that when the input is negated, the output is not just -1, but -3, so this function is not odd. In other words, it doesn't have origin symmetry. This function has no symmetry.

Increasing and Decreasing Functions:

$f(x_2) > f(x_1)$

$f(x_1) < f(x_2)$



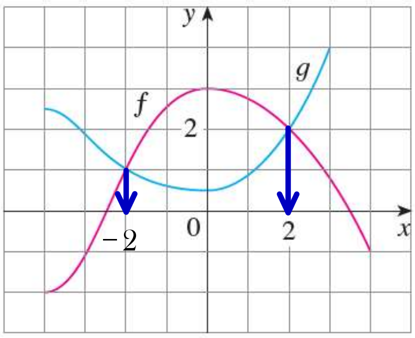
A function f is said to be increasing on an interval if $f(x_1) < f(x_2)$ whenever $x_1 < x_2$.

In the graph on the left, f increases on $(a, b) \cup (c, d)$

A function f is said to decreasing on an interval if $f(x_1) > f(x_2)$ whenever $x_1 < x_2$

In this picture, where does f decrease?
(b,c)

4. The graphs of f and g are given.



(a) State the values of $f(-4)$ and $g(3)$. $f(-4) = 2.5$, $g(3) = 4$

(b) For what values of x is $f(x) = g(x)$? $-2, 2$

evaluate f(x)=3x^2 -x+2

$f(2a) = 3(2a)^2 - (2a) + 2 = 3 \cdot 2^2\, a^2 - 2a + 2 = 3 \cdot 4\, a^2 - 2a + 2 = 12a^2 - 2a + 2$

domain review:

$f(t) = \sqrt[3]{2t-1}$

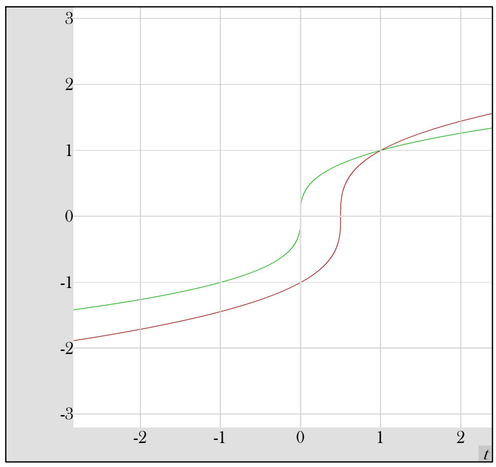
Find the domain.

remember that $\sqrt[3]{8} = 2$

$\sqrt[3]{-8} = -2$, $\sqrt[3]{0} = 0$

(cube roots can work on positive or negative values.)

$(-\infty, \infty)$ since cube roots can operate on any kind of number.



$\sqrt[3]{2t-1}$

DOmain of $\dfrac{2}{1-\dfrac{1}{x}}$

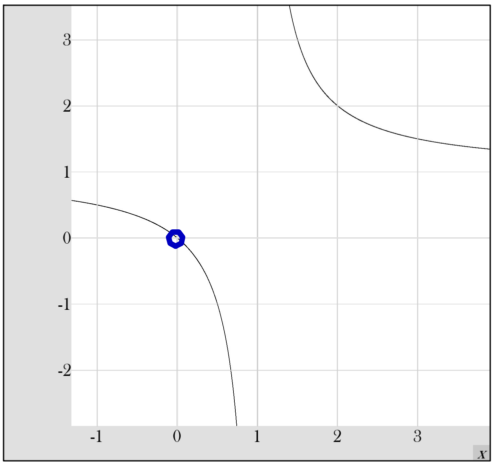
look only at $\dfrac{1}{x}$. What value cannot be plugged in? 0

$\dfrac{2}{1-\dfrac{1}{0}} \rightarrow$ undefined

$\dfrac{2}{0}$ cannot have this. *What's* the value of x that makes $1 - \dfrac{1}{x}$ equal to 0 ? x cannot be 1.

So the domain excludes 0 and 1 for two different reasons.

$$\frac{2}{1-\frac{1}{1}} = \frac{2}{1-1} = \frac{2}{0} = \text{undefined}$$



key to differential calculus (calc 1)
difference quotient:

$$\frac{f(x+h)-f(x)}{h}$$

$f(x)=3x^2$

$f(x+h)=3(x+h)^2$

$f(x)=3x^2$

h

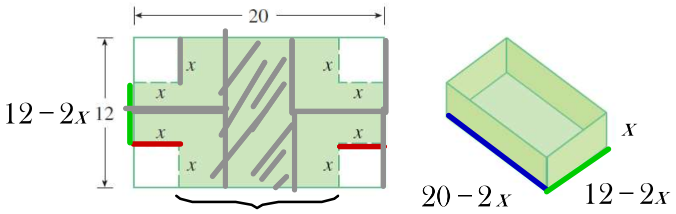
note: expand means multiply out

$$(x+h)^2=(x+h)(x+h)$$
$$=x^2+xh+hx+h^2$$
$$=x^2+2xh+h^2$$

$$\frac{3(x+h)^2-3x^2}{h}=\frac{3[x^2+2xh+h^2]-3x^2}{h}$$
$$=\frac{3x^2+6xh+3h^2-3x^2}{h}$$
$$=\frac{6xh+3h\cdot h}{h}$$
$$=\frac{h[6x+3h]}{h}$$

$=6x+3h$ left over. (in Calc, you take the limit as h goes to 0)

63. A box with an open top is to be constructed from a rectangular piece of cardboard with dimensions 12 in. by 20 in. by cutting out equal squares of side x at each corner and then folding up the sides as in the figure. Express the volume V of the box as a function of x.



$20-x-x$

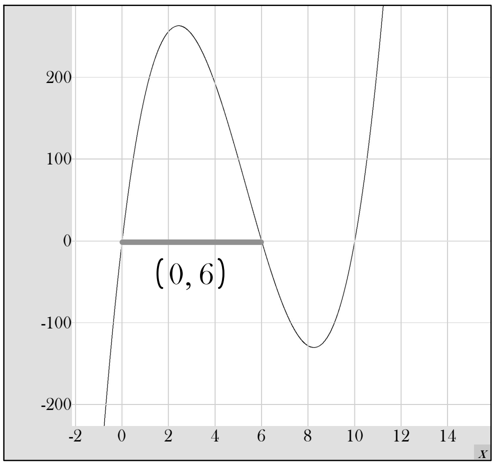
$=20-2x$

$V(0)=(20-2\cdot0)(12-2\cdot0)\cdot0=0$

$V(6)=(20-2\cdot6)(12-2\cdot6)\cdot6=8\cdot0\cdot6=0$

$V(x)$, V of x

Volume as a function of x.
$V(x)=(20-2x)(12-2x)x$



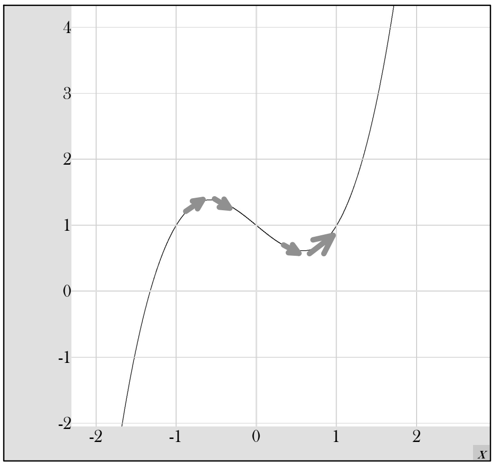
Which interval of x gives the domain?

As a reminder, a linear function is a function of the form $f(x)=mx+b$, $m=$ slope, x variable, b y intercept

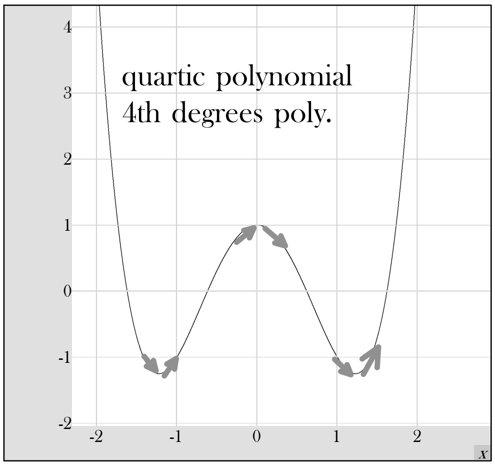
Polynomials:
$P(x)=a_n x^n+a_{n-1}x^{n-1}+\ldots+a_2x^2+a_1x^1+a_0$
$a_n, a_{n-1}, \ldots, a_2, a_1, a_0$ are called the coefficients.
Domains of polynomials are always pretty much $(-\infty,\infty)$, unless the polynomial represents a physical situtation.

$P(x)=ax^2+bx+c$ quadratic because of x^2

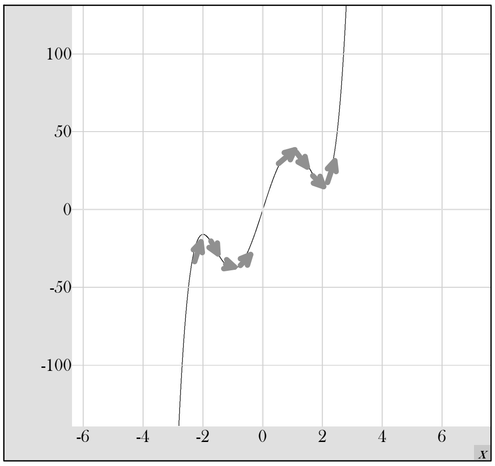
$P(x)=ax^3+bx^2+cx+d$ cubic polynomial b/c of x^3



3rd degree has 2
turning points



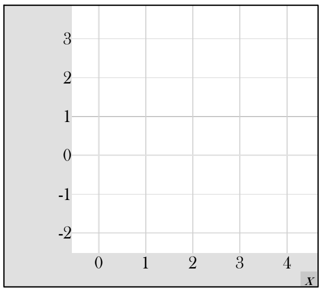
4th degree has
3 turning points
not 4



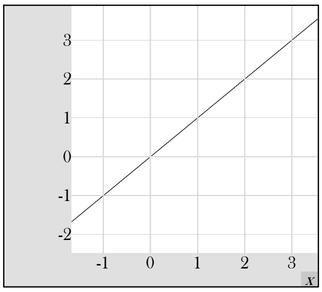
this is a 5th degree
(quntic polys.)
there 4 turns

If the degree is n, what's an expression for the number of turning points? n-1

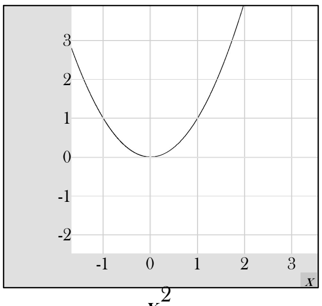
Power Functions: x^n
What happens as we change n?



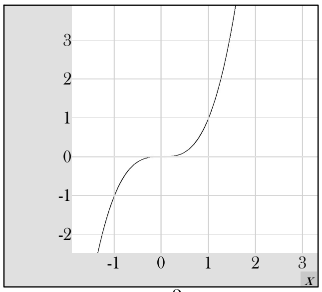
$x^0 = 1$



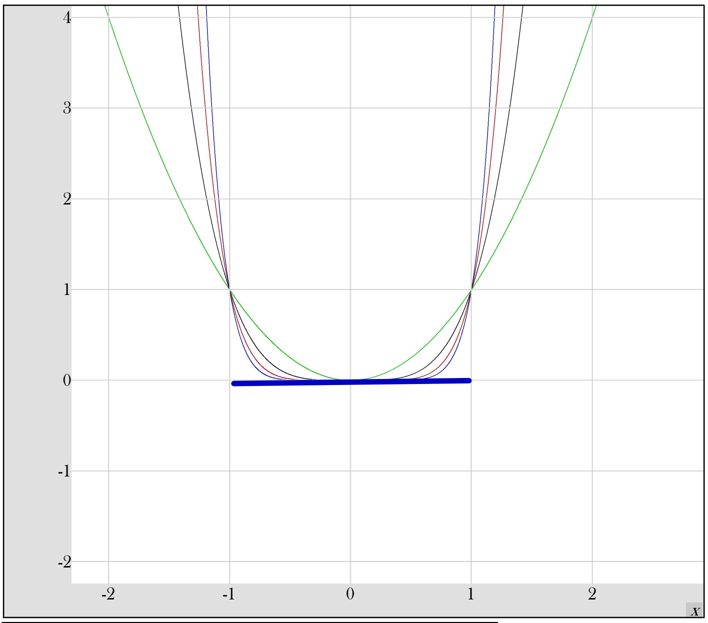
x^1



x^2

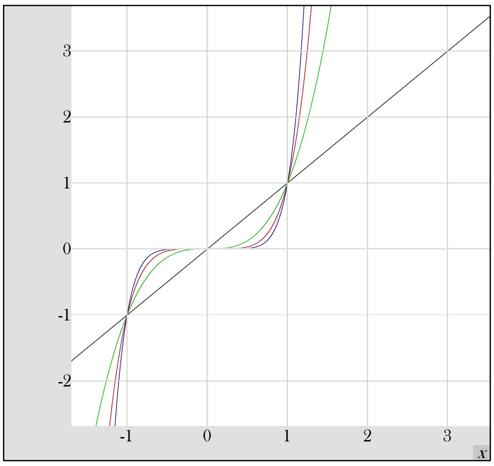


x^3



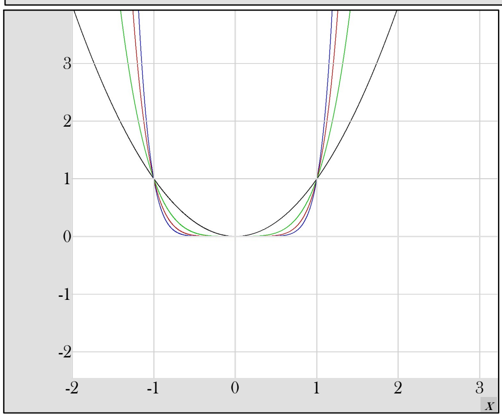
Which is closer to the x-axis from x=-1 to x=1?
$x^4 < x^2$ for $-1 < x < 1$
$x^8 < x^6 < x^4 < x^2$ for $-1 < x < 1$

As we increase the exponent through even values 2,4,6,8,10,12, and so on, the graph gets flatter and flatter around 0.
What's the value of all the functions at x=1 or-1?
$1^8 = 1^6 = 1^4 = 1^2 = 1$



x^1, x^3, x^5, x^7

are all going through the x-axis.
WHen the exponents are even, that doesn't happen



So for even expo nents, we don't cross the x-axis.

Rational Functions:

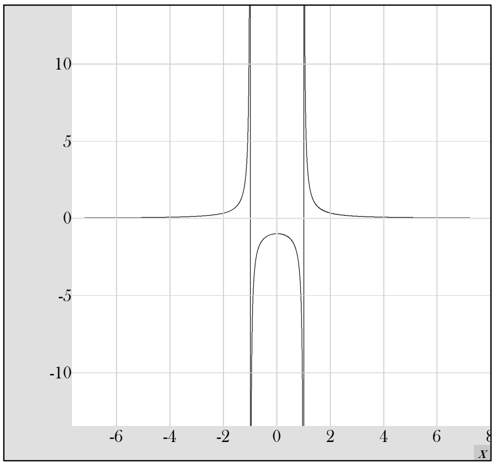
A rational function is a ratio of two polynomials.

$$f(x)=\frac{P(x)}{Q(x)}$$

Examples: $f(x)=\dfrac{x^2}{x^3-5}$, P(x)= x^2 , $Q(x)=x^3-5$

With rational functions, the big concern is the domain. Since we're dividing, we could potentially divide by 0,which is not allowed.

$f(x)=\dfrac{1}{x^2-1}=\dfrac{1}{(x-1)(x+1)}$, $x\neq 1$, $x\neq -1$ b/c *each* value would give division by 0.



The equations of the vertical asymptotes are x=1 and x=-1.

Algebraic Functions:

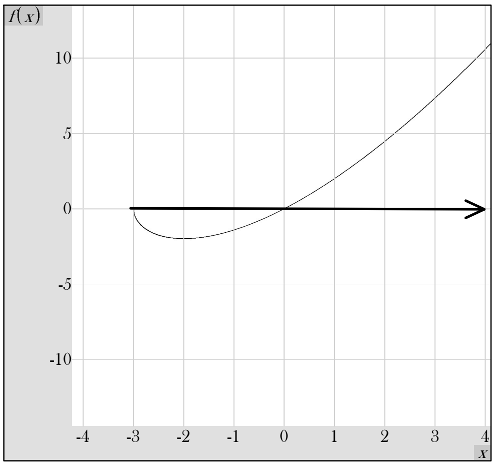
A function f is called algebraic if it can be constructed using algebraic operations(such as addition, subtraction, division, multiplication and taking roots)

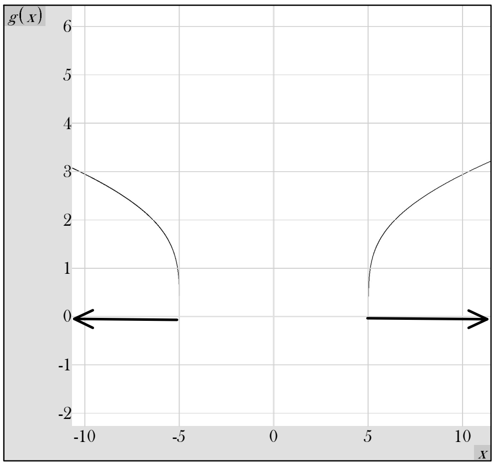
examples:

$f(x)=\sqrt{x^2+1}$ (*squaring*, adding, and taking a root)

g(x)= $\dfrac{x^4}{x+\sqrt{x}}$, (exponents x^4 =x· x· x· x,division, roots)

Algebraic can assume a variety of different forms. You will study how to sketch these more carefully in calculus.





$f(x)=x\sqrt{x+3}$

$x+3\geq 0$ since $\sqrt{-1}$ doesn't work

$x\geq -3$

$\sqrt[3]{8}$ What the value of a number we multiply by itself 3 times to get 8.

$\sqrt[4]{16}$ = what's the number we multiply by itself 4 times to get 16.

$\sqrt[\text{index}]{\text{radicand}}$

$g(x)=\sqrt[4]{x^2-25}$ radicand is x^2 -25, the root is the 4th root

$g(0)=\sqrt[4]{0^2-25}=\sqrt[4]{-25}$

cannot take the fourth root of a negative number.

$x\cdot x\cdot x\cdot x=-25$

$+\cdot+\cdot+\cdot+=+$

$(-)(-)(-)(-)=+$

$D=(-\infty,-5\,]\cup[5,\infty)$

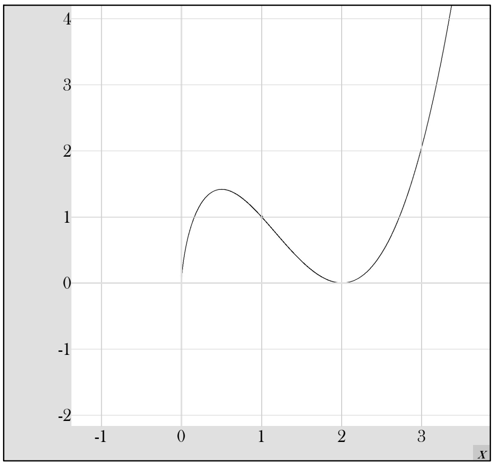
$g(5)=\sqrt[4]{5^2-25}=\sqrt[4]{25-25}$

$=\sqrt[4]{0}$

$=0$ b/c 0· 0· 0· 0=0

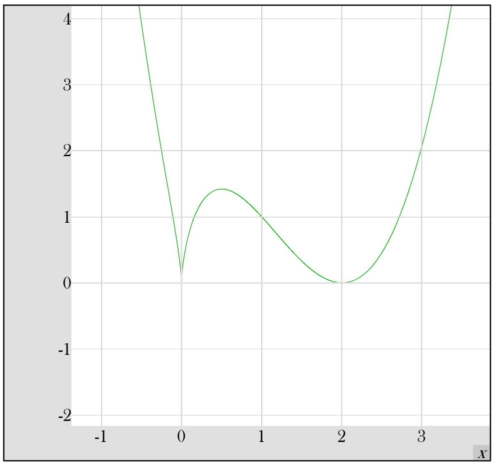
$f(x) = x^{2/3}(x-2)^2$

note: $x^{a/b} = \sqrt[b]{x^a} = (\sqrt[b]{x})^a$



$f(-1) = (-1)^{2/3}(1-2)^2 = (-1)^{0.67}$

sometimes computers/calculators are not equipped to handle certain rare cases.

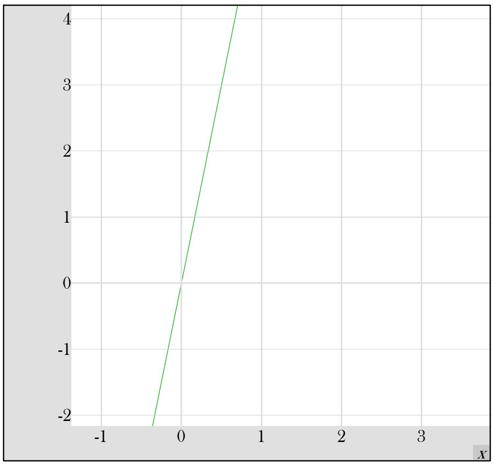


The two graphs overlap perfectly, so writing $x^{2/3}(x-2)^2$

$= \sqrt[3]{x^2}\,(x-2)^2$

$= (\sqrt[3]{x})^2(x-2)^2$

$2x + 4x = 6x$

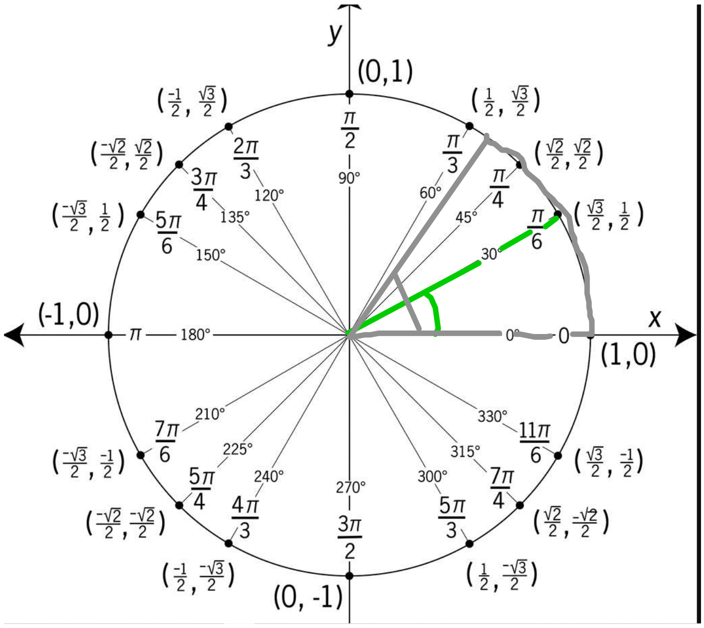


You will see a variety of applications in calculus, using different types of functions.

Trigonometric Functions:

Trig is the study of triangles and the relationship between the sides of the triangles.

Unwrapping the Unit Circle:



In a circle, there are 360 degrees.

1 degree is $\frac{1}{360}$ th of a circle.

1 radian is the angle made when we travel along the arc a distance equal to the radius.

For a circle, degrees are also used to measure off arc. For a circle of radius 1, the $C = 2\pi(1) = 2\pi$

So the circumference has the value 2π

However, we also know a circle is $360°$. So we can equate these to get $360° = 2\pi$ radians

$30°\cdot$ conversion factor

$30° \cdot \frac{\pi}{180°}$

$= 30 \cdot \frac{\pi}{30 \cdot 6}$

$= \frac{\pi}{6}$ radians

$45° \cdot \frac{\pi}{180°} = \cancel{45°} \cdot \frac{\pi}{4 \cdot \cancel{45°}}$

$= \frac{\pi}{4}$ radians

$360 = 2\pi$

$\frac{360}{360} = \frac{2\pi}{360}$

$1 = \frac{\pi}{180}$ bottom has 180 degrees, top represents π radians

$360 = 2\pi$

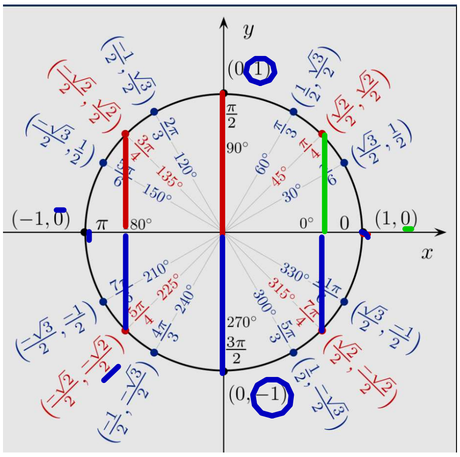
$\frac{360}{2\pi} = 1$

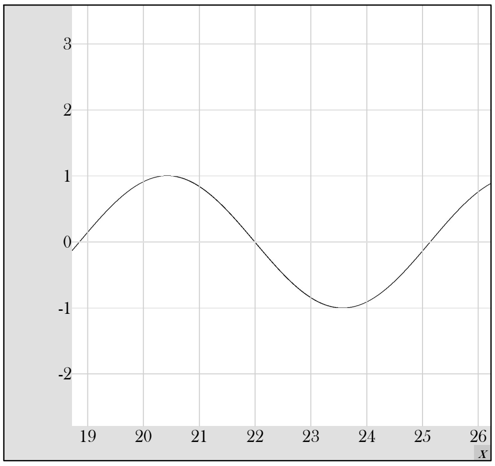
$\frac{180}{\pi} = 1$

$\cancel{\pi}$ ~~radians~~ $\cdot \frac{180°}{\cancel{\pi}\ \text{~~radians~~}} = 180°$

$300° = \frac{5\pi}{3}$ radians

The sine function is the y-coordinate of each point on the unit circle.





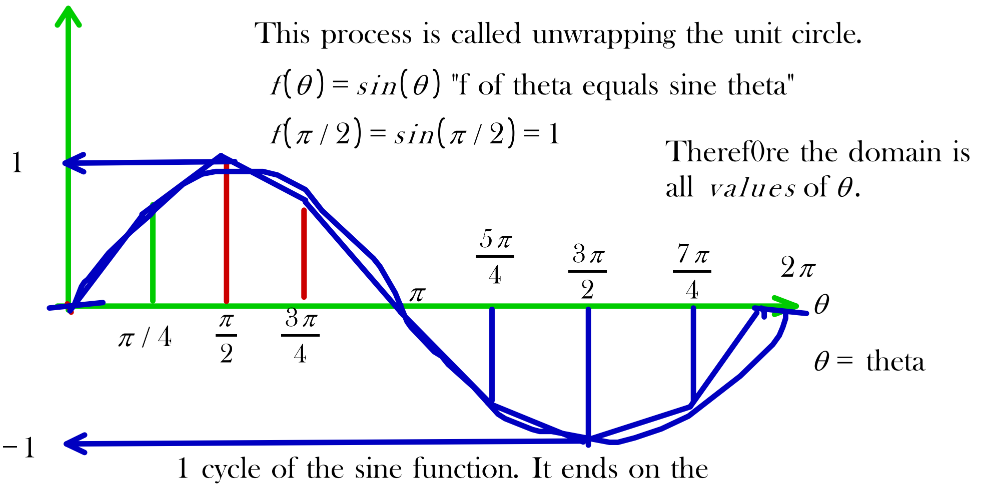
How to create a picture of the sine function.

This process is called unwrapping the unit circle.

$f(\theta) = sin(\theta)$ "f of theta equals sine theta"

$f(\pi/2) = sin(\pi/2) = 1$

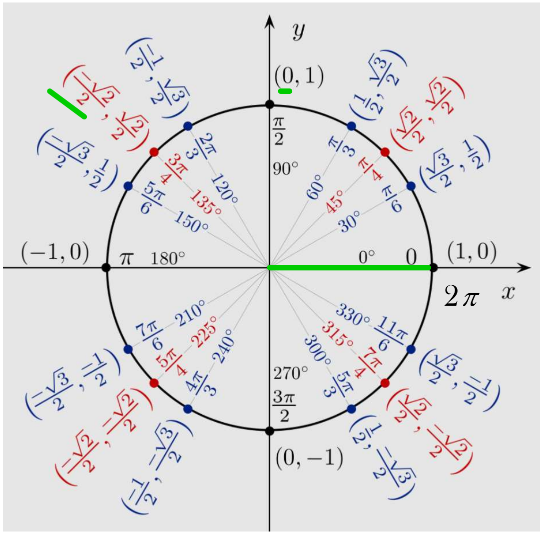
Theref0re the domain is all *values* of θ.



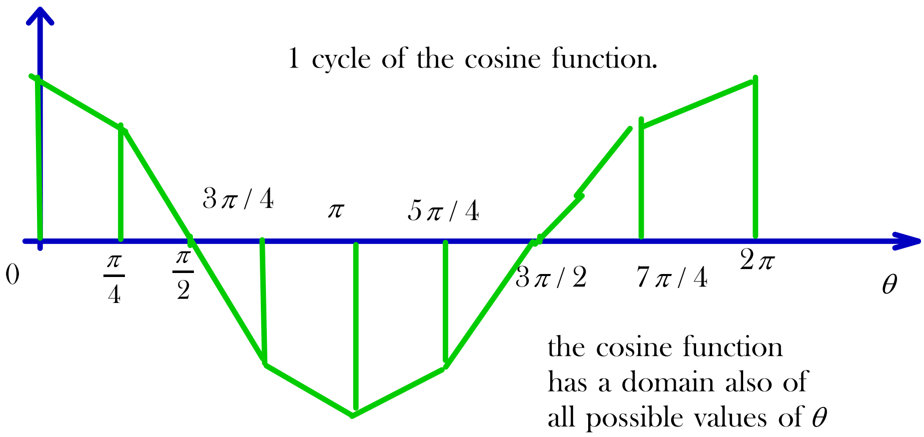
1 cycle of the sine function. It ends on the same horizontal level as it begins and includes a peak and a trough.

The period is the minimum angle required to complete one cycle. For the sine function, it's 2π.

The cosine function is defined as the x-coordiante of each point on the unit circle.

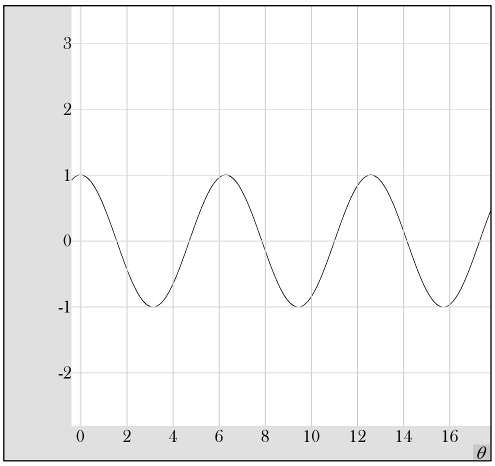


1 cycle of the cosine function.



the cosine function has a domain also of all possible values of θ

In other words, any number can be plugged ito the cosine and sine functions.

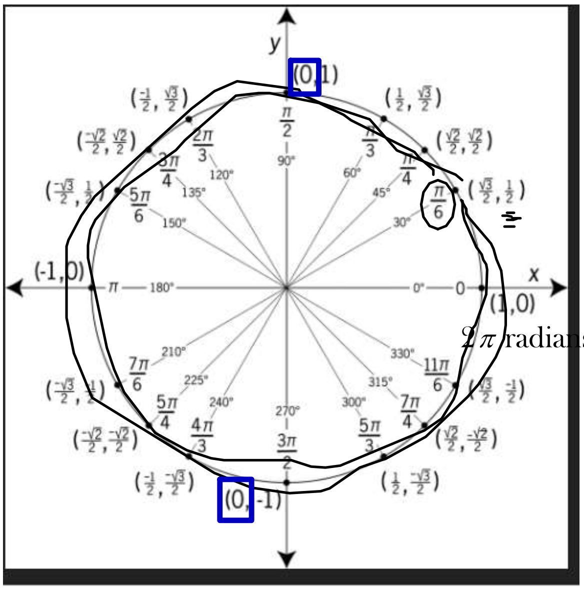


$sin(100000) =$

$cos(799767\pi)$

$sin(0.0001)$

$cos(-0.00001)$



Periods of trig functions.
sin(θ) is defined as the y coordinate of each point on the unit circle.

$$sin(\theta+2\pi)$$

$$sin\left(\frac{\pi}{6}\right)=\frac{1}{2}$$

$$sin\left(\frac{\pi}{6}+2\pi\right)=\frac{1}{2}$$

$$sin\left(\frac{\pi}{6}+2\pi+2\pi\right)=\frac{1}{2}$$

$$sin\left(\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}$$

$$sin\left(\frac{\pi}{4}+2\pi\right)=\frac{\sqrt{2}}{2}$$

$$sin(\theta+2\pi)=sin(\theta)$$

$$\left.\begin{aligned}cos\left(\frac{\pi}{6}\right)&=\frac{\sqrt{3}}{2}\\ cos\left(\frac{\pi}{6}+2\pi\right)&=\frac{\sqrt{3}}{2}\end{aligned}\right\}$$

$$cos\left(\frac{\pi}{6}\right)=cos\left(\frac{\pi}{6}+2\pi\right)$$

b/c the x coordiantes at both angles are the same.

In general, we can say that $cos(\theta+2\pi)=cos(\theta)$

The cosine and sine functions are useful for representing things that repeat.
tides,vibrating springs, and sound waves.

Whatis the domain of f(θ)= $\frac{1}{cos(\theta)}$, $f\left(\frac{\pi}{2}\right)=\frac{1}{cos(\pi/2)}=\frac{1}{0}=$ undefined

Domain is set of all θ such that $\theta\neq\pi/2, \theta\neq 3\pi/2$

$$f(\theta)=\frac{1}{1-cos(\theta)}$$

$f(0)=\frac{1}{1-cos(0)}=\frac{1}{1-1}=\frac{1}{0}=$ undefined

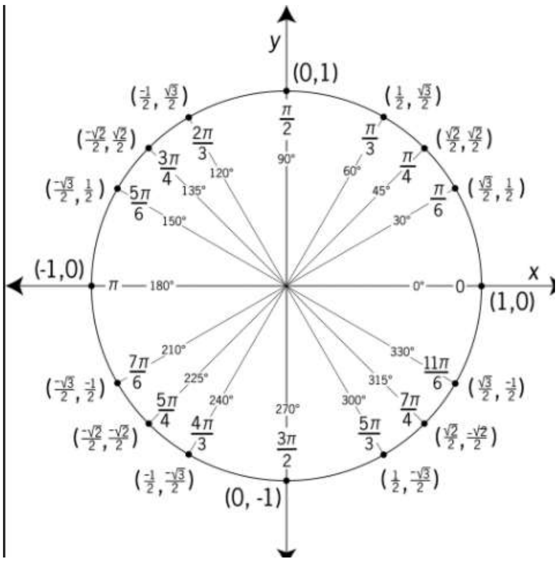
cannot again have 0 in the bottom.

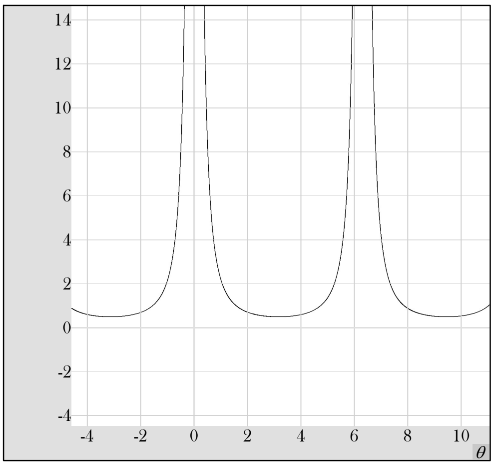
$1-cos(\theta)\neq 0$

$1-cos(\theta)+cos(\theta)\neq cos(\theta)$

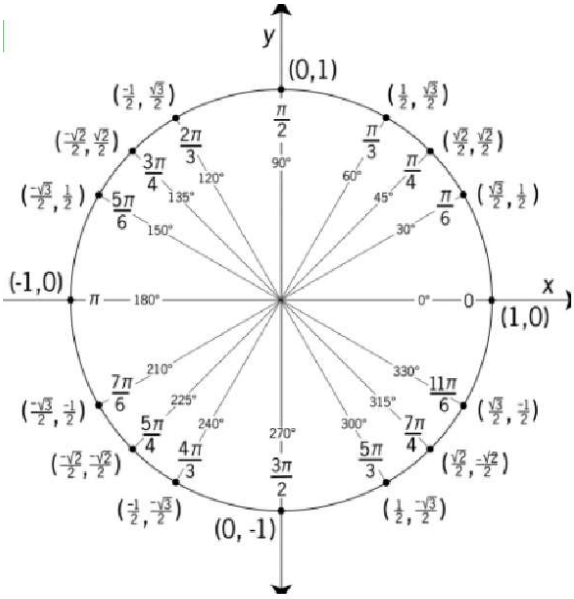
$1\neq cos(\theta)$ Domain is all θ except $\theta=0$ or $\theta=2\pi$

The angle must not give a value for cosine that is equal to 1.





Next, we define the tangent function as $tan(\theta)=\dfrac{sin(\theta)}{cos(\theta)}$, it's a ratio, so we can't divide by 0.
in other words, can't have $cos(\theta)=0$



$$tan(\pi/6)=\frac{1/2}{\sqrt{3}/2} \to \text{keep change flip on fractions}$$
$$=\frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{\sqrt{3}}$$
$$\sqrt{\ }=(\ )^{1/2}$$
$$=\frac{1}{\sqrt{3}}$$

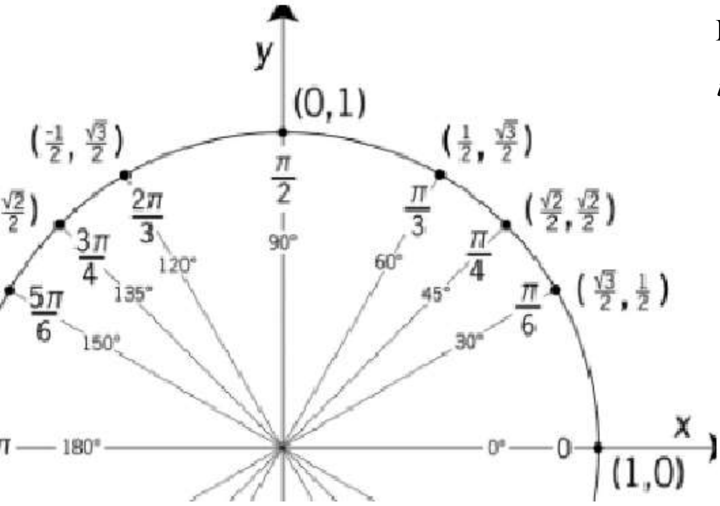
Often, you're expected to rationalize, which means to write it so the bottom doesn't contain a radical.

$$\frac{1}{\sqrt{3}}=\frac{1}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\frac{\sqrt{3}}{(\sqrt{3})^2}=\frac{\sqrt{3}}{(3^{1/2})^2}=\frac{\sqrt{3}}{3^{2/2}}=\frac{\sqrt{3}}{3^1}=\frac{\sqrt{3}}{3}$$
$$\frac{\sqrt{3}}{\sqrt{3}}=1$$
$$\frac{1}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=\frac{\sqrt{2}}{2},\ \frac{1}{\sqrt{5}}=\frac{\sqrt{5}}{5},$$

rule:
$\sqrt{a}\sqrt{b}=\sqrt{ab}$

$$\frac{1}{\sqrt{7}}\frac{\sqrt{7}}{\sqrt{7}}=\frac{\sqrt{7}}{\sqrt{49}}=\frac{\sqrt{7}}{7}$$

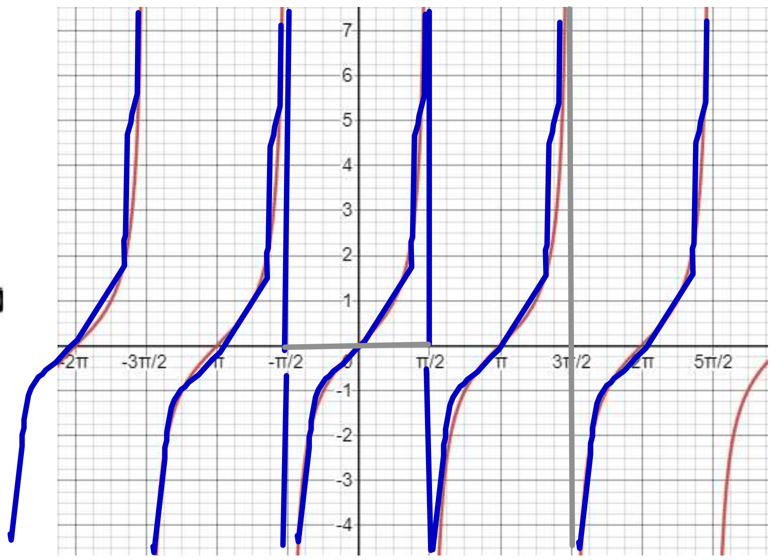
In math, we're allowed to multiply by 1, or add 0.
In calculus these are used a variety of ways that allow solving problems that otherwise can't be solved.



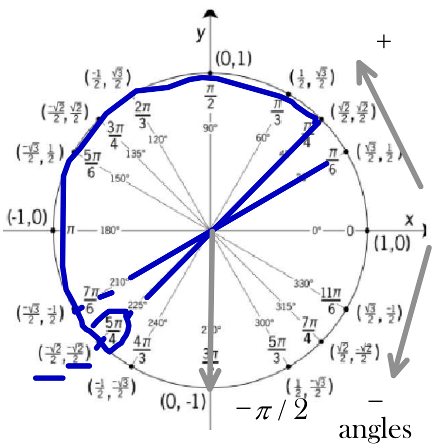
$$tan(\pi/2)=\frac{sin(\pi/2)}{cos(\pi/2)}=\frac{1}{0}=\text{not defined}$$
$$tan(-\pi/2)=\frac{sin(-\pi/2)}{cos(-\pi/2)}=\frac{-1}{0}=\text{undefined}$$

Imagine we want to find

$$tan\left(\frac{\pi}{2}\right)=\frac{sin(\pi/2)}{cos(\pi/2)}=\frac{1}{0}=\text{undefined}$$



Period of the Tangent Function(Period is the smallest angle required for the values to repeat).



$$tan(\pi/4)=\frac{\sqrt{2}/2}{\sqrt{2}/2}=1$$
$$tan(5\pi/4)=\frac{-\sqrt{2}/2}{-\sqrt{2}/2}=1$$
$$tan\left(\frac{\pi}{4}\right)=tan\left(\frac{\pi}{4}+\pi\right)\ (\text{not } 2\pi)$$
$$1=1$$

In general, the period of the tangent function is π.

$$tan\left(\frac{\pi}{6}\right)=\frac{1/2}{\sqrt{3}/2}$$
$$tan\left(\frac{\pi}{6}+\pi\right)=\frac{-1/2}{-\sqrt{3}/2}=\frac{1/2}{\sqrt{3}/2}$$

Next we come to a new type of function, called an exponential function.

Exponential functions are functions of the form $f(x) = b^x$, $b =$ is a number.

f(x)=2^x

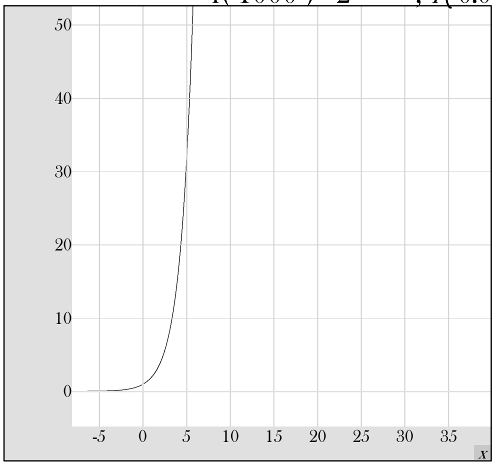
g(x)=$\left(\frac{1}{3}\right)^x$

$p(t) = 5\left(\frac{1}{9}\right)^t$

So for exponential functions the domain is $(-\infty, \infty)$

Imagine we have $f(x) = 2^x$, f(-1)=2^{-1}

f(-1000) =2^{-1000}, $f(0.0001) = 2^{0.0001}$



Range: set of outputs

$f(x) = 2^x$, $f(-45) = 2^{-45}$ it's very tiny but it's not 0.

y= 2^x can never be equal to 0.

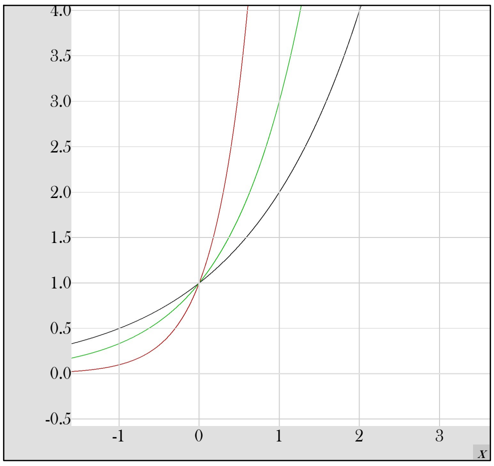
In other words, $(x, 2^x)$. So $y = 2^x$ cannot be 0.

Which means the range is (0, ∞) and not

[0, ∞)

$f(x) = 2^x$

$f(-10000) = 2^{-10000} = \frac{1}{2^{10000}}$ super tiny but not 0.



$f(x) = 2^x$

$f(1) = 2$

$f(-1) = 2^{-1} = \frac{1}{2} = 0.5$

$g(x) = 3^x$

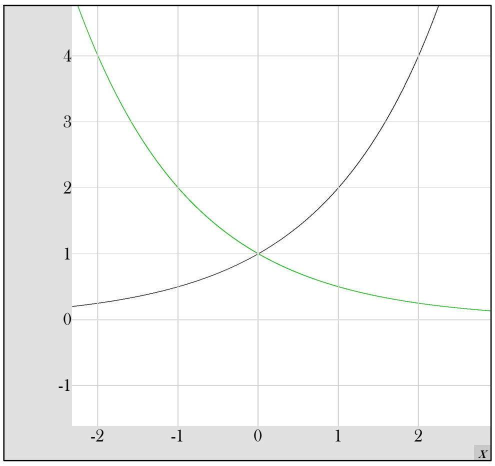
$g(1) = 3^1 = 3$

$g(-1) = 3^{-1} = \frac{1}{3} \approx 0.33$

$h(x) = 10^x$

$h(1) = 10$

$h(-1) = 10^{-1} = 0.1$



black graph is 2^x

green graph is $\left(\frac{1}{2}\right)^x$

$x = 1:$

$2^1 = 2$

$\left(\frac{1}{2}\right)^1 = \frac{1}{2}$

$x = -1:$

$2^{-1} = \frac{1}{2}$

$\left(\frac{1}{2}\right)^{-1} = \frac{1}{\left(\frac{1}{2}\right)^1} = \frac{1}{\frac{1}{2}} = 2$ by keep change flip

Log Functions are used as the inverses of exponential function. This means whatever the exponential function does, the log function reverses.

Inverse Functions:

$f(x) = 2x + 4$

addition, multiplication

inverse function reverses operations and reverses the order of the operations

division and subtract, done in the opposite order

f^{-1} (x), f inverse

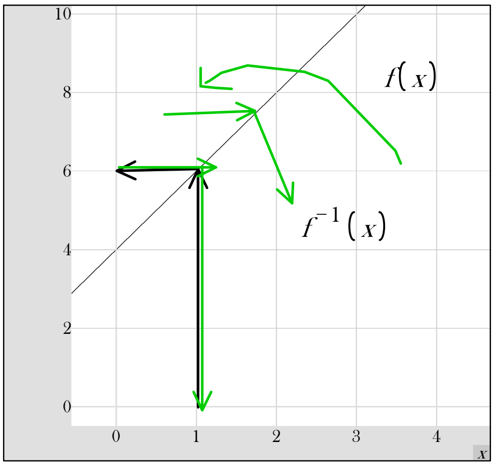
$f^{-1}(x) = \frac{x-4}{2}$

$f(1) = 2 \cdot 1 + 4 = 2 + 4 = 6$ (1,6)

f^{-1} (6)= $\frac{6-4}{2} = \frac{2}{2} = 1, (6, 1)$

$f(x) = x^3$ $\quad f(2) = 2^3 = 8, (2, 8)$

$f^{-1}(x) = \sqrt[3]{x}$ $\quad f^{-1}(8) = \sqrt[3]{8} = 2, (8, 2)$



$f(x) = 2x + 4$

$(1, 6)$

the inverse would do 6 in, 1 out.

$f^{-1}(10) = 3$

$f(3) = 10$

Inverses can be found by the following steps:

1. $f(x) = 2x + 4$
2. $y = 2x + 4$
3. interchange y and x: $x = 2y + 4$
4 solve for y: $x - 4 = 2y$

$$\frac{x-4}{2} = y$$

5. replace the y with the inverse symbol: $f^{-1}(x) = \frac{x-4}{2}$

Find the inverse of $f(x) = \sqrt[3]{x-2}$

first subtract and then take the cube root

f^{-1} (x)=x^3 +2

$f(x) = \sqrt[3]{x-2}$

$y = \sqrt[3]{x-2}$

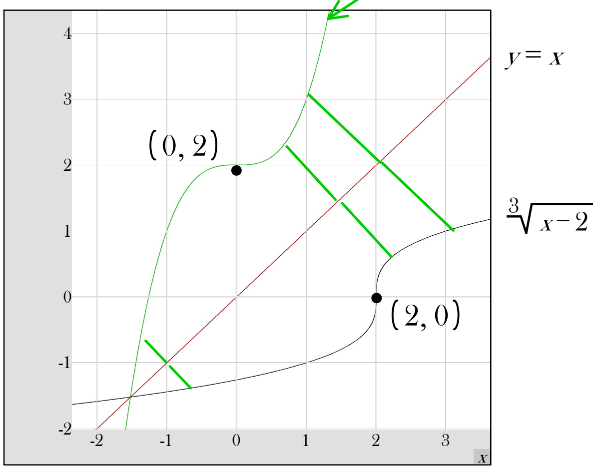
$x = \sqrt[3]{y-2}$

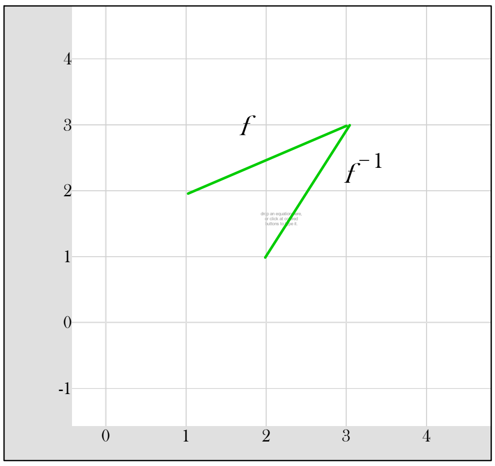
$x^3 = (\sqrt[3]{y-2})^3$

$x^3 = y - 2$

$x^3 + 2 = y$

$f^{-1}(x) = x^3 + 2$





Making a picture of an inverse function from a graph using points.

f has the points (1,2) and (3,3) on it.

f^{-1} has on it the points (2,1) and (3,3)

$f(x) = \frac{1}{x}$

find the inverse :

$y = \frac{1}{x}$

interchange x and y

$x = \frac{1}{y}$

now solve for y

$xy = 1$

$y = \frac{1}{x} = f^{-1}(x)$

So the function 1/x has the same expression as its inverse.

$f(x) = 10^x$

$f^{-1}(x) = log_{10}(x)$

$f(2) = 10^2 = 100$ (2,100)

$f^{-1}(100) = log_{10}(100) =$ 2, (100, 2)

Going back to exponential functions
and logarithmic functions.

$f(x) = 2^x$

exponent goes in, power comes out.
inverse reverses that:
power goes in and exponent comes out

$f(x) = 2^x$

the inverse is $f^{-1}(x) = log_2 x$

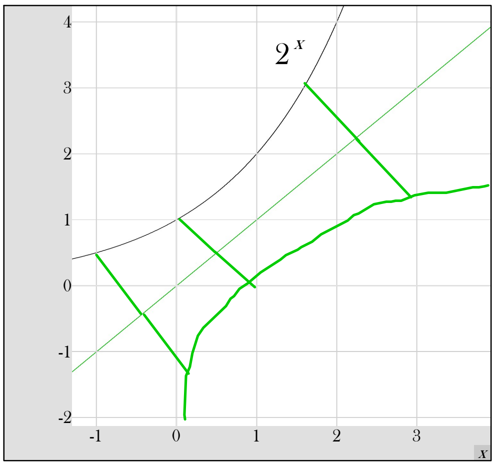
$f(3) = 2^3 = 8, (3, 8)$

$f^{-1}(8) = log_2(8) \Leftarrow$ What is the exponent

that 2, the base, has to be raised to , to get 8? 3

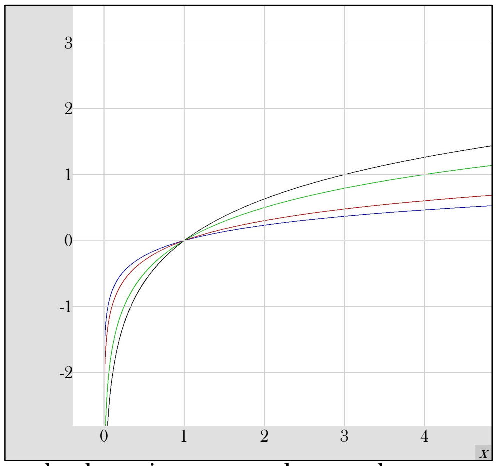
f^{-1} (8)=$\log_2$ (8)=3, (8,3)

logarithm, just think exponent



$y = x$

$log_2(x)$, the inverse of 2^x

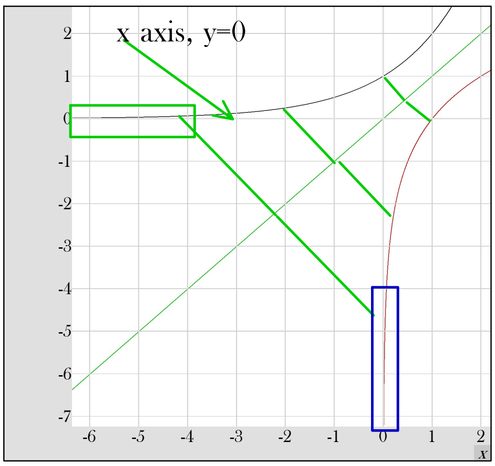


as the base increases, the graph
gets
pulled more quickly to the y axis
between 0 and 1 and faster to the x axis
to the right of x=1

$f(x) = 2^x$, domain is anything, all real numbers, **R**

$f(-1), f(0), f(1), f(10000)$

$x = 0$ 2^x



$y = x$

$log_2 x$

the domain of the $\log_2$ x function is $x > 0$.

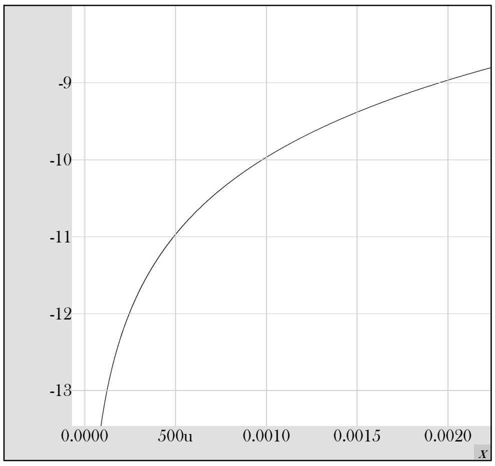
$log_2(0) =$ error

for the function 2^x, y=0(the x axis) is the horizontal asymptote
(asymptotate are lines a graph approaches)

for the function $log_2(x)$, the domain is x>0, and the axis
x=0 is the vertical asymptote.

y axis is called also the line x=0

$f(-10) = 2^{-10} = \frac{1}{2^{10}}$ tiny number but not 0



$log_2(0) =$ undefined(doesn't exist)

EXAMPLE 6 Classify the following functions as one of the types of functions that we have discussed.

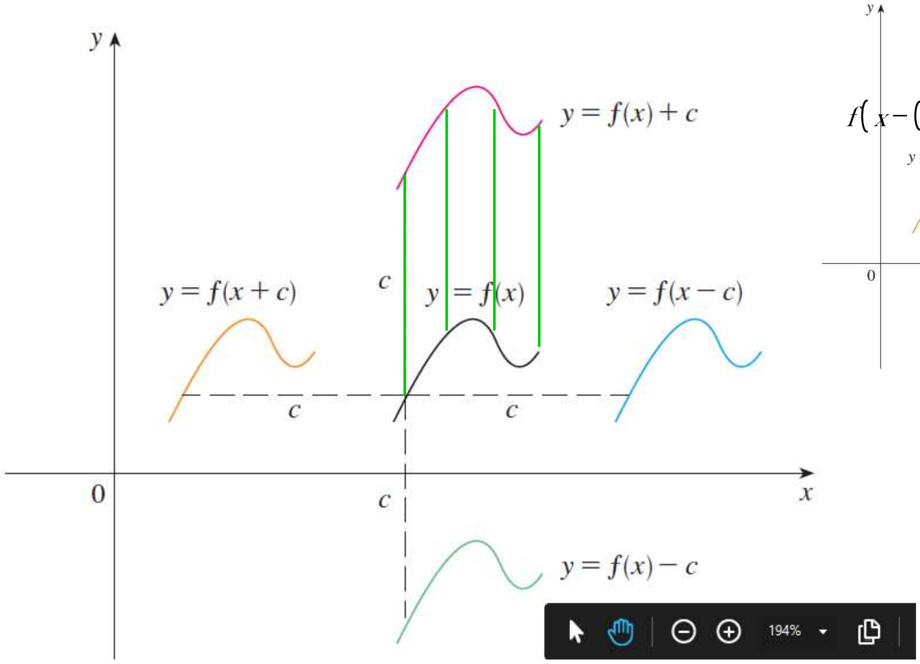
(a) $f(x) = 5^x$ exponential

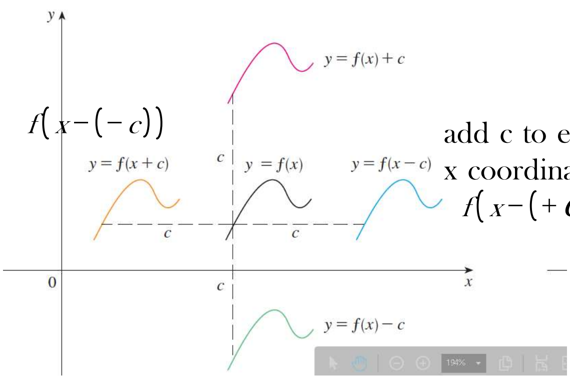
(b) $g(x) = x^5$ power function

(c) $h(x) = \dfrac{1 + x}{1 - \sqrt{x}}$ algebraic function

(d) $u(t) = 1 - t + 5t^4$ polynomial of degree 4

new functions from old functions:
transformations of functions:
translations(just moving left or right)

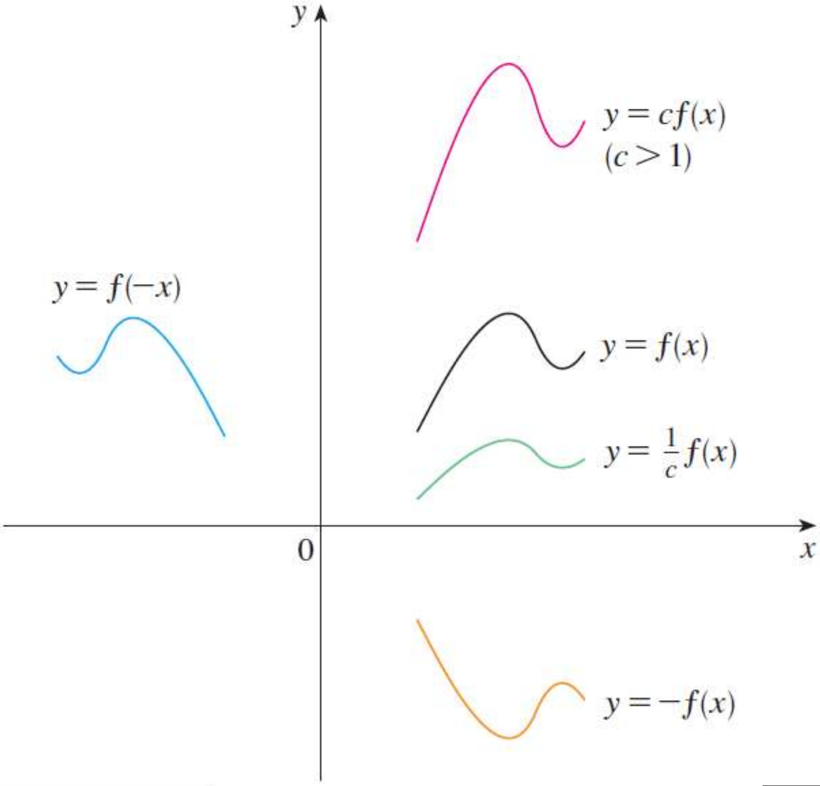




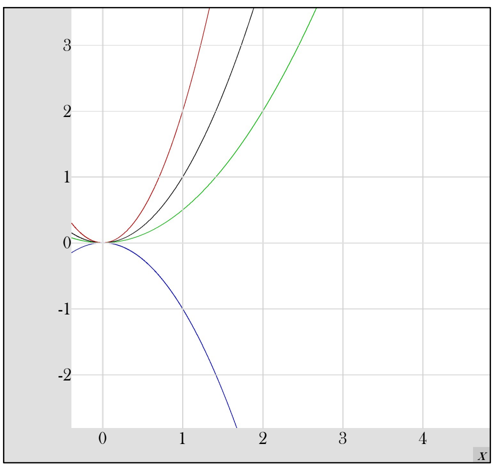
$f(x-(-c))$

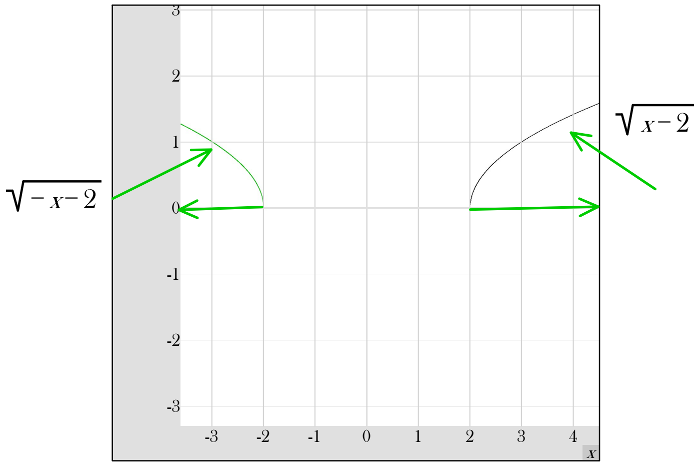
add c to every x coordinate

$f(x-(+C))$



$f(x) = x^2$

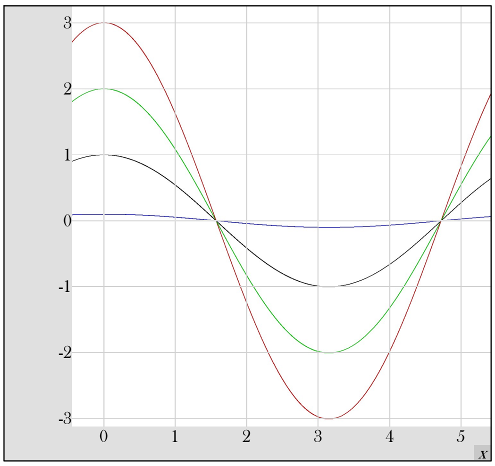




$f(x) = \sqrt{x-2},\ x \geq 2$

$f(-x) = \sqrt{-x-2},\ -x \geq 2$

$x \leq -2$



$f(x) = a\cos(x)$

a dependening on its size, pulls the graph away from the x axis or towards it

A limit is what the values of a function approach as x approaches some value of its own along the x axis.
$f(x) = x^2$

What happens to the values of the function as x approaches 1 , that means from the left and then from the right.

From the left side, x values are smaller than 1.

$x=\boxed{.9}, f(0.9) = 0.9^2 = \boxed{0.81}$

$x=\boxed{0.99}, f(0.99) = 0.99^2 = \boxed{0.9801}$

$x=\boxed{0.999}, f(0.999) = 0.999^2 = \boxed{0.998001}$

$x=\boxed{0.9999}, f(0.9999) = 0.9999^2 = \boxed{0.99980001}$

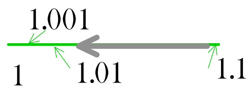
The values of f(x) approach 1 as x approaches 1 from the left.

Mathematically, we write this as $\lim\limits_{x \to 1^-} f(x) = 1$

the limit of f(x) as x approaches 1 from the left is 1

What value does f(x)=x ^{2} approach when x approaches 1 from the right side of x.

$x = 1.1$ (this number is a little more than 1), $f(1.1) = 1.1^2 = \boxed{1.21}$



$x = 1.01$ (this number is closer to 1 than 1.1), $f(1.01) = 1.01^2 = \boxed{1.0201}$

$x = 1.001$ (this number is closer to 1 than 1.01), $f(1.001) = 1.001^2 = \boxed{1.002001}$

The values of f(x) approach 1 as x approaches 1 from the right side.

Mathematically, we write this as $\lim\limits_{x \to 1^+} f(x) = 1$

When the left and right limits are the same, as they here because are equal to 1, the overall limit exists.

When $\lim\limits_{x \to 1^+} f(x) = \lim\limits_{x \to 1^{-1}} (f(x))$, then we say the overall limit exists. $\lim\limits_{x \to 1} f(x) = 1$

$f(x) = \frac{1}{x}$

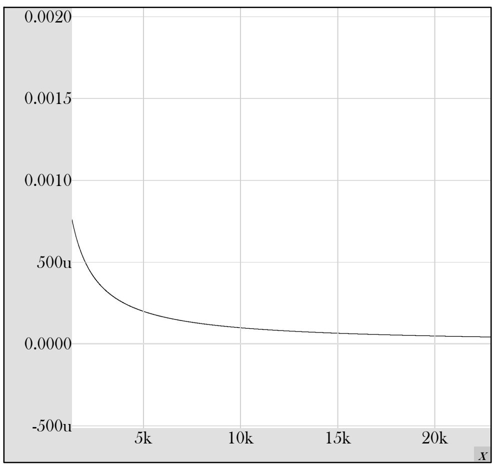
∞, this is the infinity symbol.

What happens when x approaches positive infinity?

$x = 100,\ f(100) = \frac{1}{100} = 0.01$ the output is a small number.

$x = 1000000,\ f(1000000) = \frac{1}{1000000} = 10^{-6}$ this is a very tiny numbe

In this case, we can say that $\lim\limits_{x \to \infty} \frac{1}{x} = 0$



This doesn't mean that $\frac{1}{x}$ is equal to 0.

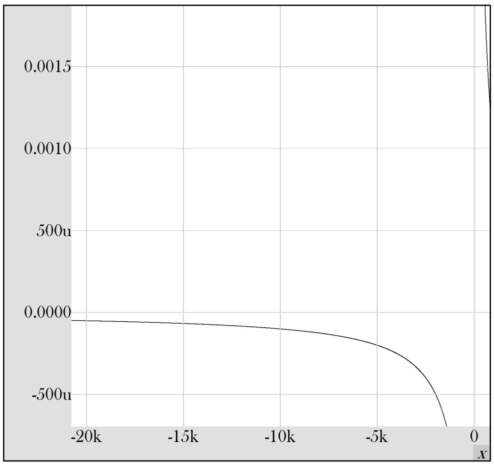
It means the y coordinates get ever closer to 0 without ever actually being 0.

What value is approached by the outputs of f(x)= $\frac{1}{x}$ when x goes to negative infinity?

In this case, plug in very negative numbers.

$x = -100000,\ f(-100000) = \frac{1}{-100000}$ this is a number that is negative but very close to 0.

$x = -1,000,000,$ then we get $f(-1,000,000) = \frac{1}{-1,000,000}$, this number is negative but it's even closer to 0 .



The graph shows that as x approaches negative infinity, the values of 1/x approach 0 also.

$$\lim_{x \to -\infty} \frac{1}{x} = 0$$

This 0 doesn't means that $\frac{1}{x}$ equals 0. It means that 1/x approaches 0 or , in other words, gets pulled ever closer to the x axis where y=0.

$$f(x) = \frac{1}{x}$$

What happens to the values of f(x) as x approaches 0 from the right side?

Plug in number bigger than 0, but very close to 0.

0.1 (to the right of 0 and close to 0), f(.1)= $\frac{1}{0.1}$ = 10 point (0.1, 10)

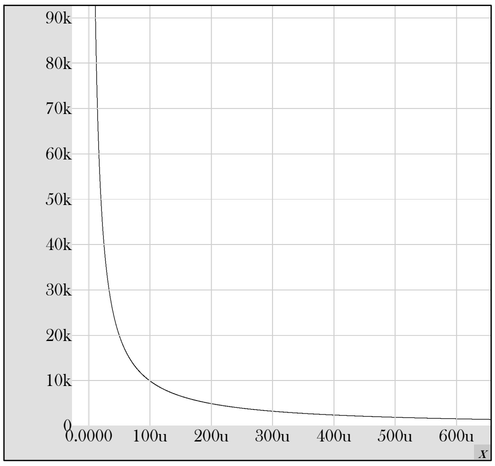
.01 (to the right of 0 and closer to 0 than .1) $f(0.01) = \frac{1}{0.01} = 100$ point: (0.01, 100)

0.001 (to the right of 0 still and closer to 0 than .01), $f(0.001) = \frac{1}{0.001} = 1000$ point(0.001, 1000)

As x approaches 0 from the right side, does f(x)=1/x approach any particular value? Since the y coordinates get ever bigger, f(x) approaches infinity. In math we say $\lim_{x \to 0^+} \frac{1}{x} = \infty$

This doesn't say that $\frac{1}{x}$ ever equals infinity. Infinity is not a number. It's concept.

Here, the y axis is called the vertical asymptote.
It's a line that graph approches but never crosses.



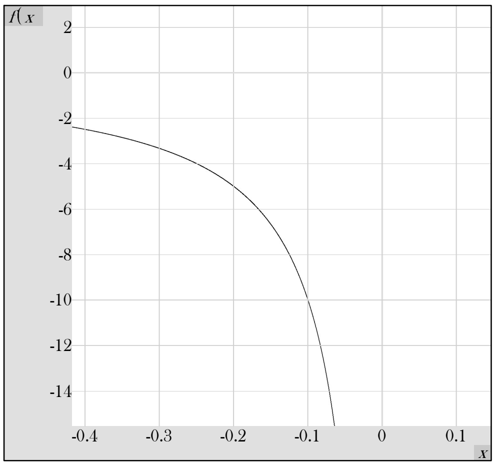
What happens to the values of f(x)=1/x as x approaches 0 from the left side?

$x = -0.1$ (value is to the left of 0, but closer to it): $f(-0.1) = \dfrac{1}{-0.1} = -10$

$x = -0.000001$ (value is to the left of 0 and very close to it): $f(-0.000001) = \dfrac{1}{-0.000001} = -1000000$

As x approaches 0 from the left side, the values of f(x) (or you could say y), approach $-\infty$.

$$\lim_{x \to 0^-} \frac{1}{x} = \lim_{x \to 0^-} f(x) = -\infty$$



$$\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)$$

$x = 1000,\ 1 + \dfrac{1}{1000} = 1 + 0.001 = 1.001$

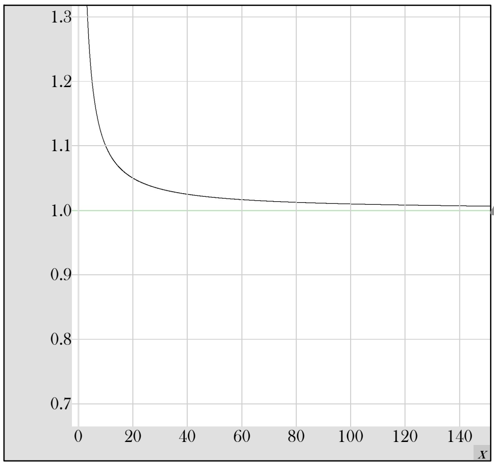
$x = 100000,\ 1 + \dfrac{1}{100000} = 1.00001$

$x = 1000000,\ 1 + \dfrac{1}{1000000} = 1.000001$

What's the number the outputs are approaching? The outputs are approaching the number 1.

$$\lim_{x \to \infty} \left(1 + \frac{1}{x}\right) = 1$$

This doesn't say that $1 + \dfrac{1}{x}$ actual ever equals 1. It says the values of 1+1/x APPROACH 1.



In this case, there is again always a gap between y=1 and the curve. So the the y coordinates of the graph approach 1 but never reach 1.

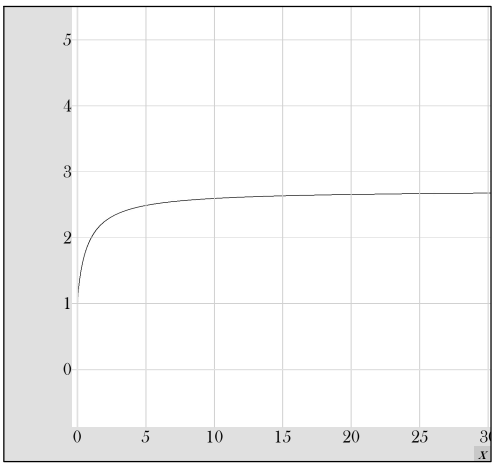
$x = 1000,\ 1 + \dfrac{1}{1000} = 1 + 0.001 = 1.001,\ (1000, 1.001)$

$x = 100000,\ 1 + \dfrac{1}{100000} = 1.00001,\ (100000, 1.00001)$

read this way

Really famous limit:

$$f(x) = \left(1 + \frac{1}{x}\right)^x$$



$x = 1000: \left(1 + \dfrac{1}{1000}\right)^{1000} = \boxed{2.71}692$

$x = 10000: \left(1 + \dfrac{1}{10000}\right)^{10000} = \boxed{2.718}1459$

$x = 100000: \left(1 + \dfrac{1}{100000}\right)^{100000} = \boxed{2.718}27$

$\lim_{x \to \infty} \left[\left(1 + \dfrac{1}{x}\right)^x\right] \approx 2.718$. *This* value you will learn in calculus is given the special symbol e.