Make sure to take very detailed notes.
Please remember my notes are loaded with each homework assignment. You can copy them from the PDF if you just want to pay attention in class. Regardless, your version of the notes must be loaded with the homework solutions and please be sure to solve the questions as the directions state in MyOpenMath.

Section 5.4:

bottom page 403:

$\int e^x dx = e^x + C$

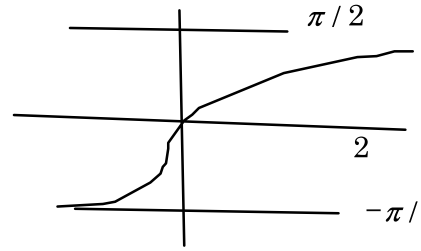
$\int \sin x dx = -\cos x + c$

$\int \sec x \tan x dx = \sec x + c$

$\int \frac{1}{x^2+1} dx = \tan^{-1}(x) + C$

$\int b^x dx = \frac{1}{\ln b} b^x + C$

reason: $\frac{d}{dx}\left(\frac{b^x}{\ln b} + C\right) = \frac{b^x \ln(b)}{\ln b} + 0 = b^x =$ integrand



Example 2/Page 404:

$\int \frac{\cos\theta}{\sin^2\theta} d\theta = \int \frac{1}{\sin\theta} \frac{\cos\theta}{\sin\theta} d\theta = \int \csc\theta \cot(\theta) d\theta = -\csc\theta + C$

example 4/page 405:

$\int_0^2 2x^3 - 6x + \frac{3\cdot 1}{x^2+1} dx = \left(\frac{2}{4}x^4 - 6\frac{x^2}{2} + 3\tan^{-1}(x)\right)\Big|_0^2 = \frac{1}{2}\cdot 2^4 - 3\cdot 2^2 + 3\tan^{-1}(2) - (0)$

$= 2^3 - 3\cdot 4 + 3\tan^{-1}(2)$

$= 8 - 12 + 3\tan^{-1}(2)$

$= -4 + 3\tan^{-1}(2)$

Page 406: Net Change THeorem:
The integral of a rate of change is the net change.

$\int_a^b F'(x) dx = F(b) - F(a)$

total distance traveled= $\int_{t_1}^{t_2} |v(t)| dt$ (integrate speed with respect to time)

$v(t) =$ velocity, |v(t) |=speed

displacement= $\int_a^b v(t) dt$ (integral of velocity with respect to time)

change in velocity = $\int_a^b a(t) dt$ (integral of acceleration: m/s^2 from $\frac{m/s}{s}$)

Page 409/Ex 45:

$\int_{-1}^{2} (x - 2|x|) dx = \int_{-1}^{0} x - 2(-x) dx + \int_{0}^{2} x - 2(x) dx = \int_{-1}^{0} 3x dx + \int_{0}^{2} -x dx = \frac{3}{2}x^2 \Big|_{-1}^{0} - \frac{x^2}{2}\Big|_{0}^{2}$

$x < 0, |x| = -x$ $\qquad$ $x > 0, |x| = x$

$= \frac{3}{2}\cdot 0^2 - \frac{3}{2}(-1)^2 - \left(\frac{2^2}{2} - \frac{0^2}{2}\right)$

$= -\frac{3}{2} - \frac{4}{2} = -\frac{7}{2} = -3.5$

$\lvert x \rvert = -x$		$\lvert x \rvert = x$
$x < 0$	0	$x > 0$

page 409: $\int \frac{\sin\theta+\sin\theta\tan^2\theta}{\sec^2\theta}d\theta = \int \frac{\sin\theta(1+\tan^2\theta)}{\sec^2\theta}d\theta = \int \frac{\sin\theta(\sec^2\theta)}{\sec^2\theta}d\theta = \int \sin\theta d\theta$

menacing! — factor sin(θ) out — trig identity! — simple!

$= -\cos(\theta)+C$

page 409: $\int \frac{\sin 2x}{\sin x}dx \xrightarrow{\text{trig identity for sin2x}} \int \frac{2\cancel{\sin x}\cos x}{\cancel{\sin x}}dx = \int 2\cos x dx = 2\int \cos x dx = 2\sin x + C$

page 409: $\int \frac{1+\cos^2\theta}{\cos^2\theta} = \int \frac{1}{\cos^2\theta}+\frac{\cos^2\theta}{\cos^2\theta}\ d\theta = \int \sec^2\theta + 1 d\theta = \tan\theta+\theta+C$

split into two pieces and each piece can be done relatively easily!

Section 5.5/The Substitution Rule:

example 1: $\int 2x\sqrt{1+x^2}\,dx$

key observation: $2x$ is the derivative of $1+x^2$

variable substitution..relies on seeing a function and something that is a multiple of its derivatrive!

$\int \sqrt{1+x^2}\cdot 2xdx$

$u = 1+x^2$ (why...b/c its derivative is 2x)

$\frac{du}{dx} = 2x \rightarrow$ here we write $\boldsymbol{du} = 2xdx$

replace 2xdx with du, $1+x^2$ with u: $\int \sqrt{u}\,du = \int u^{1/2}du = \frac{u^{1/2+1}}{1/2+1}+C = \frac{u^{3/2}}{3/2}+C = \frac{2}{3}u^{3/2}+C$

recall u= $1+x^2$, so go back to x: $\frac{2}{3}(1+x^2)^{3/2}+C$

example 1/page 413:

$\int x^3\cos(x^4+2)dx$, do we have a function and its derivative?

$u = x^4+2 \quad \leftarrow$ transforms x space to u space

$du = 4x^3dx \quad \leftarrow$ stretch

$\frac{du}{4} = x^3dx$

$\int \cos(x^4+2)x^3dx$

$\int \cos(u)\frac{du}{4} \leftarrow$ only u remains

$\frac{1}{4}\int \cos u du = \frac{1}{4}\sin(u)+C \rightarrow$ replace u back with $x^4+2 \rightarrow \frac{1}{4}\sin(x^4+2)+C$

example 3/page 414:

Find $\int \frac{x}{\sqrt{1-4x^2}}dx$

do we have a function and a scalar multiple of its derivative?

$u = 1-4x^2$

$du = -8xdx$

$\frac{du}{-8} = xdx$

$\int \frac{1}{\sqrt{u}}\frac{du}{-8} = -\frac{1}{8}\int u^{-1/2}du = -\frac{1}{8}\frac{u^{-1/2+1}}{-1/2+1}+C = -\frac{1}{8}\frac{u^{1/2}}{1/2}+C = -\frac{1}{8}\cdot\frac{2}{1}\sqrt{u}+C = -\frac{2}{8}\sqrt{u}+C = -\frac{1}{4}\sqrt{u}+C$

recall u= $1-4x^2$, so go back to x: $-\frac{1}{4}\sqrt{1-4x^2}+C$

example 4: Calculate $\int e^{5x}dx$, $u=5x, du=5dx \to \frac{du}{5}=dx$

$\to \int e^u \frac{du}{5} = \frac{1}{5}\int e^u du = \frac{1}{5}e^u + C \to$ replace u with 5x: $\frac{1}{5}e^{5x}+C$

example 5/page 415:

$\int \sqrt{1+x^2}\, x^5 dx$ try u= $1+x^2$, then $du=2xdx$ but we have x^5? ??

$\int \sqrt{1+x^2}\, x^4 xdx$ $u=1+x^2 \to du=2xdx$ since $u=1+x^2$

$\frac{du}{2}=xdx$ $u-1=x^2$

$\int \sqrt{u}\,(x^4)\frac{du}{2}$ we have x^4, so square?

$(u-1)^2=x^4$

$\int \sqrt{u}\,(u-1)^2\frac{du}{2}$ replace x^4 with $(u\text{-}1)^2$ in the integrand.

expand: $\frac{1}{2}\int \sqrt{u}\,(u^2-2u+1)\; du = \frac{1}{2}\int u^{2+1/2}-2u^{1+1/2}+u^{1/2}du$

$= \frac{1}{2}\int u^{5/2}-2u^{3/2}+u^{1/2}du$

$= \frac{1}{2}\left(\frac{u^{5/2+1}}{5/2+1}-2\frac{u^{3/2+1}}{3/2+1}+\frac{u^{1/2+1}}{1/2+1}\right)+C$

$= \frac{1}{2}\left(\frac{2}{7}u^{7/2}-2\left(\frac{2}{5}u^{5/2}\right)+\frac{2}{3}u^{3/2}\right)+C$

$= \frac{1}{2}\left(\frac{2}{7}(1+x^2)^{7/2}-\frac{4}{5}(1+x^2)^{5/2}+\frac{2}{3}(1+x^2)^{3/2}\right)+C$

example 6/page 415:

$\int tanxdx = \int \frac{sinx}{cosx}dx$ do we have a function and its derivative?

u=sinx,

$du=cosxdx$ tried..but we have sinxdx, and not $cosxdx$

$= \int \frac{-du}{u}$

$u=cosx$

$= -\int \frac{1}{u}du$

$du=-sinxdx$ (we have sinxdx)

$-du=sinxdx$

$= -ln|u|+C$ (rule)

$= -ln|cosx|+C$ does cos(x) assume negative values? yes...so we can't drop the bars!!

ln (stuff) is such that stuff>0

common form of rewriting this: $-ln|cosx|+C$ power rule for logs $= ln|(cosx)^{-1}|+C$

$= ln\left|\frac{1}{cosx}\right|+C$

$= ln|secx|+C$

power rule for logs: $ln(\text{stuff}^b)=b\,ln$ (stuff) (works both ways)

be careful: $ln(x^2)$ has domain $(-\infty,0)\cup(0,\infty)$

power rule: $2lnx$, has domain only $x>0.(0,\infty)$

Substitution Rule With Def. Integrals:

abstract form: $\int_a^b f(g(x))g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\, du$

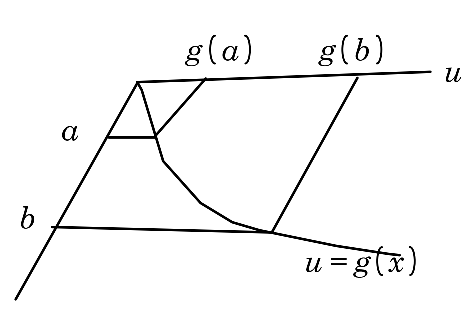
transforms x space to u space

$u = g(x)$

$u(a) = g(a)$

$u(b) = g(b)$

$u = g(x) \rightarrow du = g'(x)\,dx$



..can just integrate with respec to u and go back to x!

where do the integrands go in the picture above?? puzzle!!

example 9/page 417:

$\int_1^e \frac{lnx}{x}\,dx$

$u = lnx$

$du = \frac{1}{x}\,dx$

$\int_1^e lnx \ \frac{1}{x}\,dx$

change limits: $u(1) = ln(1) = 0$ (new lower limit)

$u(e) = ln(e) = 1$ (new upper limit)

$$\int_0^1 u\,du = \frac{u^2}{2}\Bigg|_0^1 = \frac{1^2}{2} - \frac{0^2}{2} = \frac{1}{2} - 0 = \frac{1}{2}$$

example 11/page 418:

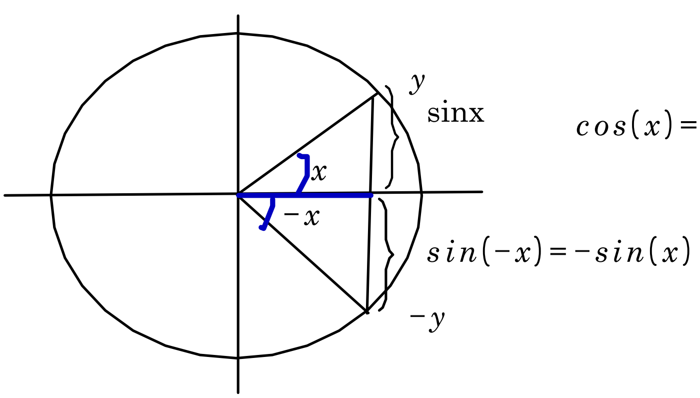
$$\int_{-1}^{1} \frac{tanx}{1 + x^2 + x^4}\,dx$$

even functions:
$f(-x) = f(x) = y$ (same vertical output) $(x, y) \rightarrow (-x, y)$

odd functions:
$f(-x) = -f(x)$ (y-coord gets negated) $(x, y) \rightarrow (-x, -y)$

check for evenness/oddness:

$$\frac{tan(-x)}{1 + (-x)^2 + (-x)^4} = \frac{\frac{sin(-x)}{cos(-x)}}{1 + x^2 + x^4} = \frac{\frac{-sin(x)}{cos(x)}}{1 + x^2 + x^4} = \frac{-tan(x)}{1 + x^2 + x^4} = -\left(\frac{tanx}{1 + x^2 + x^4}\right) \text{ (it's odd)}$$

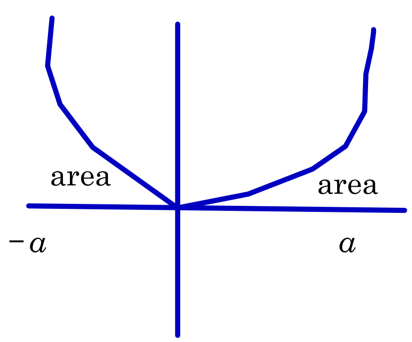


$cos(x) = cos(-x)$ (even)

when we have a symmetric integration interval, like -1 to 1, odd functions integrate to 0!

$\int_{-1}^{1} \frac{tanx}{1 + x^2 + x^4}\,dx = \mathbf{0}$ b/c our integrand is odd!

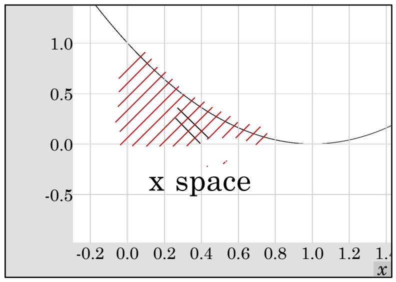
and the limits are symmetric about x=0.

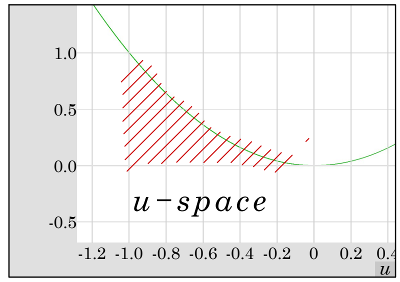


integrand is even:

$\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$ by symmetry

$$\int_0^1 (x-1)^2\,dx$$





$u = x-1$
$du = dx$
$u(0) = -1$

$u(1) = 0$

$$\int_{-1}^{0} u^2\,du = \frac{u^3}{3}\Big|_{-1}^{0} = \frac{0^3}{3} - \left(\frac{1}{3}(-1)^3\right) = -\frac{1}{3}(-1) = \frac{1}{3}$$

so

$$\int_0^1 (x-1)^2\,dx = \int_{-1}^{0} u^2\,du$$

another example:

Consider a function f that is continuous over $[-29, 29]$ and let

$$\int_0^{29} f(x)dx = -2$$

Use the above information to evaluate each of the following definite integrals.

$\int_0^{29} [f(x) - 13]dx =$ ☐

$\int_{13}^{42} f(x-13)dx =$ ☐

$f(x-13)$ means $f(x-(+13))$

, so we add 13 to x and not subtract 13 .

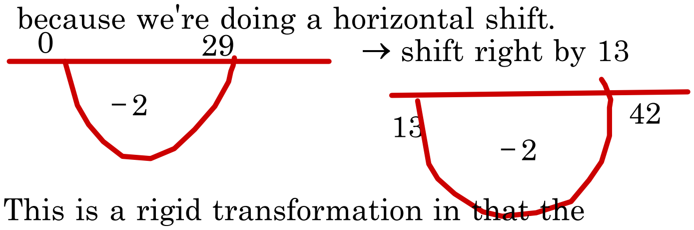
We're told $\int_0^{29} f(x)dx = -2$

Writing $f(x-13)$ means x gets shifted to the right by 13 units, so we get

$29 + 13 = 42 =$ new upper limit

0+13=13=new lower limit

We thus get $\int_{13}^{42} f(x-13)dx = -2$. The result is the same because we're doing a horizontal shift.



This is a rigid transformation in that the graph shape gets shifted but the shape doesn't get changed, so value of integral stays the same.