Effective Rate Calculation for Doubling an Investment

Problem Statement

Given the compound interest formula

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

where:

- P is the principal (initial amount),
- r is the annual interest rate,
- n is the number of compounding periods per year,
- t is the number of years,

we want to determine the effective rate per compounding period, $\frac{r}{n}$, required to double the initial amount P (i.e., $A = 2P$), while keeping n and t constant.

Solution

To find $\frac{r}{n}$ that will double P over time t with n compounding periods per year, start by setting up the equation:

$$2P = P\left(1 + \frac{r}{n}\right)^{nt}$$

Dividing both sides by P:

$$2 = \left(1 + \frac{r}{n}\right)^{nt}$$

Next, take the nt-th root of both sides to remove the exponent:

$$1 + \frac{r}{n} = 2^{\frac{1}{nt}}$$

Then, isolate $\frac{r}{n}$ by subtracting 1 from both sides:

$$\frac{r}{n} = 2^{\frac{1}{nt}} - 1$$

Conclusion

Thus, the effective interest rate per compounding period required to double the principal P is:

$$\frac{r}{n} = 2^{\frac{1}{nt}} - 1$$

This formula provides the rate that applies at the end of each compounding period to double the initial investment over the specified time.