

Illustration of an essential singularity. As $z \rightarrow 0$, $\frac{1}{z} \rightarrow \infty$

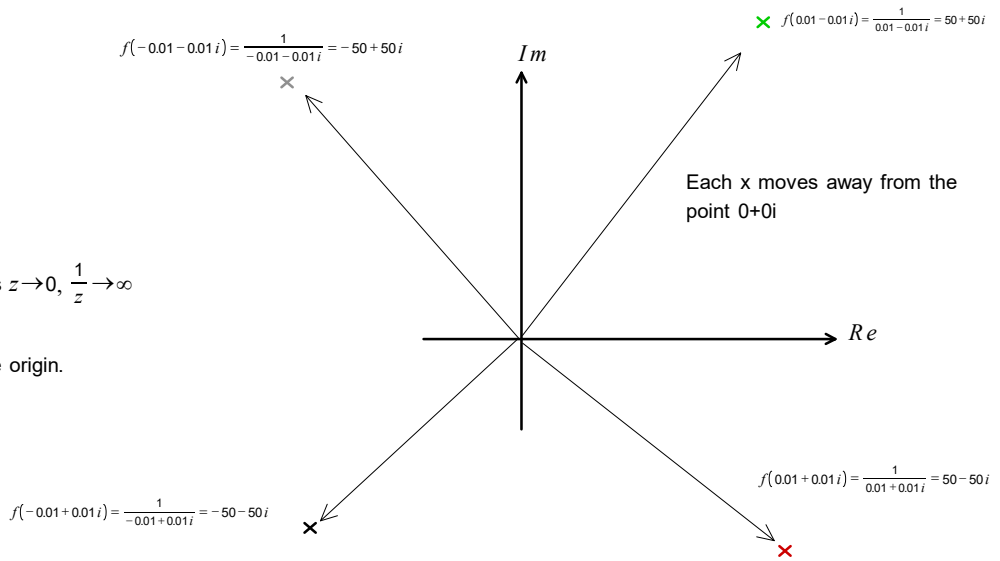
In other words, $\frac{1}{z}$ shoots away from the origin.

So $z=0$ is an essential singularity. So

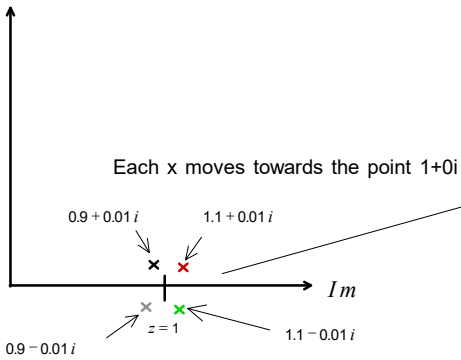
As $z \rightarrow 0$, $\frac{1}{z} \rightarrow \infty$

Note: The color coding is meaningful. Follow the colored x 's carefully.

C = complex plane/ output plane for $f(z) = \frac{1}{z}$ where $z \in C$



C = complex plane /input plane



$$f(z) = \frac{z-1}{(z-1)(z+2)}$$

The singularity at $z=1$ is removable. As the graph on the right shows, the values around $z=1$, from the left graph, are mapped in such a way that they stay closely together.

The effect of the $\frac{z-1}{z-1}$ in $\frac{z-1}{(z-1)(z+2)}$ is to make the values stable around $z=1$.

We can see the output points are moving towards a certain value. Thus the singularity that arises from the presence of $z-1$ can be removed simply by redefining the value of the function to be equal to the limit that $f(z)$ approaches as $z \rightarrow 1$.

C = complex output plane

