Find the Laurent expansion for $f(z)=\frac{1}{(1-z)(z-2)}$

Partial fractions:

$$\frac{1}{(1-z)(z-2)} = \frac{A}{1-z}+\frac{B}{z-2}$$

on $\begin{cases} D_1: |z|<1 \\ D_2: 1<|z|<2 \\ D_3: |z|>2 \end{cases}$

$$1 = A(z-2)+B(1-z)$$

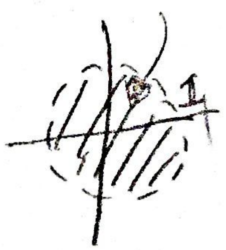
$z=1$: $1 = A(1-2)+B(0) \Rightarrow 1=-A \Rightarrow A=-1$

$z=2$: $1 = A(2-2)+B(1-2) \Rightarrow 1 = 0-B \Rightarrow B=-1$

So $$\frac{1}{(1-z)(z-2)} = \frac{\overset{A}{-1}}{1-z} - \frac{\overset{B}{1}}{z-2} = \frac{-1}{1-z} - \frac{1}{-2+z} = \boxed{-\frac{1}{1-z}+\frac{1}{2-z}}$$ use this...

On D_1: $f(z) = -\frac{1}{1-z}+\frac{1}{2\left(1-\frac{z}{2}\right)}$ note: $\frac{1}{2-z} = \frac{1}{2\left(1-\frac{z}{2}\right)}$

Valid on



$$= -\left[1+z+z^2+\cdots\right]+\frac{1}{2}\left[1+\frac{z}{2}+\frac{z^2}{2^2}+\frac{z^3}{2^3}+\cdots\right]$$

$$= -\sum_{n=0}^{\infty} z^n + \sum_{n=0}^{\infty}\frac{z^n}{2^{n+1}} = \boxed{\sum_{n=0}^{\infty} z^n\left(\frac{1}{2^{n+1}}-1\right)}$$

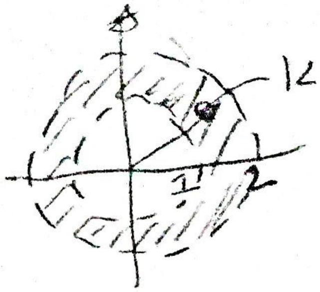
answer in Σ form

note: $$\frac{1}{2}\left[1+\frac{z}{2}+\frac{z^2}{2^2}+\frac{z^3}{2^3}+\cdots\right]$$

$$= \frac{1}{2}+\frac{z^1}{2^2}+\frac{z^2}{2^3}+\frac{z^3}{2^4}+\cdots$$

$=$

On D_2:



$1<|z|<2$

$$1<|z| = \frac{1}{|z|}<1$$

$$|z|<2 \Rightarrow \frac{|z|}{2}<1$$

$$f(z) = -\frac{1}{1-z} + \frac{1}{2-z}$$

$$= -\frac{1}{z\left(\frac{1}{z}-1\right)} + \frac{1}{2\left(1-\frac{z}{2}\right)}$$

$$= -\frac{1}{z\left(-1+\frac{1}{z}\right)} + \frac{1}{2\left(1-\frac{z}{2}\right)}$$

$$= \frac{\cancel{-}1}{z\cancel{-1}\left(1-\frac{1}{z}\right)} + \frac{1}{2\left(1-\frac{z}{2}\right)}$$

$$= \frac{1}{z}\frac{1}{1-1/z} + \frac{1}{2\left(1-\frac{z}{2}\right)}$$

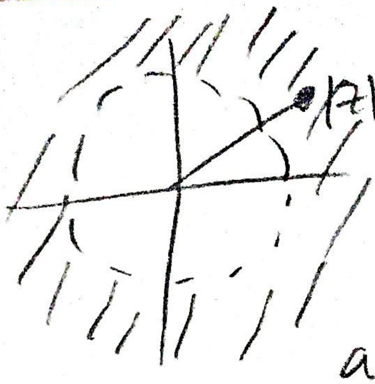
So $\frac{1}{z}\left[1+\frac{1}{z}+\frac{1}{z^2}+\frac{1}{z^3}+\cdots\right]+\frac{1}{2}\left[1+\frac{z}{2}+\frac{z^2}{2^2}+\frac{z^3}{2^3}+\cdots\right]$

$$= \left[\frac{1}{z}+\frac{1}{z^2}+\frac{1}{z^3}+\frac{1}{z^4}+\cdots\right]+\left[\frac{1}{2}+\frac{z^1}{2^2}+\frac{z^2}{2^3}+\frac{z^3}{2^4}+\cdots\right]$$

$$= \sum_{n=0}^{\infty}\frac{1}{z^n}+\sum_{n=0}^{\infty}\frac{z^n}{2^{n+1}}$$

answer

on D_3:



$|z|>2 \Rightarrow 2<|z|$

$\Rightarrow \frac{2}{|z|}<1$

also, $\frac{1}{|z|}<1$, since $\frac{2}{|z|}<1$

So $$f(z) = -\frac{1}{1-z} + \frac{1}{2-z}$$

$$= -\frac{1}{z\left(\frac{1}{z}-1\right)} + \frac{1}{z\left(\frac{2}{z}-1\right)}$$

$$= -\frac{1}{z\left(-1+\frac{1}{z}\right)} + \frac{1}{z\left(-1+\frac{2}{z}\right)}$$

$$= \frac{1}{z\left(1-\frac{1}{z}\right)} - \frac{1}{z\left(1-\frac{2}{z}\right)}$$

$$= \frac{1}{z}\left[1+\frac{1}{z}+\frac{1}{z^2}+\frac{1}{z^3}+\dots\right] - \frac{1}{z}\left[1+\frac{2}{z}+\frac{2^2}{z^2}+\frac{2^3}{z^3}+\dots\right]$$

$$= \left[\frac{1}{z}+\frac{1}{z^2}+\frac{1}{z^3}+\frac{1}{z^4}+\dots\right] + \left[-\frac{1}{z}-\frac{2}{z^2}-\frac{2^2}{z^3}-\frac{2^3}{z^4}+\dots\right]$$

answer